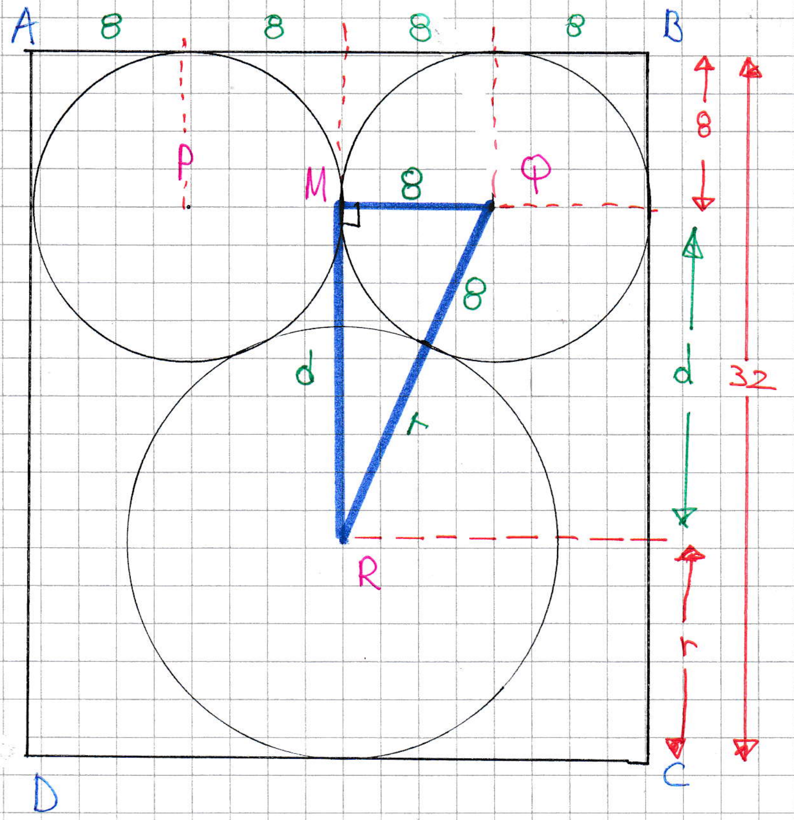
IYGB - MP1 PAPER S - QUESTION 1

LOOKING AT THE DIAGRAM



$d = 32 - 8 - r$

$d = 24 - r$

BY PYTHAGORAS

$$|MQ|^2 + |MR|^2 = |QR|^2$$

$$8^2 + d^2 = (8+r)^2$$

$$64 + (24-r)^2 = (8+r)^2$$

$$\cancel{64} + 576 - 48r + \cancel{r^2} = \cancel{64} + 16r + \cancel{r^2}$$

$$576 = 64r$$

$$r = 9$$

IYGB - MP1 PAPER S - QUESTION 2

$$\boxed{f(x) = 2^{ax} + b \quad , \quad x \in \mathbb{R}}$$

- USING $f(2) = \frac{5}{2}$

$\Rightarrow 2^{a \times 2} + b = \frac{5}{2}$

$\Rightarrow 2^{2a} + b = \frac{5}{2}$

$\Rightarrow b = \frac{5}{2} - 2^{2a}$

- USING $f(-2) = 4$

$\Rightarrow 2^{a(-2)} + b = 4$

$\Rightarrow 2^{-2a} + b = 4$

$\Rightarrow b = 4 - 2^{-2a}$

- SOLVING SIMULTANEOUSLY

$\Rightarrow 4 - 2^{-2a} = \frac{5}{2} - 2^{2a}$

$\Rightarrow 4 - \frac{1}{2^{2a}} = \frac{5}{2} - 2^{2a}$

$\Rightarrow 4 - \frac{1}{A} = \frac{5}{2} - A \qquad (A = 2^{2a})$

$\Rightarrow 4A - 1 = \frac{5}{2}A - A^2$

$\Rightarrow 8A - 2 = 5A - 2A^2$

$\Rightarrow 2A^2 + 3A - 2 = 0$

$\Rightarrow (2A - 1)(A + 2) = 0$

$\Rightarrow A = \frac{1}{2}, \cancel{-2}$

IYGB - MP1 PAPER S - QUESTION 2

$$\Longrightarrow 2^{2a} = \frac{1}{2}$$

$$\Longrightarrow 2^{2a} = 2^{-1}$$

$$\Longrightarrow 2a = -1$$

$$\Longrightarrow \underline{a = -\frac{1}{2}} \ //$$

● & USING FINALLY $b = 4 - 2^{-2a}$

$$\Longrightarrow b = 4 - 2^{-2(-\frac{1}{2})}$$

$$\Longrightarrow b = 4 - 2^{1}$$

$$\Longrightarrow \underline{b = 2} \ //$$

IYGB - MP1 PAPER S - QUESTION 3

APPLY THE DEFINITION OF A COMBINATION

$$\binom{n}{m}\binom{n-m}{r-m} = \frac{n!}{m!\,(n-m)!} \times \frac{(n-m)!}{(r-m)!\,[(n-m)-(r-m)]!}$$

