IYGB - MP1 PAPER R - QUESTION 1

EXPANDING IN THE "USUAL" MANNER

$$\left(x+\frac{2}{x}\right)^6 = \binom{6}{0}(x)^6\left(\frac{2}{x}\right)^0 + \binom{6}{1}(x)^5\left(\frac{2}{x}\right)^1 + \binom{6}{2}(x)^4\left(\frac{2}{x}\right)^2 + \binom{6}{3}(x)^3\left(\frac{2}{x}\right)^3$$

$$+\binom{6}{4}(x)^2\left(\frac{2}{x}\right)^4 + \binom{6}{5}(x)^1\left(\frac{2}{x}\right)^5 + \binom{6}{6}(x)^0\left(\frac{2}{x}\right)^6$$

$$= \left(1\times x^6\times 1\right) + \left(6\times x^5\times\frac{2}{x}\right) + \left(15\times x^4\times\frac{4}{x^2}\right) + \left(20\times x^3\times\frac{8}{x^3}\right)$$

$$+\left(15\times x^2+\frac{16}{x^4}\right) + \left(6\times x\times\frac{32}{x^5}\right) + \left(1\times 1\times\frac{64}{x^6}\right)$$

$$= x^6 + 12x^4 + 60x^2 + 160 + \frac{240}{x^2} + \frac{192}{x^4} + \frac{64}{x^6}$$

IYGB - MP1 PAPER R - QUESTION 2

a) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left.\begin{array}{l} y = x^2 - 5x + 9 \\ y = 5 \end{array}\right\} \Longrightarrow 5 = x^2 - 5x + 9$$

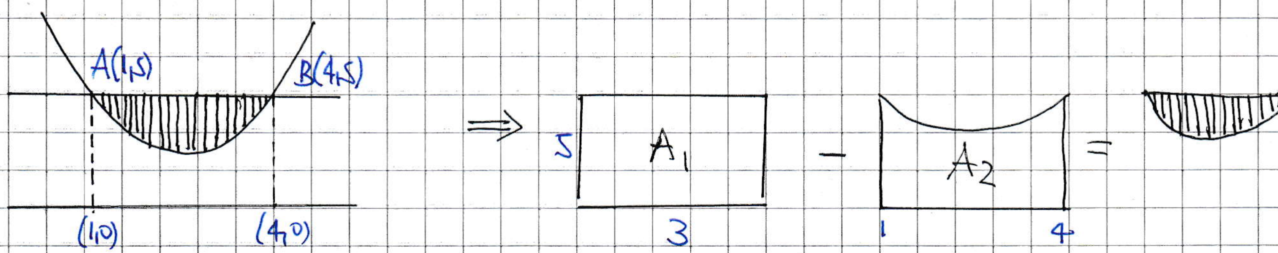
$$\Longrightarrow 0 = x^2 - 5x + 4$$

$$\Longrightarrow 0 = (x-1)(x-4)$$

$$\Longrightarrow x = \begin{cases} 1 \\ 4 \end{cases}$$

$\therefore$ $A(1,5)$ & $B(4,5)$

b) LOOKING AT THE DIAGRAM BELOW



- $A_1 = 5 \times 3 = 15$
- $A_2 = \int_1^4 x^2 - 5x + 9 \, dx = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 9x \right]_1^4$

$$= \left(\frac{1}{3} \times 4^3 - \frac{5}{2} \times 4^2 + 9 \times 4 \right) - \left(\frac{1}{3} \times 1^3 - \frac{5}{2} \times 1^2 + 9 \times 1 \right)$$

