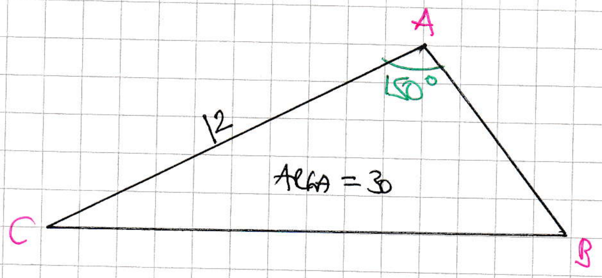
IYGB - MP1 PAPER F - QUESTION 1

a) LOOKING AT THE DIAGRAM



$\Rightarrow \text{AREA} = \frac{1}{2}|AC||AB|\sin 150°$

$\Rightarrow 30 = \frac{1}{2} \times 12 \times |AB| \times \frac{1}{2}$

$\Rightarrow 30 = 3|AB|$

$\Rightarrow |AB| = 10$

AS REQUIRED

b) BY THE COSINE RULE

$\Rightarrow |BC|^2 = |AC|^2 + |AB|^2 - 2|AC||AB|\cos 150$

$\Rightarrow |BC|^2 = 12^2 + 10^2 - 2 \times 12 \times 10 \times \left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow |BC|^2 = 144 + 100 + 120\sqrt{3}$

$\Rightarrow |BC|^2 = 451.846...$

$\Rightarrow |BC| \simeq 21.26 \text{ m}$

c) THE SMALLEST ANGLE LIES OPPOSITE THE SHORTEST SIDE, IE AB

BY THE SINE RULE

$$\frac{\sin \hat{ACB}}{|AB|} = \frac{\sin \hat{CAB}}{|BC|} \Longrightarrow \frac{\sin\theta}{10} = \frac{\sin 150}{21.26...}$$

$$\Longrightarrow \sin\theta = \frac{10\sin 150}{21.25667...}$$

$$\Longrightarrow \sin\theta = 0.23522...$$

$$\Longrightarrow \theta \simeq 13.6°$$

IYGB – MP1 PAPER F – QUESTION 2

a) USING THE STANDARD EXPANSION FORMULA

$$\Rightarrow (2-3x)^{10} = \binom{10}{0}(2)^{10}(-3x)^{0} + \binom{10}{1}(2)^{9}(-3x)^{1} + \binom{10}{2}(2)^{8}(-3x)^{2} + \dots$$

$$\Rightarrow (2-3x)^{10} = (1 \times 1024 \times 1) + [10 \times 512 \times (-3x)] + (45 \times 256 \times 9x^2) + \dots$$

$$\Rightarrow (2-3x)^{10} = 1024 - 15360x + 103680x^2 + \dots$$

b) CREATING 1.97^{10} OUT OF $(2-3x)^{10}$

$$\Rightarrow 1.97 = 2 - 3x$$

$$\Rightarrow 3x = 0.03$$

$$\Rightarrow x = 0.01$$

USING PART (a)

$$\Rightarrow [2 - 3(0.01)]^{10} = 1024 - 15360(0.01) + 103680(0.01)^2 + \dots$$

$$\Rightarrow 1.97^{10} = 1024 - 153.6 + 10.368 + \dots$$

$$\Rightarrow 1.97^{10} \approx 880.768 \simeq 881$$

c) USING PART (b)