$$= \frac{n!}{m!\,(n-m)!} \times \frac{(n-m)!}{(r-m)!\,(n-r)!}$$

$$= \frac{n!}{m!} \times \frac{1}{(r-m)!\,(n-r)!}$$

$$= \frac{n!}{m!\,(r-m)!\,(n-r)!}$$

$$= \frac{n!\,r!}{m!\,(r-m)!\,(n-r)!\,r!}$$

$$= \frac{n!}{r!\,(n-r)!} \times \frac{r!}{m!\,(r-m)!}$$

$$= \binom{n}{r}\binom{r}{m}$$

AS REQUIRED

IYGB - MP1 PAPER S - QUESTION 4

TREATING A POINT ON l AS A LOCUS — LET $C(k,h)$ SO C MUST LIE ON THE PERPENDICULAR BISECTOR OF AB SO EQUIDISTANT FROM A & B

$A(3,2)$ $\quad B(7,8)$ $\quad C(k,h)$

- $|AC|^2 = (k-3)^2 + (h-2)^2$
- $|BC|^2 = (k-7)^2 + (h-8)^2$

THUS WE HAVE

$$(k-3)^2 + (h-2)^2 = (k-7)^2 + (h-8)^2$$

$$\cancel{k^2} - 6k + 9 + \cancel{h^2} - 4h + 4 = \cancel{k^2} - 14k + 49 + \cancel{h^2} - 16h + 64$$

$$-6k - 4h + 13 = -14k - 16h + 113$$

$$8k + 12h = 100$$

$$2k + 3h = 25$$

SOLVING SIMULTANEOUSLY WITH $|AC|^2 = 26$

$$\left.\begin{array}{l} (k-3)^2 + (h-2)^2 = 26 \\ 2k + 3h = 25 \end{array}\right\} \Rightarrow 4(k-3)^2 + 4(h-2)^2 = 104$$

$$\Rightarrow (2k-6)^2 + (2h-4)^2 = 104$$

$$\Rightarrow \left[(25-3h) - 6\right]^2 + (2h-4)^2 = 104$$

$$\Rightarrow (19-3h)^2 + (2h-4)^2 = 104$$

$$\Rightarrow \left(\begin{array}{l} 361 - 114h + 9h^2 \\ 16 - 16h + 4h^2 \end{array} \right. = 104$$

$$\Rightarrow 13h^2 - 130h + 377 = 104$$

$$\Rightarrow 13h^2 - 130h + 273 = 0 \quad \div 13$$

$$\Rightarrow h^2 - 10h + 21 = 0$$

$$\Rightarrow (h-3)(h-7) = 0$$

$h = 3$ $\quad (2 < h < 5)$

$k = 8$ $\quad$ (USING $2k+3h=25$)

IYGB - MP1 PAPER S - QUESTION 4

FINALLY THE EQUATION OF l IS THE LOCUS FOUND IN x & y

I.E $\quad 2k + 3h = 25$

$$2x + 3y = 25$$

A GEOMETRIC APPROACH IS ALSO POSSIBLE

E.g
- GRAD AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{7-3} = \frac{6}{4} = \frac{3}{2}$
- GRAD OF l MUST BE $-\frac{2}{3}$
- MIDPOINT OF AB MUST BE $M\left(\frac{7+3}{2}, \frac{8+2}{2}\right) = M(5,5)$
- EQUATION OF l MUST BE $y - y_0 = m(x - x_0)$

$$y - 5 = -\frac{2}{3}(x - 5)$$
$$3y - 15 = -2x + 10$$
$$2x + 3y = 25$$

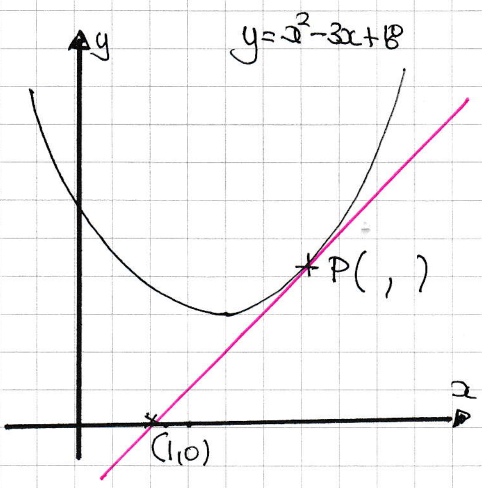
THEN SOLVE SIMULTANEOUS EQUATIONS

$|AC|$ OR $|BC|$ IS $\sqrt{26}$ & $\quad 2k + 3h = 25$

IYGB - MP1 PAPER S - QUESTION 5

FIRSTLY WE REQUIRE TO FIND THE CO-ORDINATES OF THE POINT OF TANGENCY

METHOD A (PARAMETRIC APPROACH)