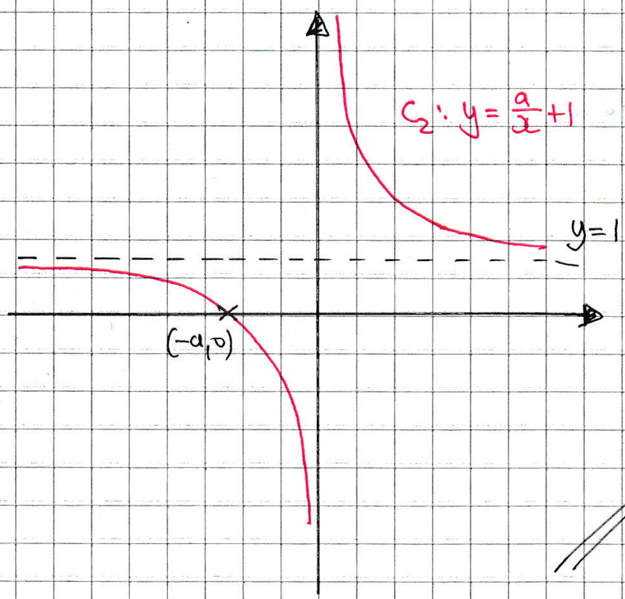
$$= \left(\frac{64}{3} - 40 + 36 \right) - \left(\frac{1}{3} - \frac{5}{2} + 9 \right) = \frac{52}{3} - \frac{41}{6} = \frac{21}{2}$$

$\therefore$ REQUIRED AREA $= A_1 - A_2 = 15 - \frac{21}{2} = \frac{9}{2}$

IYGB – MP1 PAPER R – QUESTION 3

a) IT IS A TRANSLATION BY THE VECTOR $\begin{pmatrix}0\\1\end{pmatrix}$, IE TRANSLATION "UPWARDS" BY ONE UNIT

b)



- NO y INTERCEPTS
- $x=0$ & $y=1$ ARE THE TWO ASYMPTOTES
- $x=-a$ IS THE x INTERCEPT SINCE $y=0$ GIVES

$$0 = \frac{a}{x} + 1$$
$$\frac{a}{x} = -1$$
$$-x = a$$
$$x = -a$$

c) SOLVING SIMULTANEOUS EQUATIONS

$$y = \frac{a}{x} + 1$$
$$y = x$$

I) USING $(-2,-2)$ IN THE FIRST EQUATION

$$-2 = \frac{a}{-2} + 1$$
$$4 = a - 2$$
$$a = 6$$

II) NOW

$$\left.\begin{array}{l} y = \frac{6}{x} + 1 \\ y = x \end{array}\right\} \Rightarrow x = \frac{6}{x} + 1$$
$$\Rightarrow x^2 = 6 + x$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x+2)(x-3) = 0$$

$\therefore x = \begin{cases} -2 \\ 3 \end{cases}$ POINT A (for -2)

$\therefore B(3,3)$

IYGB - MP1 PAPER R - QUESTION 4

ASSERTION: $m^2 - n^2 \neq 102$ IF $m \in \mathbb{N}, n \in \mathbb{N}$

PROOF BY EXHAUSTION

REWRITE THE LHS AS A DIFFERENCE OF SQUARES

$$f(m,n) = m^2 - n^2 = (m+n)(m-n)$$

SUPPOSE THAT

(I) BOTH m, n ARE EVEN $\Longrightarrow$ $m+n$ AND $m-n$ WILL ALSO BE EVEN

$$\Longrightarrow \begin{pmatrix} m+n = 2\alpha \\ m-n = 2\beta \end{pmatrix} \quad \alpha, \beta \in \mathbb{N}$$

$$\Longrightarrow f(m,n) = (2\alpha)(2\beta) = 4\alpha\beta$$

$\Longrightarrow$ $f(m,n)$ DIVIDES BY 4

BUT 102 DOES NOT

(II) BOTH m, n ARE ODD $\Longrightarrow$ $m+n$ AND $m-n$ WILL BE

$\Longrightarrow$ BY IDENTICAL ARGUMENT AS IN (I)

THIS IS NOT POSSIBLE

(III) IF m IS ODD & n IS EVEN (OR THE OTHER ROUND), THEN BOTH $m+n$ AND $m-n$ WILL BE ODD

$$\Longrightarrow \begin{pmatrix} m+n = 2\lambda+1 \\ m-n = 2\mu+1 \end{pmatrix} \quad \lambda, \mu \in \mathbb{N}$$