$$3.94^{10} = (2 \times 1.97)^{10}$$

$$= 2^{10} \times 1.97^{10}$$

$$\approx 1024 \times 880.768$$

$$\simeq 901906.432$$

$$\simeq 900000$$

IYGB - MP1 PAPER F - QUESTION 3

LET THE ODD POSITIVE INTEGER BE $2n+1$, $n=0,1,2,3,4,\ldots$

$$(2n+1)^2 = 4n^2+4n+1 = 4(n^2+n)+1$$

$$= 4m+1 \quad (\text{WHERE } m = n^2+n)$$

IE LEAVES REMAINDER 1, WHEN DIVIDED BY 4

IYGB – MP1 PAPER F – QUESTION 4

a) SUBSTITUTE $x=3$ INTO THE FUNCTION

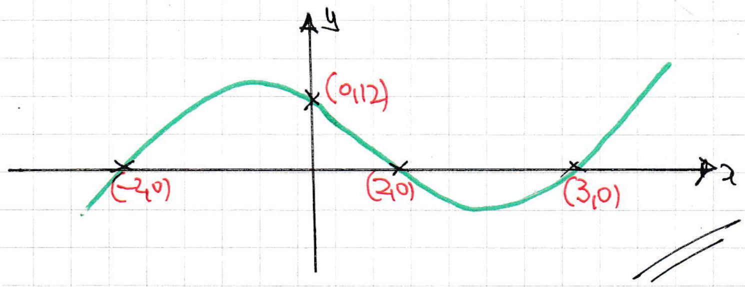
$$f(x) = x^3 - 3x^2 - 4x + 12$$
$$f(3) = 3^3 - 3\times3^2 - 4\times3 + 12$$
$$f(3) = 27 - 27 - 12 + 12$$
$$f(3) = 0$$

$\therefore$ $(x-3)$ IS A FACTOR

BY LONG DIVISION OR INSPECTION

$$f(x) = x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x-3)(x^2-4)$$
$$f(x) = (x-3)(x-2)(x+2)$$

b)



$+x^3 \Rightarrow \sim$

$y=0 \Rightarrow x = \begin{cases} 2 \\ -2 \\ 3 \end{cases}$

$x=0 \Rightarrow y=12$

c) SOLVING SIMULTANEOUSLY $f(x) = g(x)$

$$\Rightarrow (x-3)(x-2)(x+2) = (x-2)(x-4)^2$$

DIVIDE BY $(x-2)$ & NOTE $x=2$ IS A SOLUTION

$$\Rightarrow (x-3)(x+2) = (x-4)^2$$
$$\Rightarrow \cancel{x^2} - 3x + 2x - 6 = \cancel{x^2} - 8x + 16$$
$$\Rightarrow 7x = 22$$
$$\Rightarrow x = \frac{22}{7}$$

$$\therefore x = \begin{cases} 2 \\ \frac{22}{7} \end{cases}$$

IYGB - MP1 PAPER F - QUESTION 5

$$p = \log_6 25 \quad \& \quad q = \log_6 2$$

a) $\log_6 200 = \log_6 (25 \times 8)$
$= \log_6 25 + \log_6 8$
$= \log_6 25 + \log_6 2^3$
$= \log_6 25 + 3\log_6 2$
$= \underline{p + 3q}$

b) $\log_6 (3.2) = \log_6 \left(\frac{32}{10}\right) = \log_6 \left(\frac{16}{5}\right)$
$= \log_6 16 - \log_6 5$
$= \log_6 2^4 - \log_6 25^{\frac{1}{2}}$
$= 4\log_6 2 - \frac{1}{2}\log_6 25$
$= \underline{4q - \frac{1}{2}p}$

c) $\log_6 75 = \log_6 (25 \times 3)$
$= \log_6 \left(\frac{25 \times 6}{2}\right)$
$= \log_6 25 + \log_6 6 - \log_6 2$
$= \underline{p + 1 - q}$