- LET THE COORDINATES OF P BE $(a, a^2 - 3a + 18)$, FOR SOME $a > 0$
- THEN WE HAVE

$$\Rightarrow \frac{dy}{dx} = 2x - 3$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=a} = 2a - 3$$

- EQUATION OF THE TANGENT AT P WILL BE

$$\Rightarrow y - (a^2 - 3a + 18) = (2a - 3)(x - a)$$

- AS THE TANGENT PASSES THROUGH (1,0) WE HAVE

$$\Rightarrow -(a^2 - 3a + 18) = (2a - 3)(1 - a)$$

$$\Rightarrow a^2 - 3a + 18 = (2a - 3)(a - 1)$$

$$\Rightarrow a^2 - 3a + 18 = 2a^2 - 5a + 3$$

$$\Rightarrow 0 = a^2 - 2a - 15$$

$$\Rightarrow (a + 3)(a - 5) = 0$$

$$\Rightarrow a = \begin{cases} \cancel{-3} \\ 5 \end{cases}$$

IE $P(5, 28)$

METHOD B (DISCRIMINANT APPROACH)

- LET THE EQUATION OF THE TANGENT BE $y = mx + c$
- USING THE CONSTRAINT (1,0) WE HAVE $0 = m + c$
- SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = x^2 - 3x + 18 \\ y = mx + c \end{array}\right\} \Rightarrow \left.\begin{array}{l} y = x^2 - 3x + 18 \\ y = m \quad - m \end{array}\right\} \Rightarrow x^2 - 3x + 18 = mx - m$$

$$\Rightarrow x^2 - (3 + m)x + (m + 18) = 0$$

LOOKING FOR REPEATED ROOTS

IYGB - MP1 PAPER S - QUESTION 5

$\Longrightarrow [-(3+m)]^2 - 4\times1\times(m+18) = 0$

$\Longrightarrow m^2+6m+9-4m-72=0$

$\Longrightarrow m^2+2m-63=0$

$\Longrightarrow (m-7)(m+9)=0$

$\Longrightarrow m = \begin{cases} 7 \\ \cancel{-9} \end{cases}$ (POSITIVE GRADIENT)

- HENCE WE HAVE

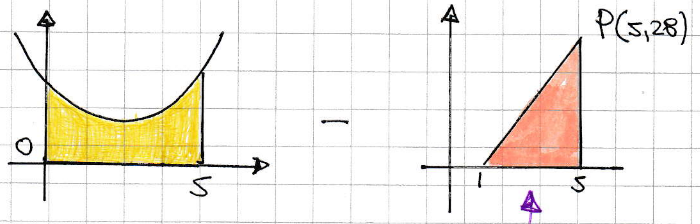
$x^2-(3+m)x+(m+18)=0$

$x^2-10x+25=0$

$(x-5)^2=0$

$x=5$ & $y=28$ AS BEFORE

- NOW WE HAVE



= REQUIRED AREA

$\frac{1}{2}\times4\times28 = \underline{56}$

$\int_0^5 x^2-3x+18\,dx = \left[\frac{1}{3}x^3-\frac{3}{2}x^2+18x\right]_0^5 = \left(\frac{125}{3}-\frac{75}{2}+90\right)-(0) = \frac{565}{6}$

- THE REQUIRED AREA IS

$\frac{565}{6}-56 = \frac{229}{6}$

IYGB - MP1 PAPER S - QUESTION 6

$$\frac{4\sqrt{2}-3\sqrt{3}-1}{1-2\sqrt{2}+\sqrt{3}} = \frac{(4\sqrt{2}-3\sqrt{3}-1)\left[1-2\sqrt{2}-\sqrt{3}\right]}{\left[(1-2\sqrt{2})+\sqrt{3}\right]\left[(1-2\sqrt{2})-\sqrt{3}\right]}$$

$$= \frac{(4\sqrt{2}-3\sqrt{3}-1)(1-2\sqrt{2}-\sqrt{3})}{(1-2\sqrt{2})^2-(\sqrt{3})^2} \quad \leftarrow \ 1-4\sqrt{2}+8-3 = 6-4\sqrt{2}$$

NOW MULTIPLYING & TIDYING UP THE NUMERATOR

$$\begin{array}{rrrr} \ldots = 4\sqrt{2} & -16 & -4\sqrt{6} & -3\sqrt{3} \\ 2\sqrt{2} & +9 & +6\sqrt{6} & +\sqrt{3} \\ & -1 & & \\ \hline \end{array}$$

$$\ldots = \frac{6\sqrt{2}+2\sqrt{6}-2\sqrt{3}-8}{6-4\sqrt{2}} = \frac{3\sqrt{2}+\sqrt{6}-\sqrt{3}-4}{3-2\sqrt{2}}$$