$$\Longrightarrow f(m,n) = (2\lambda+1)(2\mu+1)$$

IYGB - MA PAPER R - QUESTION 4

$\Longrightarrow \quad f(m,n) = 2\lambda + 2\mu + 4\lambda\mu + 1$

$\Longrightarrow \quad f(m,n) = 2[2\lambda\mu + \lambda + \mu] + 1$

$\Longrightarrow \quad f(m,n)$ IS ODD BUT 102 IS NOT

HENCE WE EXHAUSTED ALL THE POSSIBILITIES AND ALL OF THE POSSIBLE SCENARIOS CANNOT PRODUCE 102

$\therefore \quad m^2 - n^2 \neq 102$ IF $m \in \mathbb{N}$ & $n \in \mathbb{N}$

IYGB - MP1 PAPER R - QUESTION 5

a) COMPLETING THE SQUARE IN x & y

$\Rightarrow x^2+y^2-4x-2y=13$

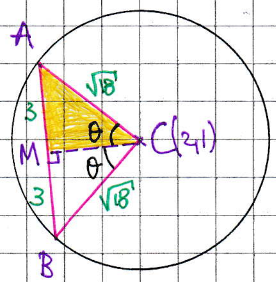
$\Rightarrow x^2-4x+y^2-2y=13$

$\Rightarrow (x-2)^2-4+(y-1)^2-1=13$

$\Rightarrow (x-2)^2+(y-1)^2=18$

$\therefore$ CENTRE $C(2,1)$, RADIUS $=\sqrt{18}$

b) LOOKING AT THE DIAGRAM BELOW



$\sin\theta = \frac{3}{\sqrt{18}}$

$\sin\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$\therefore \hat{ACB} = 2\times 45°$

$\hat{ACB} = 90°$

// AS REQUIRED

OR

$|AM|^2+|MC|^2=|AC|^2$

$3^2+|MC|^2=(\sqrt{18})^2$

$9+|MC|^2=18$

$|MC|^2=9$

$|MC|=3$

$\therefore \triangle AMC$ IS RIGHT ANGLED AND ISOSCELES

$\therefore \hat{ACM}=45°$

$\therefore \hat{ACB}=90°$

c) SOLVING SIMULTANEOUSLY THE EQUATIONS OF THE CIRCLE & THE LINE

$$\left.\begin{array}{l} x^2+y^2-4x-2y=13 \\ y=k-x \end{array}\right\} \Rightarrow x^2+(k-x)^2-4x-2(k-x)=13$$

$\Rightarrow x^2+k^2-2kx+x^2-4x-2k+2x=13$

$\Rightarrow 2x^2-2kx-2x+k^2-2k-13=0$

$\Rightarrow 2x^2-2x(k+1)+(k^2-2k-13)=0$

$\Rightarrow 2x^2-2(k+1)x+(k^2-2k-13)=0$

// AS REQUIRED

IYGB - MP1 PAPER R - QUESTION 5

d) IF THE LINE IS A TANGENT, THEN THE QUADRATIC IN x FOUND IN PART (c) MUST HAVE REPEATED ROOTS.

$$\boxed{2x^2 - 2(k+1)x + (k^2-2k-13) = 0}$$

$b^2 - 4ac = 0$ WITH $a = 2$

$b = -2(k+1)$

$c = k^2-2k-13$

$$\Rightarrow \left[-2(k+1)\right]^2 - 4\times2\times(k^2-2k-13) = 0$$

$$\Rightarrow 4(k+1)^2 - 8(k^2-2k-13) = 0$$

$\div 4$

$$\Rightarrow (k+1)^2 - 2(k^2-2k-13) = 0$$

$$\Rightarrow k^2+2k+1 - 2k^2 + 4k + 26 = 0$$

$$\Rightarrow -k^2 + 6k + 27 = 0$$

$$\Rightarrow k^2 - 6k - 27 = 0$$

$$\Rightarrow (k-9)(k+3) = 0$$

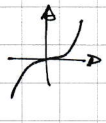
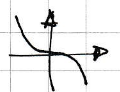
$$\Rightarrow k = \begin{cases} 9 \\ -3 \end{cases}$$

