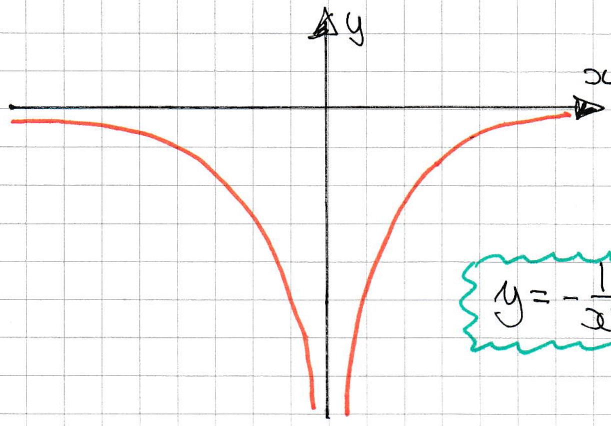
IYGB - MP1 PAPER U - QUESTION 1

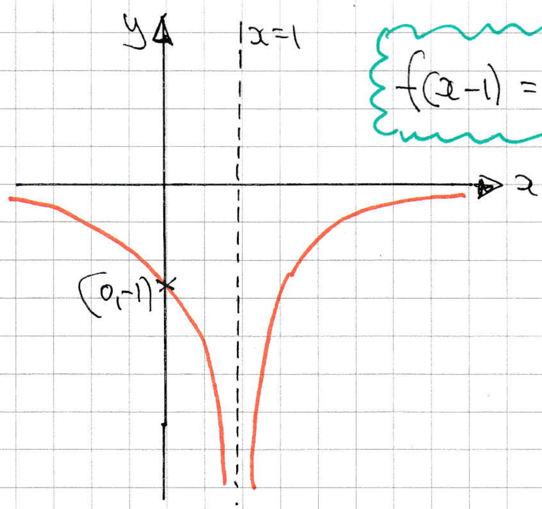
a)



$$y = -\frac{1}{x^2}$$

- THIS IS THE STANDARD $y = \frac{1}{x^2}$ "TURNED UPSIDE DOWN"
- ASYMPTOTES
  $x = 0$ (y AXIS)
  $y = 0$ (x AXIS)

b) I)

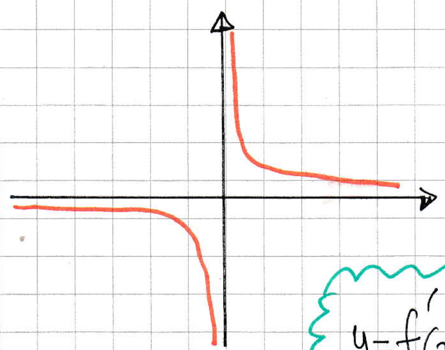


$$f(x-1) = -\frac{1}{(x-1)^2}$$

- THIS IS A TRANSLATION BY 1 UNIT TO THE "RIGHT"
- ASYMPTOTES
  $x = 1$
  $y = 0$ (x AXIS)

II)

$$f'(x) = \frac{d}{dx}\left(-\frac{1}{x^2}\right) = \frac{2}{x^3}$$



$$y = f'(x)$$

- THIS IN THE ABSENCE OF SCALE LOOKS LIKE $\frac{a}{x}$
- ASYMPTOTES
  $x = 0$ (y AXIS)
  $y = 0$ (x AXIS)

IYGB – MP1 PAPER U – QUESTION 2

FORMING THE EQUATION & TIDY TO A 3 TERM QUADRATIC

$$\Rightarrow px^2 + 4x(p+3) + 5p = -19$$

$$\Rightarrow \underbrace{p}_{a}x^2 + \underbrace{4(p+3)}_{b}x + \underbrace{(5p+19)}_{c} = 0$$

TWO DISTINCT ROOTS IMPLY $b^2-4ac>0$

$$\Rightarrow [4(p+3)]^2 - 4 \times p \times (5p+19) > 0$$

$$\Rightarrow 16(p+3)^2 - 4p(5p+19) > 0$$

$$\Rightarrow 4(p+3)^2 - p(5p+19) > 0$$

$$\Rightarrow 4(p^2+6p+9) - 5p^2 - 19p > 0$$

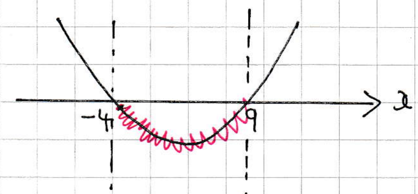
$$\Rightarrow 4p^2 + 24p + 36 - 5p^2 - 19p > 0$$