RATIONALIZING ONCE MORE

$$= \frac{(3+2\sqrt{2})(3\sqrt{2}+\sqrt{6}-\sqrt{3}-4)}{(3+2\sqrt{2})(3-2\sqrt{2})} \quad \leftarrow \ 9-8=1$$

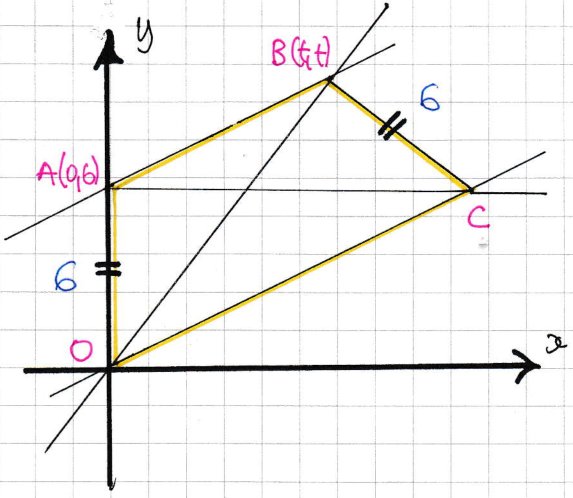
TIDY THE NUMERATOR

$$\begin{array}{rrrr} \ldots \ 9\sqrt{2} & +3\sqrt{6} & -3\sqrt{3} & -\cancel{12} \\ -8\sqrt{2} & -2\sqrt{6} & +2\sqrt{12} & +\cancel{12} \\ \hline \sqrt{2} & +\sqrt{6} & +\sqrt{3} & \end{array}$$

$$\ldots = \frac{\sqrt{6}+\sqrt{3}+\sqrt{2}}{1} = \sqrt{6}+\sqrt{3}+\sqrt{2}$$ //

IYGB - MP1 PAPER S - QUESTION 7

AS B LIES ON THE LINE $y=x$, LET $B(t,t)$



- GRADIENT "AB" = $\dfrac{t-6}{t-0} = \dfrac{t-6}{t}$
- EQUATION OF "OC" (PARRALLEL TO "AB")

$$y = \frac{t-6}{t}x$$

- AS "AC" IS HORIZONTAL, $y=6$

$$6 = \frac{t-6}{t}x$$
$$x = \frac{6t}{t-6}$$

- DISTANCE BC MUST BE 6, $B(t,t)$, $C\left(\frac{6t}{t-6},6\right)$

$$\sqrt{\left(\frac{6t}{t-6}-t\right)^2 + (6-t)^2} = 6$$

SOLVING THE ABOVE EQUATION

$$\Rightarrow \left(\frac{6t}{t-6}-t\right)^2 + (6-t)^2 = 36$$

$$\Rightarrow \left[\frac{6t-t(t-6)}{t-6}\right]^2 + (t-6)^2 = 36$$

$$\Rightarrow \left[\frac{12t-t^2}{t-6}\right]^2 + (t-6)^2 = 36$$

$$\Rightarrow \frac{t^2(12-t)^2}{(t-6)^2} + (t-6)^2 = 36$$

$$\Rightarrow t^2(t-12)^2 + (t-6)^4 = 36(t-6)^2$$

IYGB - MP1 PAPER S - QUESTION 7

SOLVING BY A SUBSTITUTION FOR SIMPLICITY, LET $w = t - 6$

$$\Rightarrow \overset{t^2}{(w+6)^2}\overset{t-12}{(w-6)^2} + w^4 = 36w^2$$

$$\Rightarrow (w^2-36)^2 + w^4 = 36w^2$$

$$\Rightarrow w^4 - 72w^2 + 1296 + w^4 - 36w^2 = 0$$

$$\Rightarrow 2w^4 - 108w^2 + 1296 = 0$$

$$\Rightarrow w^4 - 54w^2 + 648 = 0$$

BY QUADRATIC FORMULA, COMPLETING THE SQUARE, OR FACTORIZATION

$$\Rightarrow (w^2-36)(w^2-18) = 0$$

$$\Rightarrow w^2 = \left\langle \begin{array}{l} 36 \\ 18 \end{array}\right.$$

$$\Rightarrow w = \left\langle \begin{array}{l} 6 \\ -6 \\ 3\sqrt{2} \\ -3\sqrt{2} \end{array}\right. \qquad t-6 = \left\langle \begin{array}{l} 6 \\ -6 \\ 3\sqrt{2} \\ -3\sqrt{2} \end{array}\right.$$

$$\Rightarrow t = \left\langle \begin{array}{ll} 12 & \\ \cancel{0} & t>6 \text{ DUE TO POINT } C \\ 6+3\sqrt{2} & \\ \cancel{6-3\sqrt{2}} & t>6 \text{ DUE TO POINT } C \end{array}\right.$$