IYGB - MP1 PAPER F - QUESTION 6

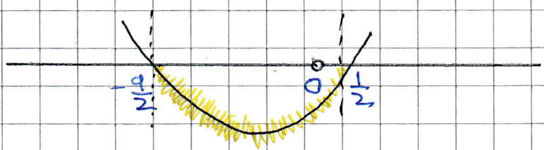
RE ORDER THE QUADRATIC FIRST

$$k(x^2+1) - 3x + 4 = 0$$
$$kx^2 + k - 3x + 4 = 0$$
$$kx^2 - 3x + (k+4) = 0$$

FOR REAL ROOTS, DISTINCT OR REPEATED, $b^2-4ac \geqslant 0$

$$\Rightarrow (-3)^2 - 4 \times k \times (k+4) \geqslant 0$$
$$\Rightarrow 9 - 4k(k+4) \geqslant 0$$
$$\Rightarrow 9 - 4k^2 - 16k \geqslant 0$$
$$\Rightarrow -4k^2 - 16k + 9 \geqslant 0$$
$$\Rightarrow 4k^2 + 16k - 9 \leqslant 0$$
$$\Rightarrow (2k+9)(2k-1) \leqslant 0$$

CRITICAL VALUES $\frac{1}{2}$, $-\frac{9}{2}$



$$-\frac{9}{2} \leqslant k \leqslant \frac{1}{2}$$

ALTHOUGH FOR $k=0$, THE QUADRATIC REDUCES TO THE LINEAR EQUATION $-3x+4=0$, IT STILL SATISFIES THE ORIGINAL CONDITION

IYGB - MP1 PAPER F - QUESTION 7

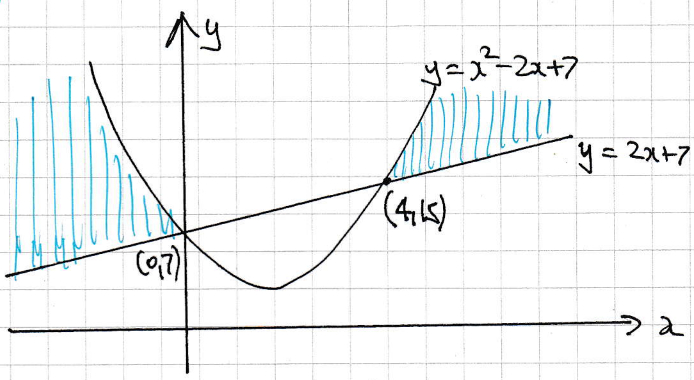
THIS IS THE REGION "ABOVE" THE LINE

$$\Longrightarrow \quad y \geqslant 2x+7$$

ALSO THE REGION BELOW THE QUADRATIC

$$\Longrightarrow \quad y \leqslant x^2-2x+7$$

HOWEVER THIS DOES NOT SATISFY JUST THIS REGION BUT THE ONE SHOWN BELOW



THUS A THIRD INEQUALITY IS NEEDED SUCH AS

$$x \geqslant 0$$
$$\text{OR } x \geqslant 1$$
$$\text{OR } x \geqslant \frac{9}{4}$$
$$\text{OR } x \geqslant 4$$

$$x^2-2x+7=2x+7$$
$$x^2-4x=0$$
$$x(x-4)=0$$
$$x = \begin{cases} 0 \\ 4 \end{cases}$$

$$\therefore \quad 2x+7 \leqslant y \leqslant x^2-2x+7 \cap \{x \geqslant a : 0 \leqslant a \leqslant 4\}$$

e.g $x \geqslant 4$

$x > 3$

$x \geqslant 2.5$

ETC

IYGB – MP1 PAPER F – QUESTION 8

MANIPULATE AS FOLLOWS

$$\Rightarrow \frac{4}{\tan^2 3\theta} + 2 = \frac{7}{\sin 3\theta}$$

$$\Rightarrow \frac{4}{\frac{\sin^2 3\theta}{\cos^2 3\theta}} + 2 = \frac{7}{\sin 3\theta}$$

$$\Rightarrow \frac{4\cos^2 3\theta}{\sin^2 3\theta} + 2 = \frac{7}{\sin 3\theta}$$