$$\Rightarrow -p^2 + 5p + 36 > 0$$

$$\Rightarrow p^2 - 5p - 36 < 0$$

$$\Rightarrow (p-9)(p+4) < 0$$

CRITICAL VALUES ARE 9 & −4



SO $-4 < p < 9$

BUT $p \neq 0$, OTHERWISE NO QUADRATIC

$$-4 < p < 0 \cup 0 < p < 9$$

IYGB - MP1 PAPER U - QUESTION 3

GETTING RID OF THE DENOMINATORS

$$\Rightarrow \frac{2+\cos 2x}{3+\sin^2 2x} = \frac{2}{5}$$

$$\Rightarrow 10+5\cos 2x = 6+2\sin^2 2x$$

USING $\cos^2 2x + \sin^2 2x \equiv 1$

$$\Rightarrow 10+5\cos 2x = 6+2(1-\cos^2 2x)$$

$$\Rightarrow 10+5\cos 2x = 6+2-2\cos^2 2x$$

$$\Rightarrow 2\cos^2 2x + 5\cos 2x + 2 = 0$$

$$\Rightarrow (2\cos 2x + 1)(\cos 2x + 2) = 0$$

$$\Rightarrow \cos 2x = \begin{cases} \cancel{-2} \\ -\frac{1}{2} \end{cases} \qquad -1 \leq \cos 2x \leq 1$$

PROCEED WITH SOLUTION

$$\arccos\left(-\tfrac{1}{2}\right) = 120°$$

$$\begin{cases} 2x = 120 \pm 360n \\ 2x = 240 \pm 360n \end{cases} \qquad n = 0,1,2,3,\ldots$$

$$\begin{cases} x = 60 \pm 180n \\ x = 120 \pm 180n \end{cases}$$

$$x = 60°, 240°, 120°, 300°$$

IYGB - MP1 PAPER U - QUESTION 4

a) $x-2$ IS A FACTOR OF $f(x)$; $2x+1$ IS A FACTOR OF $f(x)$

$f(2)=0$

$2\times2^3-9\times2^2+p\times2+q=0$

$16-36+2p+q=0$

$2p+q=20$

$q=20-2p$

$f(-\frac{1}{2})=0$

$2(-\frac{1}{2})^3-9\times(-\frac{1}{2})^2+p(-\frac{1}{2})+q=0$

$-\frac{1}{4}-\frac{9}{4}-\frac{1}{2}p+q=0$

$-\frac{5}{2}-\frac{1}{2}p+q=0$

$q=\frac{1}{2}p+\frac{5}{2}$

$20-2p=\frac{1}{2}p+\frac{5}{2}$

$40-4p=p+5$

$35=5p$

$p=7$

$q=6$

b) $2\sqrt{y}+\frac{7}{\sqrt{y}}=9-\frac{6}{y}$

LET $x=\sqrt{y}$

$\Rightarrow 2x+\frac{7}{x}=9-\frac{6}{x^2}$

$\Rightarrow 2x^3+7x=9x^2-6$

$\Rightarrow 2x^3-9x^2+7x+6=0$

THIS IS THE CUBIC OF PART (a) — FACTORIZE BY INSPECTION

$\Rightarrow (2x+1)(x-2)(x-3)=0$

$\Rightarrow x = 3,\ 2,\ -\frac{1}{2}$

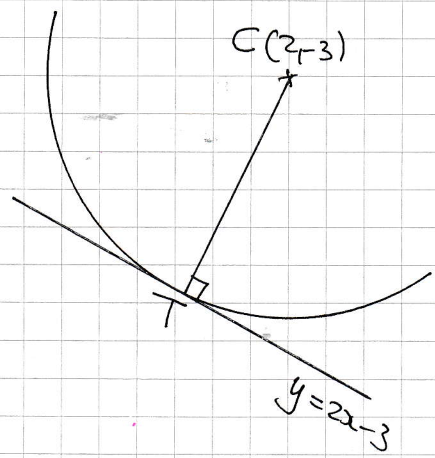
IYGB - MP1 PAPER U - QUESTION 4

$$\Rightarrow \sqrt{y} = 2, \; 3, \; \cancel{-\tfrac{1}{2}}$$

$$\Rightarrow y = 4, \; 9$$

IYGB - MP1 PAPER U - QUESTION 5

METHOD A - BY CO-ORDINATE GEOMETRY



- GRADIENT OF THE TANGENT IS 2
- GRADIENT CT MUST BE $-\frac{1}{2}$
- EQUATION OF LINE THROUGH C & T

$$y-(-3) = -\frac{1}{2}(x-2)$$
$$y+3 = -\frac{1}{2}(x-2)$$
$$2y+6 = -x+2$$
$$2y+x+4=0$$