NOW WE HAVE TO CHECK AGAINST THE x CO-ORDINATE OF C

- IF $B(12,12) \Rightarrow x_c = \frac{6t}{t-6} = \frac{6\times12}{12-6} = \frac{72}{6} = \cancel{12}$

  (AS THIS YIELDS A PARALLELOGRAM)

- IF $B(6+3\sqrt{2}, 6+3\sqrt{2}) \Rightarrow x_c = \frac{6(6+3\sqrt{2})}{6+3\sqrt{2}-6} = \frac{\overset{2}{\cancel{6}}(6+3\sqrt{2})}{\cancel{3}\sqrt{2}} = \frac{12+6\sqrt{2}}{\sqrt{2}} = 6+6\sqrt{2}$

  WHICH IS OK

IYGB - MP1 PAPER S - QUESTION 8

$$\boxed{(19+2\sin^2 2\theta)\tan 2\theta = \frac{3}{\cos 2\theta} - 17\cos 2\theta}$$

- START MANIPULATING THE EQUATION BY SWITCHING THE tan2θ INTO SINES AND COSINES AND THEN GETTING RID OF THE DENOMINATORS

$$\Rightarrow (19+2\sin^2 2\theta)\times\frac{\sin 2\theta}{\cos 2\theta} = \frac{3}{\cos 2\theta} - 17\cos 2\theta$$

$\times \cos 2\theta$

$$\Rightarrow (19+2\sin^2 2\theta)\times \sin 2\theta = 3 - 17\cos^2 2\theta$$

$$\Rightarrow (19+2\sin^2 2\theta)\sin 2\theta = 3 - 17(1-\sin^2 2\theta)$$

$$\Rightarrow 19\sin 2\theta + 2\sin^3 2\theta = -14 + 17\sin^2 2\theta$$

$$\Rightarrow 2\sin^3 2\theta - 17\sin^2 2\theta + 19\sin 2\theta + 14 = 0$$

- THIS IS A CUBIC IN sin2θ, SO LOOK FOR SOME SENSIBLE LINEAR FACTORS

$$f(x) = 2x^3 - 17x^2 + 19x + 14$$

- $f(1) = 2 - 17 + 19 + 14 \neq 0$
- $f(-1) = -2 - 17 - 19 + 14 \neq 0$
- $f(2) = 16 - 68 + 38 + 14 = 0$

$\therefore$ $(x-2)$ IS A FACTOR

IYGB - MP1 PAPER S - QUESTION 8

- FACTORIZE THE CUBIC BY USING LONG DIVISION, INSPECTION OR ANY OTHER SENSIBLE METHOD

$$f(x) = 2x^3 - 17x^2 + 19x + 14$$

$$= 2x^2(x-2) - 13x(x-2) - 7(x-2)$$

$$= (x-2)(2x^2 - 13x - 7)$$

$$= (x-2)(2x+1)(x-7)$$

$$\therefore f(x) = 0 \implies x = \begin{cases} 2 \\ 7 \\ -\frac{1}{2} \end{cases}$$

$$f(\sin 2\theta) = 0 \implies \sin 2\theta = \begin{cases} \cancel{2} \\ \cancel{7} \\ -\frac{1}{2} \end{cases}$$

- SOLVING THE EQUATION $\sin 2\theta = -\frac{1}{2}$ FOR $0 \leq \theta < 360°$

$$\implies \arcsin\left(-\tfrac{1}{2}\right) = -30°$$

$$\implies \begin{pmatrix} 2\theta = -30° \pm 360n \\ 2\theta = 210° \pm 360n \end{pmatrix} \quad n = 0, 1, 2, 3, \ldots$$

$$\implies \begin{pmatrix} \theta = -15° \pm 180n \\ \theta = 105° \pm 180n \end{pmatrix}$$

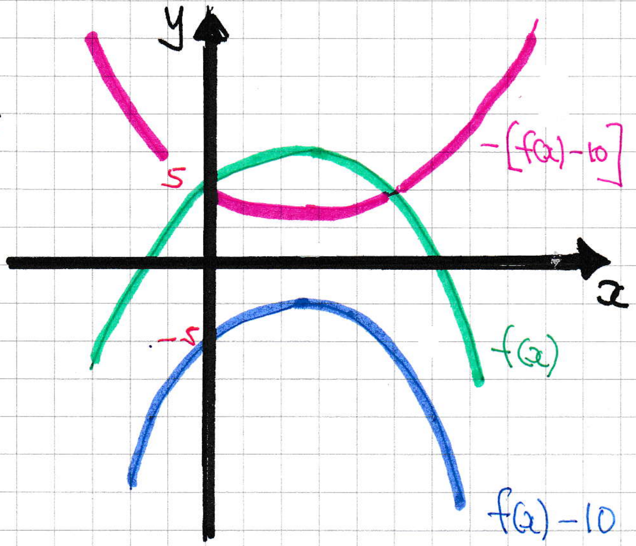
$$\implies \theta = 165°, 345°, 105°, 285°$$

