IYGB – MMS PAPER J – QUESTION 1

a) I) $\sqrt{108} + \sqrt{3}$

$= \sqrt{36 \times 3} + \sqrt{3}$

$= 6\sqrt{3} + \sqrt{3}$

$= 7\sqrt{3}$

II) $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1} = \dfrac{(\sqrt{6}+\sqrt{3})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \dfrac{\sqrt{12} - \cancel{\sqrt{6}} + \cancel{\sqrt{6}} - \sqrt{3}}{2 - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} - 1}$

$= \dfrac{\sqrt{12} - \sqrt{3}}{1}$

$= \sqrt{4}\sqrt{3} - \sqrt{3}$

$= 2\sqrt{3} - \sqrt{3}$

$= \sqrt{3}$

ALTERNATIVE FOR a II

$\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1} = \dfrac{\sqrt{3}\sqrt{2}+\sqrt{3}}{\sqrt{2}+1} = \dfrac{\sqrt{3}(\cancel{\sqrt{2}+1})}{\cancel{\sqrt{2}+1}} = \sqrt{3}$

AS BEFORE

b) PROCEED AS FOLLOWS

$\Rightarrow (5-x)^{\frac{3}{2}} = 8$

$\Rightarrow \left[(5-x)^{\frac{3}{2}}\right]^{\frac{2}{3}} = 8^{\frac{2}{3}}$

$\Rightarrow (5-x)^{1} = (\sqrt[3]{8})^{2}$

$\Rightarrow 5 - x = 4$

$\Rightarrow x = 1$

ALTERNATIVE METHOD

$\Rightarrow (5-x)^{\frac{3}{2}} = 8$

$\Rightarrow \left[\sqrt[2]{5-x}\right]^{3} = 8$

$\Rightarrow \sqrt{5-x} = \sqrt[3]{8}$

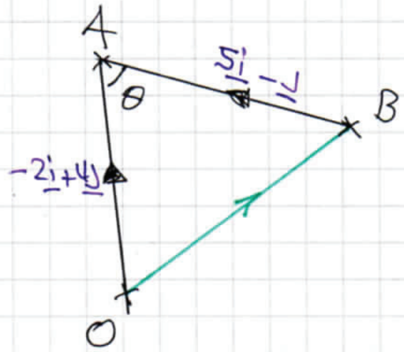
$\Rightarrow \sqrt{5-x} = 2$

$\Rightarrow 5 - x = 4$

$\Rightarrow x = 1$

IYGB - MP1 PAPER J - QUESTION 2

a) START WITH A DIAGRAM



$$\vec{OB} = \vec{OA} + \vec{AB}$$
$$\vec{OB} = \vec{OA} - \vec{BA}$$
$$\vec{OB} = (-2\underline{i}+4\underline{j})-(5\underline{i}-\underline{j})$$
$$\vec{OB} = -7\underline{i}+5\underline{j}$$

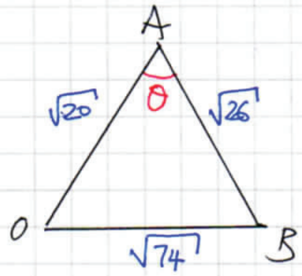
FINDING THE DISTANCE OB

$$|\vec{OB}| = |-7\underline{i}+5\underline{j}| = \sqrt{(-7)^2+5^2} = \sqrt{74} \simeq 8.60$$

b) FIND THE LENGTHS OF AB & OA

$$|\vec{OA}| = |-2\underline{i}+4\underline{j}| = \sqrt{(-2)^2+4^2} = \sqrt{4+16} = \sqrt{20}$$
$$|\vec{BA}| = |5\underline{i}-\underline{j}| = \sqrt{5^2+(-1)^2} = \sqrt{26}$$

BY THE COSINE RULE



$$|OB|^2 = |OA|^2 + |AB|^2 - 2|OA||AB|\cos\theta$$
$$74 = 20 + 26 - 2\sqrt{20}\sqrt{26}\cos\theta$$
$$2\sqrt{20}\sqrt{26}\cos\theta = -28$$
$$\cos\theta = -0.61394\ldots$$
$$\theta \simeq 128°$$

IYGB – PAPER J – QUESTION 3

WRITE IN INDEX NOTATION

$$y = \sqrt{2x} = \sqrt{2}\,x^{\frac{1}{2}}$$

DIFFERENTIATING W.R.T x & FINDING THE GRADIENT AT $x=2$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}\,x^{-\frac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{2}\sqrt{2} \times 2^{-\frac{1}{2}} = \frac{1}{2}\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} = \frac{1}{2}$$