- SOLVING SIMULTANEOUSLY WITH $y=2x-3$

$$2(2x-3)+x+4=0$$
$$5x-2=0$$
$$x=\frac{2}{5}$$

AND $y = 2\left(\frac{2}{5}\right)-3 = \frac{4}{5}-3 = -\frac{11}{5}$ $\quad\therefore \left(\frac{2}{5}, -\frac{11}{5}\right)$

- DISTANCE CT FINALLY, $C(2,-3)$ & $T\left(\frac{2}{5}, -\frac{11}{5}\right)$

$$d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$|CT| = \sqrt{\left(-3+\frac{11}{5}\right)^2+\left(\frac{2}{5}-2\right)^2}$$
$$r = \sqrt{\left(-\frac{4}{5}\right)^2+\left(-\frac{8}{5}\right)^2} = \sqrt{\frac{16}{25}+\frac{64}{25}} = \sqrt{\frac{80}{25}} = \frac{4}{5}\sqrt{5}$$

METHOD B - USING DISCRIMINANTS

- LET THE CIRCLE HAVE EQUATION

$$(x-2)^2+(y+3)^2=r^2$$

IYGB - MP1 PAPER U - QUESTION 5

- SOLVING SIMULTANEOUSLY WITH $y=2x-3$ TO "FIND" T

$$\Rightarrow (x-2)^2+(2x-3+3)^2=r^2$$
$$\Rightarrow (x-2)^2+(2x)^2=r^2$$
$$\Rightarrow x^2-4x+4+4x^2=r^2$$
$$\Rightarrow 5x^2-4x+(4-r^2)=0$$

- THIS EQUATION MUST PRODUCE REPEATED ROOTS AS THE POINT T IS A POINT OF TANGENCY

$$b^2-4ac=0 \Rightarrow (-4)^2-4\times5\times(4-r^2)=0$$
$$\Rightarrow 16-20(4-r^2)=0$$
$$\Rightarrow 16-80+20r^2=0$$
$$\Rightarrow 20r^2=64$$
$$\Rightarrow r^2=\frac{64}{20}=\frac{64}{100}\times5$$
$$\Rightarrow r=\frac{4}{5}\sqrt{5}$$

AS BEFORE

METHOD C - BY MINIMIZATION (COMPLETING THE SQUARE)

- CONSIDER A POINT ON THE LINE $y=2x-3$, IE $(x, 2x-3)$
- THE DISTANCE FROM $(x, 2x-3)$ TO THE CENTRE $(2,-3)$ IS GIVEN BY

$$\Rightarrow d=\sqrt{(x-2)^2+[2x\cancel{-3-(-3)}]^2}$$
$$\Rightarrow d=\sqrt{(x-2)^2+4x^2}$$
$$\Rightarrow d^2=x^2-4x+4+4x^2$$

IYGB - MP1 PAPER U - QUESTION 5

$$\Rightarrow d^2 = 5x^2 - 4x + 4$$

$$\Rightarrow d^2 = 5\left[x^2 - \frac{4}{5}x + \frac{4}{5}\right]$$

$$\Rightarrow d^2 = 5\left[\left(x - \frac{2}{5}\right)^2 - \frac{4}{25} + \frac{4}{5}\right]$$

$$\Rightarrow d^2 = 5\left(x - \frac{2}{5}\right)^2 - \frac{4}{5} + 4$$

$$\Rightarrow d^2 = 5\left(x - \frac{2}{5}\right)^2 + \frac{16}{5}$$

$\therefore$ MINIMUM VALUE OF d^2 IS $\frac{16}{5}$ (OCCURS AT $x = \frac{2}{5}$)

$$\therefore d_{MIN} = r = \sqrt{\frac{16}{5}} = \sqrt{\frac{16 \times 5}{25}} = \frac{4}{5}\sqrt{5}$$

IYGB - MP1 PAPER U - QUESTION 6

$$C: \; y = ax^{\frac{3}{2}} + bx^{\frac{1}{2}}, \; x>0 \quad \bullet \quad L: \; y = 8x - 32$$