MULTIPLY THROUGH BY $\sin^2 3\theta$

$$\Rightarrow 4\cos^2 3\theta + 2\sin^2 3\theta = 7\sin 3\theta$$

$$\Rightarrow 4(1-\sin^2 3\theta) + 2\sin^2 3\theta = 7\sin 3\theta$$

$\cos^2 A + \sin^2 A \equiv 1$

$$\Rightarrow 4 - 4\sin^2 3\theta + 2\sin^2 3\theta = 7\sin 3\theta$$

$$\Rightarrow 0 = 2\sin^2 3\theta + 7\sin 3\theta - 4$$

FACTORIZING YIELDS

$$\Rightarrow (2\sin 3\theta - 1)(\sin 3\theta + 4)$$

$$\Rightarrow \sin 3\theta = \begin{cases} \frac{1}{2} \\ \cancel{-4} \end{cases}$$

$$\Rightarrow \arcsin\left(\tfrac{1}{2}\right) = 30°$$

$$\begin{cases} 3\theta = 30° \pm 360n° \\ 3\theta = 150° \pm 360n° \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\begin{cases} \theta = 10° \pm 120n° \\ \theta = 50° \pm 120n° \end{cases}$$

$$\theta = 10°, 130°, 50°, 170°$$

IYGB – MP1 PAPER F – QUESTION 9

a) I) MULTIPLY OUT THE BRACKETS

$$f(x) = (x-4-\sqrt{3})(x-4+\sqrt{3}) = x^2 - 4x + \cancel{\sqrt{3}x} - 4x + 16 - \cancel{4\sqrt{3}} - \cancel{\sqrt{3}x} - 3 + \cancel{4\sqrt{3}}$$

$$f(x) = x^2 - 8x + 13$$

$\therefore\ a = -8$

$c = 13$

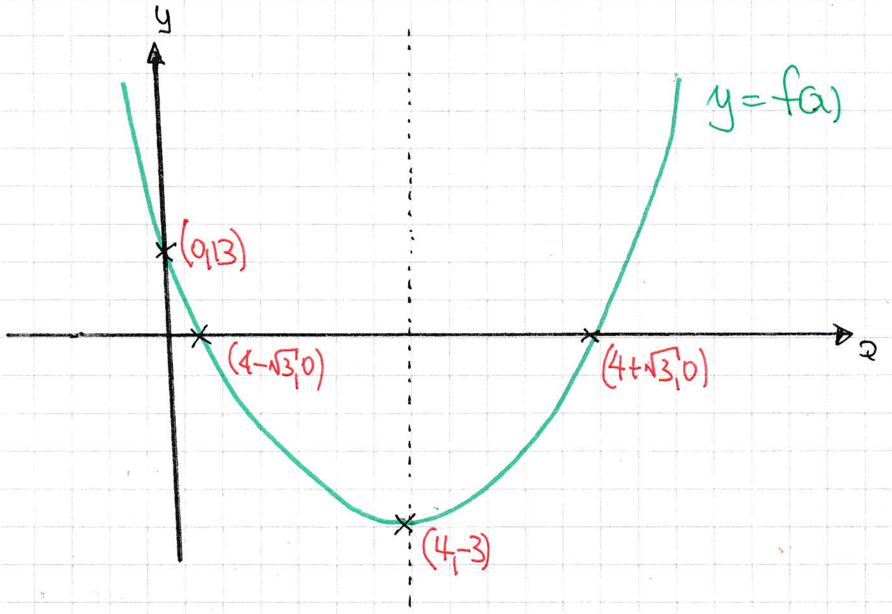
II) COMPLETING THE SQUARE

$$f(x) = x^2 - 8x + 13$$
$$f(x) = (x-4)^2 - 16 + 13$$
$$f(x) = (x-4)^2 - 3$$

ie $B = -4$

$C = -3$

b) USING THE RESULTS OF PART (a) TOGETHER WITH $x=0$ $y=13$



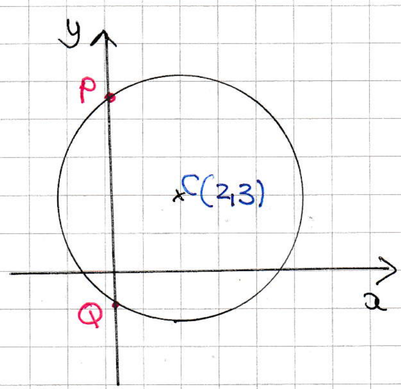
IYGB - MP1 PAPER F - QUESTION 10

a) USING THE STANDARD FORMULA

$$\Rightarrow (x-2)^2 + (y-3)^2 = 6^2$$
$$\Rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = 36$$
$$\Rightarrow x^2 + y^2 - 4x - 6y = 23$$