$$\implies \theta = 105°, 165°, 285°, 345°$$

IYGB - MP1 PAPER S - QUESTION 9

FIRSTLY, BY INSPECTION, THE y INTERCEPT OF $f(x)$ IS 5, BY SETTING $x=0$

THEN WE TRANSFORM AS FOLLOWS



- TRANSLATE $f(x)$, "DOWN" BY 10 UNITS, I.E ONTO THE "BLUE" GRAPH $f(x)-10$
- THEN REFLECT THE GRAPH OF $f(x)-10$ ABOUT THE x AXIS ONTO THE "PINK" GRAPH WITH EQUATION $y=-[f(x)-10]$ OR $y=10-f(x)$

HENCE WE CAN OBTAIN THE EQUATION OF $g(x)$

$$\Rightarrow g(x) = 10 - f(x)$$

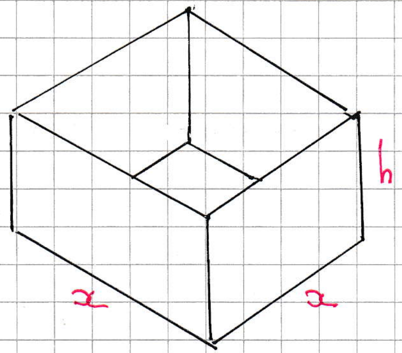
$$\Rightarrow g(x) = 10 - (x+1)(5-x)$$

$$\Rightarrow g(x) = 10 + (x+1)(x-5)$$

$$\Rightarrow g(x) = 10 + x^2 - 4x - 5$$

$$\Rightarrow g(x) = x^2 - 4x + 5$$

IYGB - MP1 PAPER S - QUESTION 10



THE BOX HAS TO HAVE A FIXED VOLUME

$$x^2 h = \text{constant volume}$$

$$x^2 h = V$$

THE SURFACE AREA OF THE BOX IS GIVEN BY

$$A = x^2 + 4xh$$

$$A = x^2 + \frac{4x^2 h}{x}$$

$$A = x^2 + \frac{4V}{x}$$

$$A = x^2 + 4Vx^{-1}$$

DIFFERENTIATING & SETTING TO ZERO

$$\frac{dA}{dx} = 2x - 4Vx^{-2}$$

$$0 = 2x - \frac{4V}{x^2}$$

$$\frac{4V}{x^2} = 2x$$

$$2x^3 = 4V$$

$$x^3 = 2V$$

$$x = \sqrt[3]{2V}$$

CHECK WHETHER THIS VALUE OF x PRODUCES A MINIMUM OR MAXIMUM

$$\frac{d^2A}{dx^2} = 2 + 8Vx^{-3} = 2 + \frac{8V}{x^3}$$

$$\left.\frac{d^2A}{dx^2}\right|_{x=\sqrt[3]{2V}} = 2 + \frac{8V}{2V} = 6 > 0$$

INDEED THIS VALUE OF x MINIMIZES

IYGB - MP1 PAPER S - QUESTION 10

USING $x^2h = V$

$$\Rightarrow h = \frac{V}{x^2}$$

$$\Rightarrow h = \frac{V}{(\sqrt[3]{2V})^2} = \frac{V}{\left[(2V)^{\frac{1}{3}}\right]^2} = \frac{V}{(2V)^{\frac{2}{3}}}$$

$$\Rightarrow h = \frac{V}{2^{\frac{2}{3}}V^{\frac{2}{3}}} = \frac{V^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{V^{\frac{1}{3}} \times 2^{\frac{1}{3}}}{2^{\frac{2}{3}} \times 2^{\frac{1}{3}}}$$

$$\Rightarrow h = \frac{2^{\frac{1}{3}}V^{\frac{1}{3}}}{2} = \frac{(2V)^{\frac{1}{3}}}{2}$$

HENCE $h = \frac{x}{2}$

$\therefore$ $x = 2h$ WHEN SURFACE IS MINIMUM

+ −

IYGB - MP1 PAPER S - QUESTION 11

ASSERTION: THE SQUARE OF AN ODD POSITIVE INTEGER IS ALWAYS OF THE FORM $8T+1$, WHERE "T" IS A TRIANGLE NUMBER

PROOF BY EXHUSTION

- LET n BE ODD

$\Rightarrow n = 2m+1$

$\Rightarrow 2n+1 = 2(2m+1)+1$

$\Rightarrow 2n+1 = 4m+3$

e.g. $7 = (4\times1)+3$
$35 = (4\times8)+3$
$47 = (4\times11)+3$
ETC

- LET n BE EVEN

$\Rightarrow n = 2m$

$\Rightarrow 2n+1 = 2(2m)+1$

$\Rightarrow 2n+1 = 4m+1$

e.g. $5 = (4\times1)+1$
$21 = (4\times5)+1$
$33 = (4\times8)+1$
ETC

- SQUARING THE ODD NUMBER IN EACH CASE YIELDS

$$(2n+1)^2 = (4m+3)^2$$
$$= 16m^2+24m+9$$
$$= 8(2m^2+3m+1)+1$$
$$= 8(2m+1)(m+1)+1$$