- "L IS A TANGENT TO THE CURVE AT $x=4$"

- $x = 4$
  $y = 8 \times 4 - 32$
  $y = 0$

- GRADIENT OF L IS 8

- $y = ax^{\frac{3}{2}} - bx^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{3}{2}ax^{\frac{1}{2}} - \frac{1}{2}bx^{-\frac{1}{2}}$$

- WHEN $x=4$, $\frac{dy}{dx} = 8$

$$\Rightarrow 8 = \frac{3}{2}a \times 4^{\frac{1}{2}} - \frac{1}{2}b \times 4^{-\frac{1}{2}}$$

$$\Rightarrow 8 = 3a - \frac{1}{4}b$$

$$32 = 12a - b$$

- WHEN $x=0$, $y=0$

$$\Rightarrow 0 = a \times 4^{\frac{3}{2}} - b \times 4^{\frac{1}{2}}$$

$$\Rightarrow 0 = 8a - 2b$$

$$\Rightarrow 2b = 8a$$

$$\Rightarrow b = 4a$$

- SOLVING THE EQUATIONS YIELDS

$$\Rightarrow 32 = 12a - (4a)$$

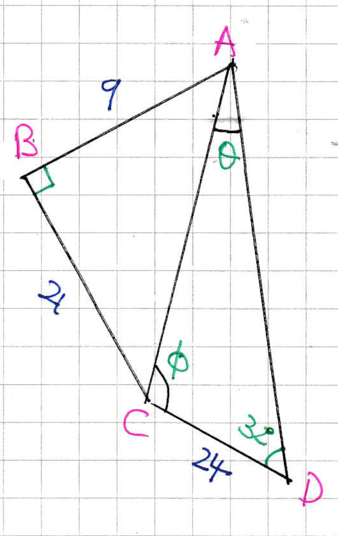
$$\Rightarrow 32 = 8a$$

$$\Rightarrow a = 4$$

&

$$b = 16$$

IYGB - MP1 PAPER U - QUESTION 7



LOOKING AT THE DIAGRAM

$\Rightarrow |AB|^2 + |BC|^2 = |AC|^2$

$\Rightarrow 9^2 + 21^2 = |AC|^2$

$\Rightarrow |AC|^2 = 522$

$\Rightarrow |AC| = \sqrt{522}$

BY THE SINE RULE ON $\hat{ACD}$

$$\frac{\sin\theta}{24} = \frac{\sin 32^\circ}{\sqrt{522}} \Rightarrow \sin\theta = \frac{24\sin 32^\circ}{\sqrt{522}}$$

$$\Rightarrow \sin\theta = 0.55665...$$

$$\Rightarrow \theta = 33.8247...^\circ$$

← NOT OBTUSE AS $\sqrt{522} \simeq 22.8$ WHICH IS COMPARABLE WITH 24

FINALLY FIND ϕ & THE AREA OF $\hat{ACD}$

$$\phi = 180 - (32 + 33.8247...)$$

$$\phi = 114.1753...^\circ$$

AREA OF $\hat{ACD}$ $= \frac{1}{2}|AC||CD|\sin\phi$

$$= \frac{1}{2}\sqrt{522} \times 24 \times \sin(114.1753...^\circ)$$

$$= 250.122... \text{ cm}^2$$

NEXT THE AREA OF $\hat{ABC}$

$$\text{AREA} = \frac{1}{2}|AB||BC| = \frac{1}{2} \times 9 \times 21 = 94.5 \text{ cm}^2$$

$\therefore$ REQUIRED AREA $= 250.122... + 94.5$

$$= 344.622...$$

$$\simeq 345 \text{ cm}^2$$

3 s.f.

IYGB - MP1 PAPER U - QUESTION 8

PROCEED AS FOLLOWS

$$\Rightarrow \frac{\log_4 x^2}{5+\log_4 x^2} + (\log_4 x)^2 = 0$$

$$\Rightarrow \frac{2\log_4 x}{5+2\log_4 x} + (\log_4 x)^2 = 0$$

LET $y = \log_4 x$

$$\Rightarrow \frac{2y}{5+2y} + y^2 = 0$$

$$\Rightarrow 2y + y^2(5+2y) = 0(5+2y)$$

$$\Rightarrow 2y + 5y^2 + 2y^3 = 0$$

$$\Rightarrow 2y^3 + 5y^2 + 2y = 0$$

$$\Rightarrow y(2y^2+5y+2) = 0$$

$$\Rightarrow y(2y+1)(y+2) = 0$$

$$\Rightarrow y = \begin{cases} 0 \\ -\frac{1}{2} \\ -2 \end{cases} \quad \text{I.E} \quad \log_4 x = \begin{cases} 0 \\ -\frac{1}{2} \\ -2 \end{cases}$$