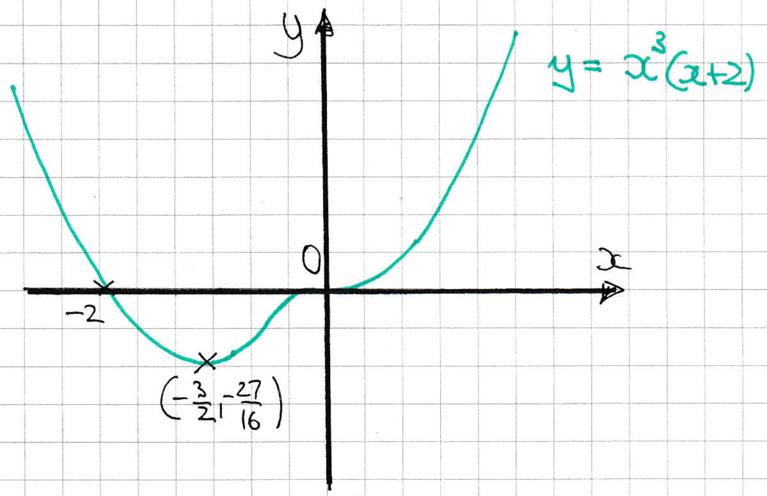
//

IYGB – MP1 PAPER R – QUESTION 6

a) $y = x^3(x+2), \quad x \in \mathbb{R}$

EXPAND AND START COLLECTING INFORMATION FOR THE SKETCH

- $y = x^4 + 2x^3 = x^3(x+2)$
- $(0,0)$,  OR , IE STATIONARY POINT OF INFLEXION
- $(-2,0)$, STANDARD x INTERCEPT
- AS $x \to +\infty$, $y \sim x^4$, IE GETS LARGE & POSITIVE
- AS $x \to -\infty$, $y \sim x^4$, IE IT GETS LARGE & POSITIVE



- $\frac{dy}{dx} = 4x^3 + 6x^2 = 2x^2(2x+3)$

SOLVING FOR ZERO YIELDS $x = \begin{cases} 0 & \text{(POINT OF INFLECTION)} \\ -\frac{3}{2} & \text{(MINIMUM AS SEEN ON THE GRAPH)} \end{cases}$

$$y\Big|_{x=-\frac{3}{2}} = \left(-\frac{3}{2}\right)^3\left(-\frac{3}{2}+2\right) = -\frac{27}{8} \times \frac{1}{2} = -\frac{27}{16}$$

//

IYGB - MP1 PAPER R - QUESTION 6

b) SET THE GRADIENT FUNCTION EQUAL TO 10

$$\Rightarrow 4x^3 + 6x^2 = 10$$
$$\Rightarrow 2x^3 + 3x^2 = 5$$
$$\Rightarrow 2x^3 + 3x^2 - 5 = 0$$

BY INSPECTION $x=1$ IS A SOLUTION — PROCEED BY LONG DIVISION OR MANIPULATION

$$\Rightarrow 2x^2(x-1) + 5x(x-1) + 5(x-1) = 0$$
$$\Rightarrow (2x^2 + 5x + 5)(x-1) = 0$$

CHECK THE DISCRIMINANT OF THE QUADRATIC

$$\Rightarrow b^2 - 4ac = 5^2 - 4 \times 2 \times 5$$
$$= 25 - 40$$
$$= -15 < 0$$

$\therefore$ NO MORE SOLUTIONS & ONLY POINT ON THE CURVE IS THE POINT $P(1,3)$

IYGB - MP1 PAPER R - QUESTION 7

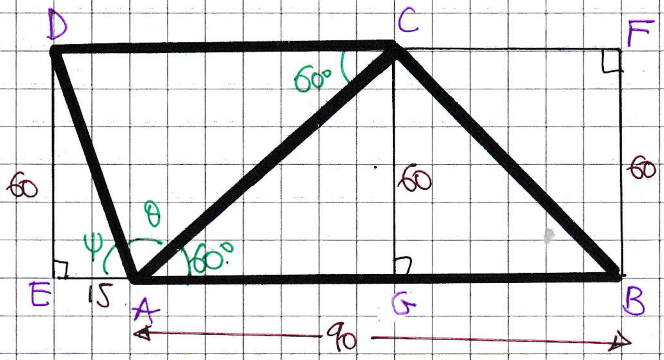
a) LOOKING AT THE DIAGRAM ON $\hat{ACG}$