NORMAL GRADIENT IS -2

WHEN $x=2$, $y = \sqrt{2 \times 2} = 2$ IE $(2,2)$

EQUATION OF NORMAL SATISFIES

$$y - y_0 = m(x - x_0)$$
$$y - 2 = -2(x-2)$$
$$y - 2 = -2x + 4$$
$$y = -2x + 6$$

IYGB - MP1 PAPER J - QUESTION 4

START BY FINDING THE POINTS OF INTERSECTION

$$\left.\begin{array}{l} y = x^2 - 2x + 4 \\ y = 3x \end{array}\right\} \Rightarrow x^2 - 2x + 4 = 3x$$

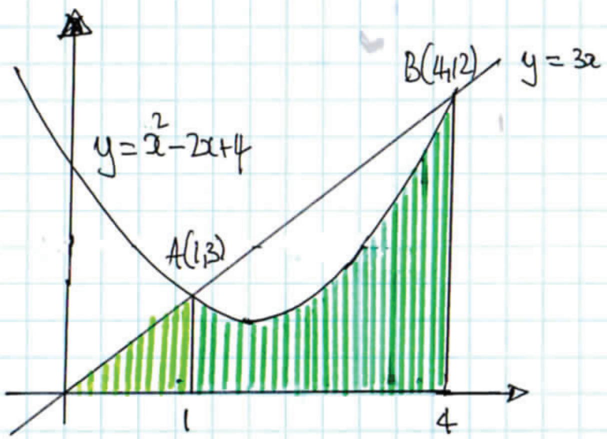
$$\Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow (x-1)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} 1 \\ 4 \end{cases} \quad y = \begin{cases} 3 \\ 12 \end{cases}$$

$$\therefore A(1,3) \ \& \ B(4,12)$$

LOOKING AT THE DIAGRAM BELOW



AREA OF TRIANGLE (LIGHT GREEN) $= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

AREA UNDER THE CURVE (DARK GREEN) $= \int_1^4 x^2 - 2x + 4 \, dx$

$$= \left[\frac{1}{3}x^3 - x^2 + 4x\right]_1^4$$

$$= \left(\frac{64}{3} \cancel{-16+16}\right) - \left(\frac{1}{3} - 1 + 4\right)$$

$$= \frac{64}{3} - \frac{1}{3} + 1 - 4$$

$$= 18$$

REQUIRED AREA $= 18 + \frac{3}{2} = \frac{39}{2}$

IYGB – MP1 – PAPER J – QUESTION 5

USING THE IDENTITY $\cos^2 A + \sin^2 A \equiv 1$

$$\Longrightarrow 3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6$$
$$\Longrightarrow 3\cos^2 2\phi - 4(1 - \cos^2 2\phi) = 15\cos 2\phi - 6$$
$$\Longrightarrow 3\cos^2 2\phi - 4 + 4\cos^2 2\phi = 15\cos 2\phi - 6$$
$$\Longrightarrow 7\cos^2 2\phi - 15\cos 2\phi + 2 = 0$$

FACTORIZING OR USE THE QUADRATIC FORMULA

$$\Longrightarrow (7\cos 2\phi - 1)(\cos 2\phi - 2) = 0$$
$$\Longrightarrow \cos 2\theta = \begin{cases} \frac{1}{7} \\ \cancel{2} \quad -1 \leq \cos 2\theta \leq 1 \end{cases}$$

$$\arccos\left(\tfrac{1}{7}\right) = 81.7867^\circ\ldots$$

$$\begin{cases} 2\phi = 81.7867^\circ\ldots \pm 360n \\ 2\phi = 278.213^\circ\ldots \pm 360n \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\begin{cases} \phi = 40.9^\circ \pm 180n \\ \phi = 139.1^\circ \pm 180n \end{cases}$$

IN THE RANGE GIVEN

$$\phi = 40.9^\circ,\ 139.1^\circ,\ 220.9^\circ,\ 319.1^\circ$$

IYGB - MP1 PAPER J - QUESTION 6

LET $y = f(x) = \frac{1}{x}$

$$f(x+h) = \frac{1}{x+h}$$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$$

