IYGB GCE

Mathematics MP1

Advanced Level

Practice Paper S

Difficulty Rating: 5.000/2.4000

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

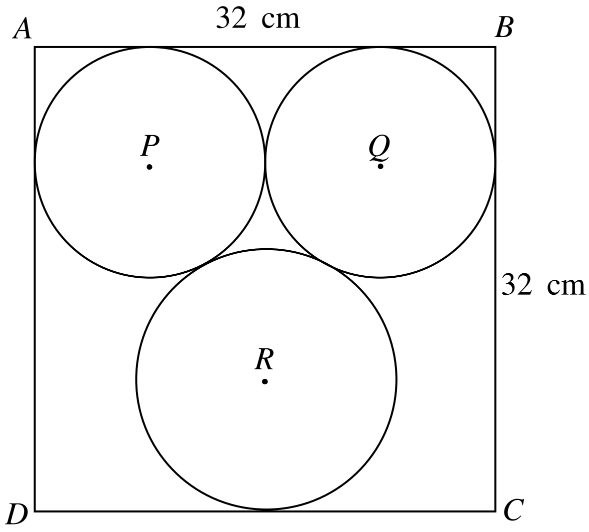
This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows, two identical circles, centred at P and Q, and a third circle, centred at R, are touching each other externally.

The three circles fit snugly inside a square $ABCD$, of side length 32 cm, so that PQ is parallel to AB.

Determine the size of radius of the circle centred at R. **(4)**

Question 2

A curve has equation

$$f(x) \equiv 2^{ax} + b, \ x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b, given further that

$$f(2) = \frac{5}{2} \quad \text{and} \quad f(-2) = 4.$$ **(8)**

Question 3

The binomial coefficients are given by

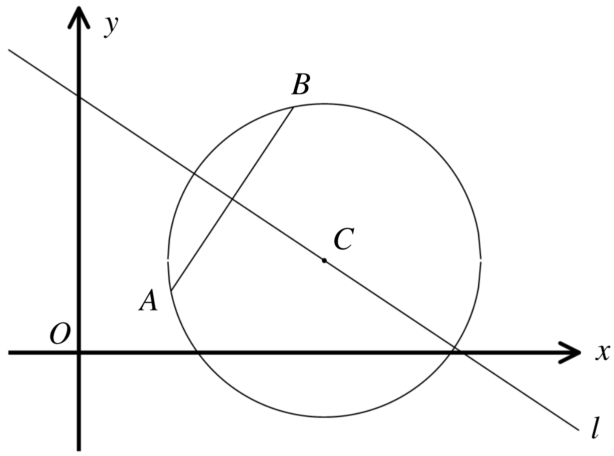
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \; k \in \mathbb{N}, \; n \in \mathbb{N} \cup \{0\}.$$

Show directly from the above definition that if $n \geq r \geq m$, then

$$\binom{n}{m}\binom{n-m}{r-m} = \binom{n}{r}\binom{r}{m}.$$

(5)

Question 4



The figure above shows a circle whose centre is located at the point $C(k,h)$, where k and h are constants such that $2 < h < 5$.

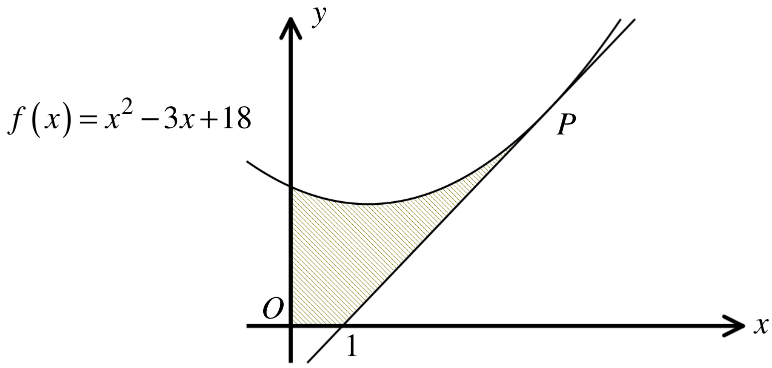
The points $A(3,2)$ and $B(7,8)$ lie on this circle.

The straight line l passes through C and the midpoint of AB.

Given that the radius of the circle is $\sqrt{26}$, find an equation for l, the value of k and the value of h.

(12)

Question 5



A quadratic curve has equation

$$y = x^2 - 3x + 18, \quad x \in \mathbb{R}.$$

The tangent to the curve at the point P meets the x axis at the point with coordinates $(1,0)$, as shown in the figure above.