$$\frac{|CG|}{|AG|} = \tan 60°$$

$$\frac{60}{|AG|} = \sqrt{3}$$

$$|AG| = \frac{60}{\sqrt{3}}$$

$$|AG| = 20\sqrt{3}$$



BY PYTHAGORAS ON $\overset{\triangle}{CGB}$

$$|CB|^2 = |CG|^2 + |GB|^2$$

$$|CB|^2 = 60^2 + (90 - |AG|)^2$$

$$|CB|^2 = 60^2 + (90 - 20\sqrt{3})^2$$

$$|CB|^2 = 6664.617093\ldots$$

$$|CB| \simeq 81.6$$

& BY SIMPLE CONSIDERATIONS

$$|CD| = |EA| + |AG|$$

$$|CD| = 15 + 20\sqrt{3}$$

$$|CD| \approx 49.6$$

b) FIRSTLY FIND ANGLE ψ IN THE DIAGRAM

$$\tan\psi = \frac{|DE|}{|EA|}$$

$$\tan\psi = \frac{60}{15}$$

$$\psi = 75.9637°\ldots$$

$$\therefore \hat{DAC} = \theta = 180 - 60 - \psi$$

$$= 180 - 60 - 75.9637°$$

$$= 44.0°$$

1 d.p.

IYGB - MP1 PAPER R - QUESTION 8

MANIPULATE THE EQUATIONS, SO WE CAN "REMOVE" THE LOGS

$$\Rightarrow \log_2(x^2y) = 2$$
$$\Rightarrow \log_2(x^2y) = 2\log_2 2$$
$$\Rightarrow \log_2(x^2y) = \log_2 4$$
$$\Rightarrow x^2y = 4$$

$$\Rightarrow 11 + \tfrac{1}{2}\log_2 y = 3\log_2 x$$
$$\Rightarrow 22 + \log_2 y = 6\log_2 x^3$$
$$\Rightarrow 22\log_2 2 + \log_2 y = \log_2 x^6$$
$$\Rightarrow \log_2 2^{22} + \log_2 y = \log_2 x^6$$
$$\Rightarrow \log(2^{22} \times y) = \log_2 x^6$$
$$\Rightarrow y \times 2^{22} = x^6$$

SOLVING BY DIVISION

$$\left.\begin{aligned} y \times 2^{22} &= x^6 \\ x^2y &= 4 \end{aligned}\right\} \Rightarrow \frac{2^{22}}{x^2} = \frac{x^6}{4}$$
$$\Rightarrow 4 \times 2^{22} = x^8$$
$$\Rightarrow 2^{24} = x^8$$
$$\Rightarrow (2^3)^8 = x^8$$
$$\Rightarrow x^8 = 8^8$$
$$\Rightarrow x = +8$$

& USING $x^2y = 4$

$$\Rightarrow 8^2 \times y = 4$$
$$\Rightarrow 64y = 4$$
$$\Rightarrow y = \frac{1}{16}$$

IYGB - MP1 PAPER R - QUESTION 8

ALTERNATIVE METHOD/APPROACH

STARTING WITH THE EQUATION & PROCEED AS FOLLOWS

$$\left.\begin{array}{l} \log_2(x^2y) = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log_2 x^2 + \log_2 y = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} 2\log_2 x + \log_2 y = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array}\right\} \Rightarrow \begin{array}{l} 2X + Y = 2 \\ 11 + \frac{1}{2}Y = 3X \quad \times(-2) \end{array}$$

$$\left.\begin{array}{l} 2X + Y = 2 \\ -22 - Y = -6X \end{array}\right\} \Rightarrow 2X - 22 = 2 - 6X$$