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{-h}{x(x+h)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{-h}{x(x+h)} \div h\right]$$

$$= \lim_{h \to 0}\left[\frac{-\cancel{h}}{x^2+xh} \times \frac{1}{\cancel{h}}\right]$$

$$= \lim_{h \to 0}\left[-\frac{1}{x^2+xh}\right]$$

TAKING LIMITS, AS $h \to 0$

$$= -\frac{1}{x^2 + \cancel{xh}}$$

$$= -\frac{1}{x^2}$$ AS REQUIRED

IYGB - MP1 PAPER J - QUESTION 7

a) USING EACH OF THE POINTS IN TURN, FORM TWO EQUATIONS IN a & b

$$(-4,7) \Longrightarrow (-4)^2+7^2+a(-4)+b\times 7+43=0$$
$$(-2,5) \Longrightarrow (-2)^2+5^2+a(-2)+b\times 5+43=0$$

$$\Longrightarrow \begin{cases} 16+49-4a+7b+43=0 \\ 4+25-2a+5b+43=0 \end{cases}$$

$$\begin{cases} -4a+7b=-108 & \times(-1) \\ -2a+5b=-72 & \times 2 \end{cases} \Longrightarrow \begin{cases} 4a-7b=108 \\ -4a+10b=-144 \end{cases}$$

ADDING $3b=-36$

$b=-12$

& $4a-7b=108$

$4a+84=108$

$4a=24$

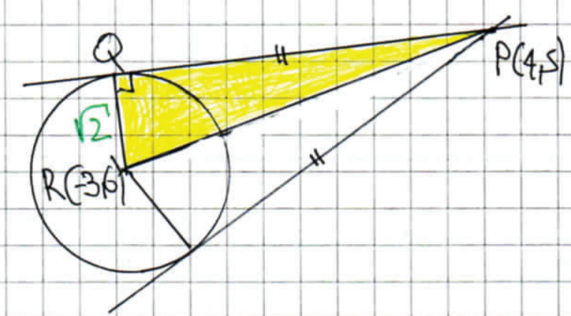
$a=6$

HENCE WE NOW HAVE

$$x^2+6x+y^2-12y+43=0$$
$$(x+3)^2-9+(y-6)^2-36+43=0$$
$$(x+3)^2+(y-6)^2=2$$

$\therefore$ CENTRE $(-3,6)$, RADIUS $=\sqrt{2}$

b) LOOKING AT THE DIAGRAM BELOW



$P(4,5)$ $R(-3,6)$

$$|PR|=\sqrt{(-3-4)^2+(6-5)^2}$$
$$|PR|=\sqrt{(-7)^2+1^2}$$
$$|PR|=\sqrt{50}$$

IYGB - MP1 PAPER J - QUESTION 7

FINALLY BY PYTHAGORAS

$$\Rightarrow |PQ|^2 + |QR|^2 = |PR|^2$$

$$\Rightarrow |PQ|^2 + (\sqrt{2})^2 = (\sqrt{50})^2$$

$$\Rightarrow |PQ|^2 + 2 = 50$$

$$\Rightarrow |PQ|^2 = 48$$

$$\Rightarrow |PQ| = \sqrt{48}$$

$$\Rightarrow |PQ| = 4\sqrt{3}$$

// AS REQUIRED

IYGB - MP1 PAPER J - QUESTION 8

EXPANDING USING THE STANDARD BINOMIAL FORMULAE

$$f(x) = (3-2x)^2(1+2x)^6$$

$$f(x) = (9-12x+4x^2)\left[1 + \frac{6}{1}(2x)^1 + \frac{6\times5}{1\times2}(2x)^2 + \frac{6\times5\times4}{1\times2\times3}(2x)^3 + \dots\right]$$

$$f(x) = (9-12x+4x^2)(1+12x+60x^2+160x^3+\dots)$$

$$\begin{aligned} f(x) = \; & 9 + 108x + 540x^2 + 1440x^3 + \dots \\ & \quad - 12x - 144x^2 - 720x^3 + \dots \\ & \quad\quad + 4x^2 + 48x^3 + \dots \end{aligned}$$

$$f(x) = 9 + 96x + 400x^2 + 768x^3 + \dots$$

IYGB – MP1 PAPER J – QUESTION 9

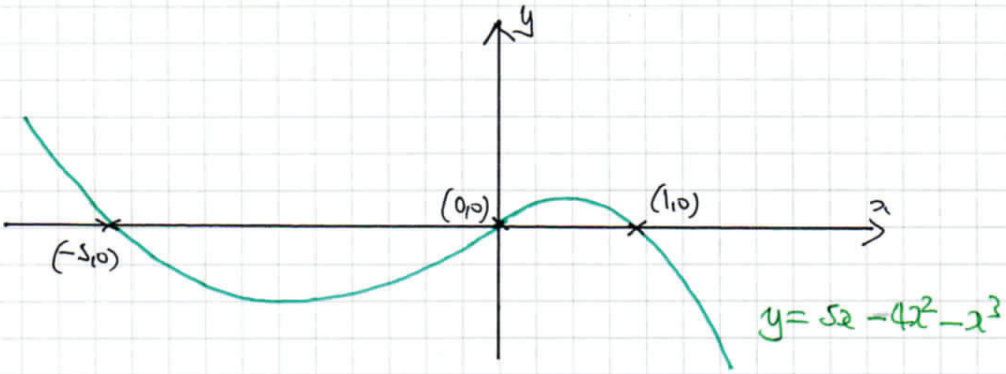
a) FACTORIZE BY COMMON FACTOR THEN THE QUADRATIC