$$(2n+1)^2 = (4m+1)^2$$
$$= 16m^2+8m+1$$
$$= 8m(2m+1)+1$$

I.E IN BOTH CASES THE OUTCOME IS OF THE FORM $8f(m)+1$

IYGB - MP1 PAPER S - QUESTION 11

NOW TO PROVE THAT $f(m)$ IS A TRIANGLE NUMBER

- TRIANGLE NUMBERS ARE 1, 3, 6, 10, 15, 21, 28, 36, ...

- 1 , 6 , 15 , 28

$$\underbrace{\underbrace{1 \quad 6}_{5} \quad \underbrace{6 \quad 15}_{9} \quad \underbrace{15 \quad 28}_{13}}$$

$$5 \quad 9 \quad 13$$
$$\underbrace{5 \quad 9}_{4} \quad \underbrace{9 \quad 13}_{4}$$

$$u_n = 2n^2 + an + b$$

$2n^2$: 2, 8, 18, 32, ...

HERE: 1, 6, 15, 28

-1, -2, -3, -4

$$\therefore u_n = 2n^2 - n$$

$$\Rightarrow u_n = n(2n-1)$$

$$n \longmapsto m+1$$

$$\Rightarrow u_m = (m+1)\left[2(m+1)-1\right]$$

$$\Rightarrow u_m = (m+1)(2m+1)$$

WHICH WE OBTAINED

- 3 , 10 , 21 , 36

$$\underbrace{3 \quad 10}_{7} \quad \underbrace{10 \quad 21}_{11} \quad \underbrace{21 \quad 36}_{15}$$
$$\underbrace{7 \quad 11}_{4} \quad \underbrace{11 \quad 15}_{4}$$

$$u_n = 2n^2 + an + b$$

$2n^2$: 2, 8, 18, 28

HERE: 3, 10, 21, 36

+1, +2, +3, +4

$$\therefore u_n = 2n^2 + n$$

$$\Rightarrow u_n = n(2n+1)$$

OR

$$\Rightarrow u_m = m(2m+1)$$

WHICH WE OBTAINED

EVERY SQUARE OF AN ODD NATURAL NUMBER GREATER THAN 3, IS OF THE FORM $8T+1$, WHERE T IS A TRIANGLE NUMBER

IYGB - MP1 PAPER S - QUESTION 12

$$3e^{2(x+1)} - (2e)^x(e^4+9) + 3e^2 4^x = 0$$

- PROCEED AS FOLLOWS

$$\Longrightarrow 3e^{2x+2} - 2^x e^x (e^4+9) + 3e^2 4^x = 0$$

$$\Longrightarrow 3e^{2x}e^2 - 2^x e^x (e^4+9) + 3e^2(2^x)^2 = 0$$

- DIVIDE THE EQUATION THROUGH BY $3e^2e^x(2^x) \neq 0$

$$\Longrightarrow \frac{\cancel{3}e^{2x}\cancel{e^2}}{\cancel{3}\cancel{e^2}e^x(2^x)} - \frac{\cancel{2^x}\cancel{e^x}(e^4+9)}{3e^2\cancel{e^x}\cancel{(2^x)}} + \frac{\cancel{3}\cancel{e^2}(2^x)^2}{\cancel{3}\cancel{e^2}e^x(2^x)} = 0$$

$$\Longrightarrow \frac{e^x}{2^x} - \frac{e^4+9}{3e^2} + \frac{2^x}{e^x} = 0$$

- LET $A = \frac{e^x}{2^x}$ TO TRANSFORM THE ABOVE EQUATION INTO A QUADRATIC IN $\frac{e^x}{2^x}$

$$\Longrightarrow A - \frac{e^4+9}{3e^2} + \frac{1}{A} = 0$$

$$\Longrightarrow A^2 - A\left(\frac{e^4+9}{3e^2}\right) + 1 = 0$$

- COMPLETING THE SQUARE (OR ATTEMPT TO FACTORIZE)

$$\Longrightarrow \left[A - \frac{e^4+9}{6e^2}\right]^2 - \frac{(e^4+9)^2}{36e^4} + 1 = 0$$

$$\Longrightarrow \left[A - \frac{e^4+9}{6e^2}\right]^2 = \frac{(e^4+9)^2}{36e^4} - 1$$