$$\Rightarrow 8X = 24$$

$$\Rightarrow X = 3$$

$$\Rightarrow \log_2 x = 3$$

$$\Rightarrow \log_2 x = 3\log_2 2 = \log_2 8$$

$$\Rightarrow x = 8$$

&

$$2X + Y = 2$$

$$6 + Y = 2$$

$$Y = -4$$

$$\Rightarrow \log_2 y = -4$$

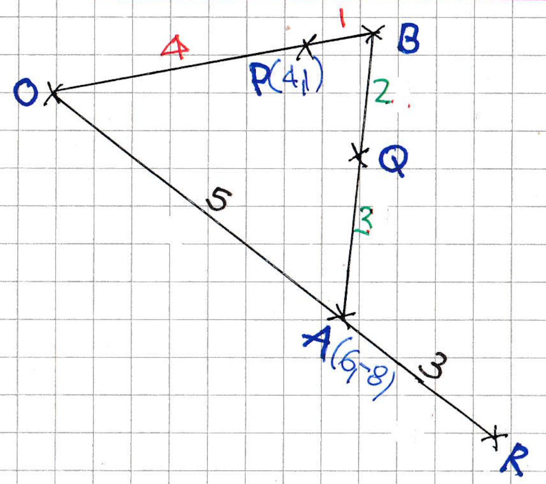
$$\Rightarrow \log_2 y = -4\log_2 2 = \log_2 2^{-4}$$

$$\Rightarrow y = 2^{-4}$$

$$\Rightarrow y = \frac{1}{16}$$

IYGB - MP1 PAPER R - QUESTION 9

a) LOOKING AT THE DIAGRAM



$\vec{OP} = 4\underline{i} + \underline{j}$

$\vec{OA} = 6\underline{i} - 8\underline{j}$

- $\vec{OB} = \frac{5}{4}\vec{OP} = \frac{5}{4}(4\underline{i} + \underline{j}) = 5\underline{i} + \frac{5}{4}\underline{j}$
- $\vec{BA} = \vec{BO} + \vec{OA} = -5\underline{i} - \frac{5}{4}\underline{j} + 6\underline{i} - 8\underline{j} = \underline{i} - \frac{37}{4}\underline{j}$
- $\vec{BQ} = \frac{2}{5}\vec{BA} = \frac{2}{5}\left(\underline{i} - \frac{37}{4}\underline{j}\right) = \frac{2}{5}\underline{i} - \frac{37}{10}\underline{j}$
- $\vec{OQ} = \vec{OB} + \vec{BQ} = 5\underline{i} + \frac{5}{4}\underline{j} + \frac{2}{5}\underline{i} - \frac{37}{10}\underline{j}$

$= \frac{27}{5}\underline{i} - \frac{49}{20}\underline{j}$

$\therefore Q(5.4, -2.45)$

b) COMPARE THE VECTORS $\vec{PQ}$ & $\vec{QR}$

- $\vec{PQ} = \vec{OQ} - \vec{OP}$ (USING POSITION VECTORS)

$= \left(\frac{27}{5}\underline{i} - \frac{49}{20}\underline{j}\right) - (4\underline{i} + \underline{j}) = \frac{7}{5}\underline{i} - \frac{69}{20}\underline{j}$

- $\vec{QR} = \vec{OR} - \vec{OQ} = \frac{8}{5}\vec{OA} - \vec{OQ}$

$= \frac{8}{5}(6\underline{i} - 8\underline{j}) - \left(\frac{27}{5}\underline{i} - \frac{49}{20}\underline{j}\right)$

$= \frac{21}{5}\underline{i} - \frac{207}{20}\underline{j}$

c) HENCE WE HAVE

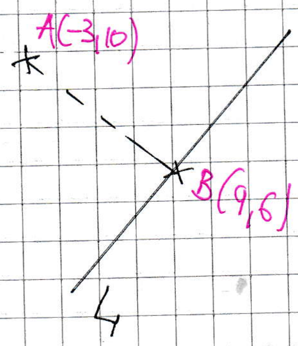
- $\vec{PQ} = \frac{7}{5}\underline{i} - \frac{69}{20}\underline{j} = \frac{1}{20}(28\underline{i} - 69\underline{j})$
- $\vec{QR} = \frac{21}{5}\underline{i} - \frac{207}{20}\underline{j} = \frac{3}{20}(28\underline{i} - 69\underline{j})$

AS $\vec{PQ}$ & $\vec{QR}$ ARE IN THE DIRECTION OF THE SAME VECTOR $(28\underline{i} - 69\underline{j})$ AND SHARE THE POINT Q, THE POINTS P, Q & R ARE COLLINEAR WITH $|\vec{PQ}| : |\vec{QR}| = 1:3$