$$y = 5x - 4x^2 - x^3$$
$$y = -x^3 - 4x^2 + 5x$$
$$y = -x(x^2 + 4x - 5)$$
$$y = -x(x-1)(x+5)$$

OR $y = x(1-x)(x+5)$

b) COLLECT THE INFORMATION & SKETCH

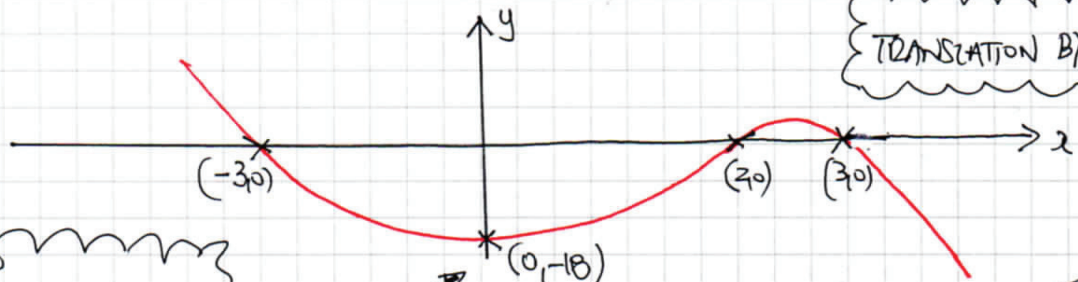
- $-x^3 \Rightarrow$ 
- $x=0 \Rightarrow y=0$ $(0,0)$
- $y=0 \Rightarrow x = 0, 1, -5$ $(0,0)$ $(1,0)$ $(-5,0)$



c) LOOKING AT "TRANSFORMATIONS"