b) y INTERCEPTS $\Rightarrow x=0$

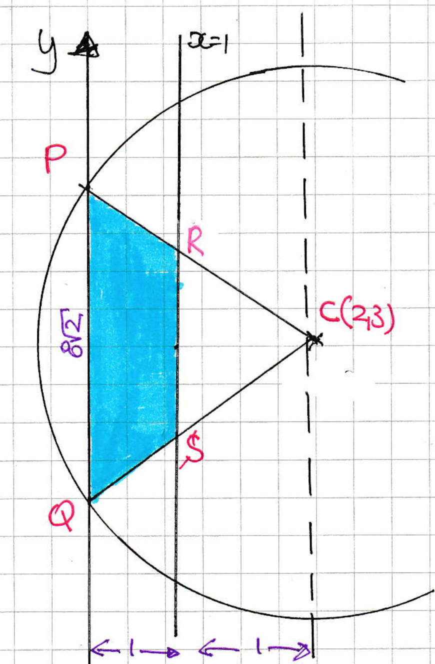
$$(x-2)^2 + (y-3)^2 = 36$$
$$(0-2)^2 + (y-3)^2 = 36$$
$$4 + (y-3)^2 = 36$$
$$(y-3)^2 = 32$$
$$y-3 = \begin{cases} \sqrt{32} \\ -\sqrt{32} \end{cases}$$
$$y = \begin{cases} 3+\sqrt{32} \leftarrow P \\ 3-\sqrt{32} \leftarrow Q \end{cases}$$



$P(0, 3+\sqrt{32})$ $\quad Q(0, 3-\sqrt{32})$

EVIDENTLY $(\cancel{3} + \sqrt{32}) - (\cancel{3} - \sqrt{32}) = 2\sqrt{32} = 2\sqrt{16\times 2} = 2\times 4\sqrt{2} = 8\sqrt{2}$

c) LOOKING AT THE DIAGRAM



METHOD A – BY DIRECT COMPUTATION

- R IS THE MIDPOINT OF $|PC|$ BECAUSE $x=1$ IS HALFWAY BETWEEN $x=2$ & THE y AXIS
- $P(0, 3+\sqrt{32})$ $\quad C(2,3)$

$$\Rightarrow R\left(\frac{0+2}{2}, \frac{3+3+\sqrt{32}}{2}\right) = R\left(1, \frac{6+4\sqrt{2}}{2}\right)$$
$$= R(1, 3+2\sqrt{2})$$

- BY ANALOGY OR BY INSPECTION $S(1, 3-2\sqrt{2})$

IYGB - MP1 PAPER F - QUESTION 10

- $|RS| = (3+2\sqrt{2}) - (3-2\sqrt{2}) = 4\sqrt{2}$

- AREA OF TRAPEZIUM $= \dfrac{|PQ|+|RS|}{2} \times \text{HEIGHT}$

$$= \frac{8\sqrt{2}+4\sqrt{2}}{2} \times 1$$

$$= \frac{12\sqrt{2}}{2}$$

$$= 6\sqrt{2}$$

METHOD B - BY SIMILAR TRIANGLES

- TRIANGLES $\overset{\triangle}{PQC}$ & $\overset{\triangle}{RSC}$ ARE SIMILAR, WITH SCALE FACTOR $\frac{1}{2}$, BY LOOKING AT THEIR RESPECTIVE HEIGHTS OF 2 & 1