IYGB - MP1 PAPER R - QUESTION 10

a) LOOKING AT THE DIAGRAM

- GRADIENT AB $= \frac{y_2-y_1}{x_2-x_1} = \frac{6-10}{9-(-3)} = \frac{-4}{12} = -\frac{1}{3}$



- GRADIENT OF L_1 $= +3$
- EQUATION OF L_1, WITH $m=3$, PASSING THROUGH (9,6)

$\Rightarrow y-y_0 = m(x-x_0)$

$\Rightarrow y-6 = 3(x-9)$

$\Rightarrow y-6 = 3x-27$

$\Rightarrow y = 3x-21$

b) EITHER USE STANDARD RESULT

$y = mx+c$ WITH $m = \tan\theta$

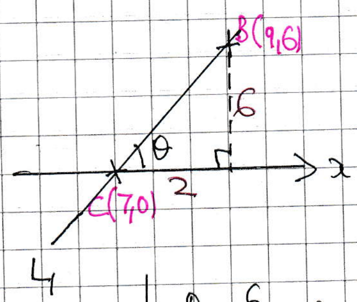
$\therefore \tan\theta = 3$

OR

FIND C FIRST

when $y=0$ $x=7$

$\therefore C(7,0)$



$\tan\theta = \frac{6}{2} = 3$

c) GRADIENT OF AB $= -\frac{1}{3}$ (PART a)

EQUATION OF L_2 WITH $m = -\frac{1}{3}$ & $C(7,0)$

$\Rightarrow y-y_0 = m(x-x_0)$

$\Rightarrow y-0 = -\frac{1}{3}(x-7)$

$\Rightarrow 3y = -(x-7)$

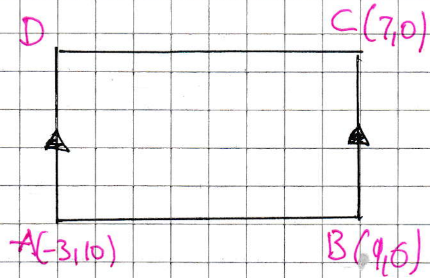
$\Rightarrow 3y = -x+7$

$\Rightarrow x+3y = 7$

IYGB - MP1 PAPER Z - QUESTION 10

d) LOOKING AT THE DIAGRAM (NOT TO SCALE)

USING "VECTOR" CONCEPTS



FROM B TO C $\Rightarrow \begin{pmatrix} 7-9 \\ 0-6 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

FROM A TO D $\Rightarrow \begin{pmatrix} -3 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$

$\therefore D(-5,4)$ // AS REQUIRED

e) $\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$

$|AB| = \sqrt{(-3-9)^2 + (10-6)^2} = \sqrt{144+16} = \sqrt{160} = 4\sqrt{10}$

$|BC| = \sqrt{(7-9)^2 + (0-6)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$\therefore$ AREA $= |AB||BC| = 4\sqrt{10} \times 2\sqrt{10} = 8 \times 10 = 80$ //

IYGB – MP1 PAPER R – QUESTION 11

FIRST WRITE THE EQUATION IN $\sin\theta$

$$y = 6 - 4\sin\theta - \cos^2\theta$$
$$y = 6 - 4\sin\theta - (1 - \sin^2\theta)$$
$$y = 5 - 4\sin\theta + \sin^2\theta$$

BY INSPECTION, LOOKING AT THE SINE GRAPH, AND NOTING THAT $\sin 270° = -1$

$$y_{MAX} = 5 - 4(-1) + (-1)^2 = 10$$

$\therefore$ $B(270°, 10)$

COMPLETING THE SQUARE IN $\sin\theta$

$$y = \sin^2\theta - 4\sin\theta + 5$$
$$y = (\sin\theta - 2)^2 + 1$$

BUT $\sin\theta \neq 2$, SO MINIMUM WILL BE ACHIEVED WHEN $\sin\theta = +1$

$$y_{MIN} = (+1-2)^2 + 1 = 1 + 1 = 2$$

$\therefore$ $A(90°, 2)$