IYGB - MP1 PAPER S - QUESTION 12

TIDY THE R.H.S FIRST

$$\frac{(e^4+9)^2}{36e^4}-1 = \frac{e^8+18e^4+81}{36e^4}-\frac{36e^4}{36e^4} = \frac{e^8-18e^4+81}{36e^4}$$

$$= \frac{(e^4-9)^2}{36e^4} = \left(\frac{e^4-9}{6e^2}\right)^2$$

THUS WE HAVE

$$\Rightarrow \left[A-\frac{e^4+9}{6e^2}\right]^2 = \left(\frac{e^4-9}{6e^2}\right)^2$$

$$\Rightarrow A-\frac{e^4+9}{6e^2} = \begin{cases} \dfrac{e^4-9}{6e^2} \\ \dfrac{9-e^4}{6e^2} \end{cases}$$

$$\Rightarrow A = \begin{cases} \dfrac{2e^4}{6e^2} \\ \dfrac{18}{6e^2} \end{cases}$$

$$\Rightarrow A = \begin{cases} \frac{1}{3}e^2 \\ 3e^{-2} \end{cases}$$

SOLVING EACH CASE SEPARATELY

$$\Rightarrow \frac{e^x}{2^x} = \frac{1}{3}e^2$$

$$\Rightarrow \left(\frac{1}{2}e\right)^x = \frac{1}{3}e^2$$

$$\Rightarrow \ln\left(\frac{1}{2}e\right)^x = \ln\left(\frac{1}{3}e^2\right)$$

$$\Rightarrow \frac{e^x}{2^x} = \frac{3}{e^2}$$

$$\Rightarrow \left(\frac{e}{2}\right)^x = \frac{3}{e^2}$$

$$\Rightarrow \ln\left(\frac{1}{2}e\right)^x = \ln\left(\frac{3}{e^2}\right)$$

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$\Longrightarrow x\ln(\frac{1}{2}e) = \ln\frac{1}{3}e^2$

$\Longrightarrow x[\ln\frac{1}{2} + \ln e] = \ln\frac{1}{3} + \ln e^2$

$\Longrightarrow x[-\ln 2 + 1] = -\ln 3 + 2$

$\Longrightarrow x = \frac{2 - \ln 3}{1 - \ln 2}$

$\Longrightarrow x\ln(\frac{1}{2}e) = \ln(\frac{3}{e^2})$

$\Longrightarrow x[\ln\frac{1}{2} + \ln e] = \ln 3 - \ln e^2$

$\Longrightarrow x[-\ln 2 + 1] = \ln 3 - 2$

$\Longrightarrow x = \frac{\ln 3 - 2}{1 - \ln 2}$

$\Longrightarrow x = -\frac{2 - \ln 3}{1 - \ln 2}$

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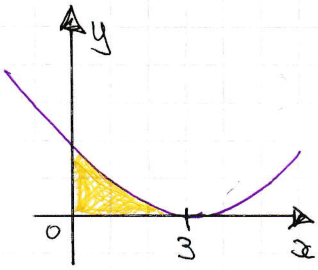
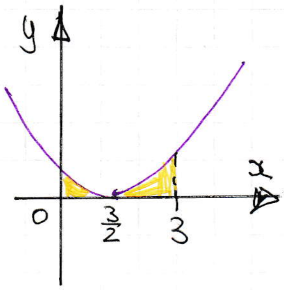
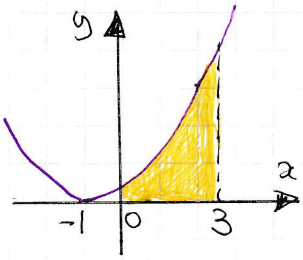
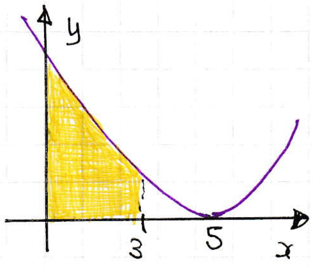
- START WITH SOME DIAGRAMS SHOWING SOME POSSIBLE CONFIGURATIONS









- GET AN EXPRESSION FOR THE AREA IN TERMS OF t, FOR $y=(x-t)^2$

$$\text{AREA} = \int_0^3 (x-t)^2\,dx = \int_0^3 x^2 - 2tx + t^2\,dx$$

$$= \left[\tfrac{1}{3}x^3 - tx^2 + t^2x\right]_0^3 = (9 - 9t + 3t^2) - (0)$$

- LET $f(t) = 3t^2 - 9t + 9$
- BY COMPLETING THE SQUARE (OR CALCULUS)

$$\text{AREA} = f(t) = 3\left[t^2 - 3t + 3\right]$$

$$= 3\left[\left(t-\tfrac{3}{2}\right)^2 - \tfrac{9}{4} + 3\right]$$

$$= 3\left[\left(t-\tfrac{3}{2}\right)^2 + \tfrac{3}{4}\right]$$

$$= 3\left(t-\tfrac{3}{2}\right)^2 + \frac{9}{4}$$

$\therefore$ MINIMUM AREA IS $\frac{9}{4}$

(IT OCCURS WHEN $t=\frac{3}{2}$, IE DIAGRAM 2)