Find the area of the finite region bounded by the curve, the coordinates axes and the tangent to the curve at P, shown shaded in figure above. **(12)**

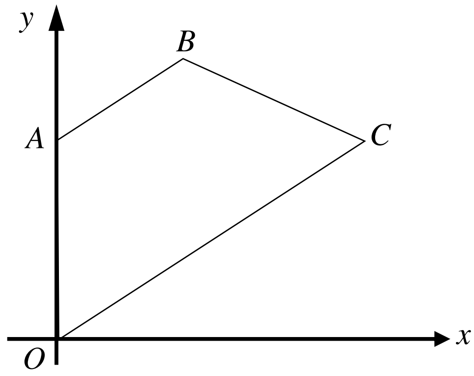
Question 6

Rationalize the denominator of the following surd.

$$\frac{4}{\sqrt{3}+\sqrt{2}+1} = 2+\sqrt{2}-\sqrt{6}.$$

Show detailed workings in this question. **(7)**

Question 7



The figure above shows an isosceles trapezium $OABC$, where O is the origin.

It is further given that

- the coordinates of A are $(0,6)$,
- the sides AB and OC are parallel,
- $|OA| = |BC|$,
- the diagonal AC is parallel to the x axis.

If B lies on the line with equation $y = x$, determine, in exact simplified surd form, the coordinates of B.

(14)

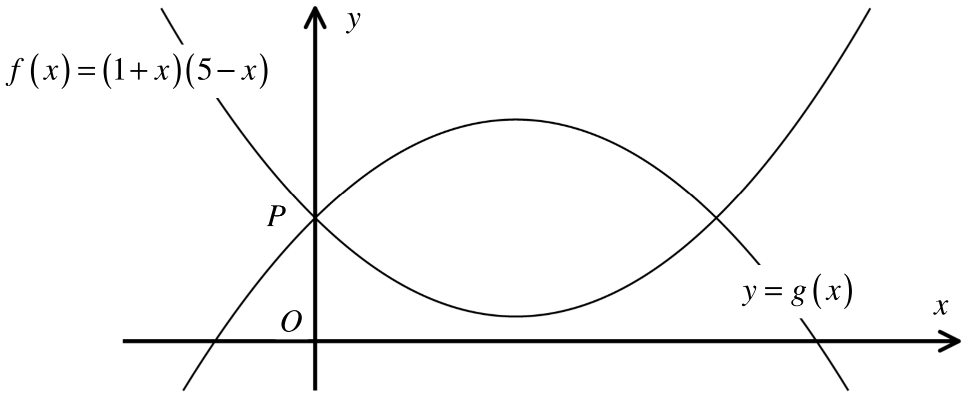
Question 8

Solve the following trigonometric equation

$$\left(19 + 2\sin^2 2\theta\right)\tan 2\theta = \frac{3}{\cos 2\theta} - 17\cos 2\theta, \quad 0° \le \theta < 360°.$$

(12)

Question 9



The figure above shows the curve with equation

$$f(x)=(1+x)(5-x),$$

and the curve with equation $y=g(x)$.

The curve with equation $y=g(x)$ can be obtained by **two** transformations of the curve with equation $y=f(x)$. It is further given that these two transformations **do not** include any stretches, shears or rotations.

If both curves meet the y axis at the same point P, find an equation for $y=g(x)$. **(6)**

Question 10

An **open** box is to be made of thin sheet metal, in the shape of a cuboid with a square base of length x and height h.

The box is to have a **fixed** volume.

Find the value of x, in terms of h, when the surface area of the box is minimum. **(11)**

Question 11

Show that the square of an odd positive integer greater than 1 is of the form

$$8T+1,$$

where T is a triangular number. **(13)**

Question 12

Solve the following equation.

$$3e^{2(x+1)} - (2e)^x\left(e^4+9\right) + 3e^2 \times 4^x, \quad x \in \mathbb{R}.$$

Give the two solutions of the equation in the form $x = \pm A$, where A is in the form $\dfrac{a-\ln 3}{b-\ln 2}$, where a and b are positive integers. **(14)**

Question 13

The curve with equation $y = x^2$, $x \in \mathbb{R}$, is translated parallel to the x axis, so that R denotes the area of the finite region bounded by the coordinated axes, the curve and the straight line with equation $x = 3$.

Determine the least value of R. **(7)**