- AREA OF $\overset{\triangle}{PQC}$ IS GIVEN BY

$$\frac{1}{2} \times 8\sqrt{2} \times 2 = 8\sqrt{2}$$

- AREA OF $\overset{\triangle}{RSC}$ IS GIVEN BY

$$\underset{\substack{\uparrow \\ \text{SCALE FACTOR SQUARED}}}{\left(\frac{1}{2}\right)^2} \times 8\sqrt{2} = \frac{1}{4} \times 8\sqrt{2} = 2\sqrt{2}$$

- AREA OF TRAPEZIUM IS

$$8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$$

AS BEFORE

IYGB – MP1 PAPER F – QUESTION 11

SOLVING THE EQUATION

$$\Rightarrow \quad 2 \times 3^{x} = 3 \times e^{-x}$$

$$\Rightarrow \quad \ln(2 \times 3^{x}) = \ln(3 \times e^{-x})$$

$$\Rightarrow \quad \ln 2 + \ln 3^{x} = \ln 3 + \ln e^{-x}$$

$$\Rightarrow \quad \ln 2 + x\ln 3 = \ln 3 - x$$

$$\Rightarrow \quad x + x\ln 3 = \ln 3 - \ln 2$$

$$\Rightarrow \quad x(1 + \ln 3) = \ln 3 - \ln 2$$

$$\Rightarrow \quad x = \frac{\ln 3 - \ln 2}{1 + \ln 3}$$

// AS REQUIRED

IYGB - MAI PAPER F - QUESTION 12

START BY FINDING THE AREA OF A_1

$$A_1 = \int_0^{\frac{3}{2}} 6x^2 - 4x^3 \, dx = \left[2x^3 - x^4\right]_0^{\frac{3}{2}}$$

$$= \left[2\left(\tfrac{3}{2}\right)^3 - \left(\tfrac{3}{2}\right)^4\right] - \left[0\right] = \frac{27}{4} - \frac{81}{16} = \frac{27}{16}$$

NOW THIS MUST BE THE SAME AREA AS A_2, BUT NOT THAT THE "SIGN" MUST BE NEGATIVE, AS IT IS BELOW THE x AXIS

$$\Rightarrow A_2 = -\frac{27}{16}$$

$$\Rightarrow \int_{\frac{3}{2}}^{k} 6x^2 - 4x^3 \, dx = -\frac{27}{16}$$

$$\Rightarrow \left[2x^3 - x^4\right]_{\frac{3}{2}}^{k} = -\frac{27}{16}$$

$$\Rightarrow \left(2k^3 - k^4\right) - \left(\frac{27}{4} - \frac{81}{16}\right) = -\frac{27}{16}$$

$$\Rightarrow 2k^3 - k^4 - \frac{27}{4} + \frac{81}{16} = -\frac{27}{16}$$

$$\Rightarrow 2k^3 - k^4 = 0$$

$$\Rightarrow k^3(2-k) = 0$$

$$\Rightarrow k = \begin{cases} 0 \\ 2 \end{cases}$$

$$\therefore k = 2$$

IYGB - MP1 PAPER F - QUESTION 13

REWRITE IN INDEX NOTATION AND DIFFERENTIATE TWICE

$$\Rightarrow y = 7\sqrt{x} - \frac{3}{\sqrt{x}}$$

$$\Rightarrow y = 7x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{7}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{7}{4}x^{-\frac{3}{2}} - \frac{9}{4}x^{-\frac{5}{2}}$$

SUBSTITUTE INTO THE R.H.S. OF THE REQUIRED EXPRESSION

$$4x\left(x\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) = 4x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx}$$

$$= 4x^2\left[-\frac{7}{4}x^{-\frac{3}{2}} - \frac{9}{4}x^{-\frac{5}{2}}\right] + 4x\left[\frac{7}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}\right]$$

$$= -7x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 14x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$$

$$= 7x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$= 7\sqrt{x} - \frac{3}{\sqrt{x}}$$

$$= y$$

AS REQUIRED