IF $y = f(x) = 5x - 4x^2 - x^3$, THEN $f(x-2) = 5(x-2) - 4(x-2)^2 - (x-2)^3$

TRANSLATION BY $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$



$x=0$
$y = -10 - 16 + 8 = -18$

IYGB – MP1 PAPER J – QUESTION 10

PROCEED AS FOLLOWS

$\Rightarrow 3kx^2 - 2kx - 4x + 3 = 0$

$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$

$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$

TWO DIFFERENT REAL ROOTS $b^2 - 4ac > 0$

$\Rightarrow [-(2k+4)]^2 - 4 \times 3k \times 3 > 0$

$\Rightarrow (2k+4)^2 - 36k > 0$

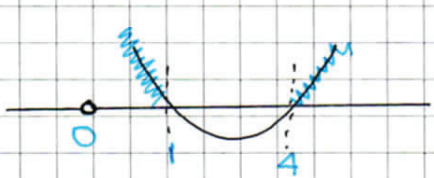
$\Rightarrow 4k^2 + 16k + 16 - 36k > 0$

$\Rightarrow 4k^2 - 20k + 16 > 0$

$\Rightarrow k^2 - 5k + 4 > 0$

$\Rightarrow (k-4)(k-1) > 0$

CRITICAL VALUES $\left\langle \begin{array}{l} 1 \\ 4 \end{array} \right.$



$\therefore\ k < 1,\ k \neq 0$ OR $k > 4$

NOTE THAT IF $k=0$, THE QUADRATIC SEIZES TO EXIST & REDUCES TO A LINEAR EQUATION WITH ONE ROOT ONLY

IYGB - MP1 PAPER J - QUESTION 11

PROCEED AS FOLLOWS

$\Rightarrow \ln(x+1)^2 - 2y = 2$ AND $x + e^y = 1$

$\Rightarrow 2\ln(x+1) - 2y = 2$ $\quad\Rightarrow e^y = 1 - x$

$\Rightarrow \ln(x+1) - y = 1$ $\quad\Rightarrow y = \ln(1-x)$

$$\ln(x+1) - \ln(1-x) = 1$$

$$\Rightarrow \ln\left(\frac{x+1}{1-x}\right) = 1$$

$$\Rightarrow \frac{x+1}{1-x} = e^1$$

$$\Rightarrow \frac{x+1}{1-x} = e$$

$$\Rightarrow x+1 = e - ex$$

$$\Rightarrow x + ex = e - 1$$

$$\Rightarrow x(1+e) = e-1$$

$$\Rightarrow x = \frac{e-1}{e+1} \quad //$$

AND $y = \ln(1-x)$

$$y = \ln\left(1 - \frac{e-1}{e+1}\right)$$

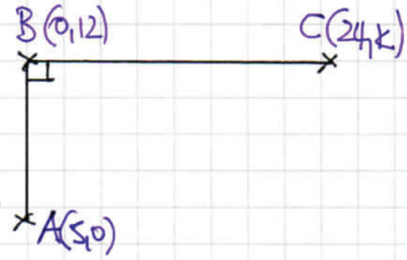
$$y = \ln\left(\frac{\cancel{e}+1-\cancel{e}+1}{e+1}\right)$$

$$y = \ln\left(\frac{2}{e+1}\right)$$

$$\therefore (x,y) = \left[\frac{e-1}{e+1}, \ln\left(\frac{2}{e+1}\right)\right]$$

IYGB – MP1 PAPER J – QUESTION 12

a) WORK WITH GRADIENTS FIRST



$$\text{GRADIENT } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{12-0}{0-5} = -\frac{12}{5}$$

$$\text{GRADIENT } BC = \frac{k-12}{24-0} = \frac{k-12}{24}$$

NOW AS $AB \perp BC$, GRAD $BC = +\frac{5}{12}$

$$\Rightarrow \frac{k-12}{24} = \frac{5}{12}$$

$$\Rightarrow 12k - 144 = 120$$

$$\Rightarrow 12k = 264$$

$$\Rightarrow k = 22 \quad \text{AS REQUIRED}$$

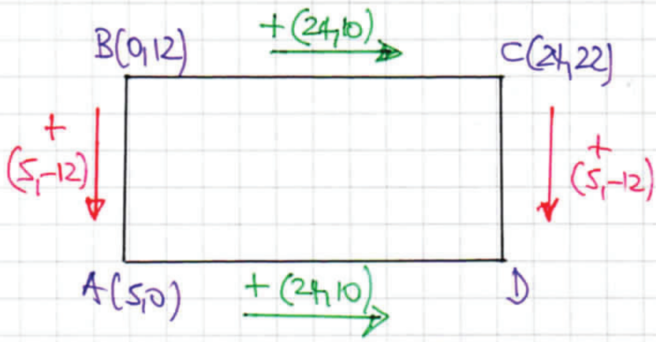
NEXT THE LENGTHS, USING $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$|AB| = \sqrt{(12-0)^2+(0-5)^2} = \sqrt{144+25} = \sqrt{169} = 13$$

$$|BC| = \sqrt{(22-12)^2+(24-0)^2} = \sqrt{100+576} = \sqrt{676} = 26$$

$$\therefore \text{AREA} = 13 \times 26 = 338$$

b) BY INSPECTION, OR "VECTORS"



$$\therefore D(29,10)$$

IYGB - MP1 PAPER J - QUESTION 13

a) COMPLETING THE SQUARE

$$y = 4x^2 + 24x + A$$

$$y = 4\left[x^2 + 6x + \frac{A}{4}\right]$$

$$y = 4\left[(x+3)^2 - 9 + \frac{A}{4}\right]$$

$$y = 4(x+3)^2 - 36 + A$$

If $p = 4$, $q = 3$, $r = A - 36$

b) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 4x^2 + 24x + A \\ y = Bx + 10 \end{array}\right\} \Rightarrow 4x^2 + 24x + A = Bx + 10$$

$$\Rightarrow 4x^2 + (24 - B)x + (A - 10) = 0$$

If $x = -1$

$$4 - (24 - B) + (A - 10) = 0$$

$$4 - 24 + B + A - 10 = 0$$

$$A + B = 30$$

$$B = 30 - A$$

If $x = -\frac{21}{4}$

$$4\left(-\frac{21}{4}\right)^2 + (24 - B)\left(-\frac{21}{4}\right) + (A - 10) = 0$$

$$\frac{441}{4} - 126 + \frac{21}{4}B + A - 10 = 0$$

$$441 - 504 + 21B + 4A - 40 = 0$$

$$4A + 21B = 103$$

$$4A + 21(30 - A) = 103$$

$$4A + 630 - 21A = 103$$

$$527 = 17A$$

$$A = 31$$

$$\& \ B = -1$$