INVERTING BY INSPECTION

$$x = \begin{cases} 4^0 \\ 4^{-\frac{1}{2}} \\ 4^{-2} \end{cases} \quad \text{IE} \quad x = \begin{cases} 1 \\ \frac{1}{2} \\ \frac{1}{16} \end{cases} \quad //$$

IYGB - MP1 PAPER U - QUESTION 9

$A(-1,4)$ • $B(2,3)$ • $C(8,1)$

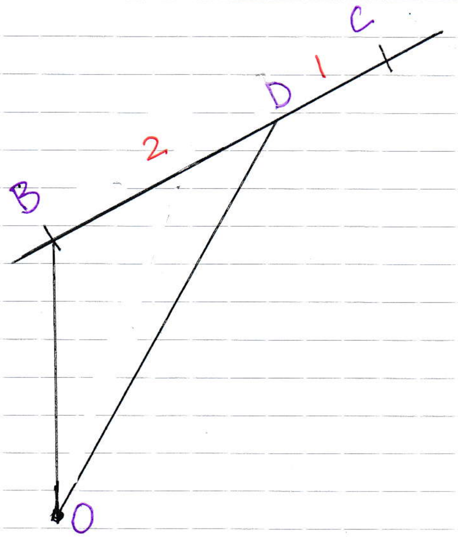
a) FIND THE VECTORS $\overrightarrow{AB}$ & $\overrightarrow{BC}$

$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}-1\\4\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix}$$

$$\overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix}8\\1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}6\\-2\end{pmatrix} = 2\begin{pmatrix}3\\-1\end{pmatrix}$$

AS $\overrightarrow{AB}$ & $\overrightarrow{BC}$ ARE IN THE SAME DIRECTION & SHARE THE POINT B, A, B & C MUST BE COLLINEAR.

b)



LOOKING AT THE DIAGRAM

$$\Longrightarrow \overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$$

$$\Longrightarrow \overrightarrow{OD} = \overrightarrow{OB} + \frac{2}{3}\overrightarrow{BC}$$

$$\Longrightarrow \underline{d} = \underline{b} + \frac{2}{3}(\underline{c} - \underline{b})$$

$$\Longrightarrow 3\underline{d} = 3\underline{b} + 2\underline{c} - 2\underline{b}$$

$$\Longrightarrow 3\underline{d} = \underline{b} + 2\underline{c}$$

$$\Longrightarrow 3\underline{d} = \begin{pmatrix}2\\3\end{pmatrix} + 2\begin{pmatrix}8\\1\end{pmatrix}$$

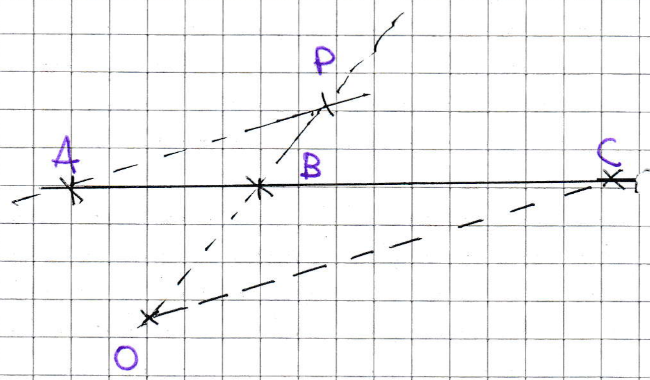
$$\Longrightarrow 3\underline{d} = \begin{pmatrix}18\\5\end{pmatrix}$$

$$\Longrightarrow \underline{d} = \begin{pmatrix}6\\5/3\end{pmatrix}$$

$$\therefore D\left(6, \tfrac{5}{3}\right)$$

IYGB - MP1 PAPER U - QUESTION 9

b) LOOKING AT THE DIAGRAM BELOW



$$\Rightarrow \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\Rightarrow \lambda \overrightarrow{OB} = \overrightarrow{OA} + \mu \overrightarrow{OC}$$

$$\Rightarrow \lambda \underline{b} = \underline{a} + \mu \underline{c}$$

$$\Rightarrow \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2\lambda \\ 3\lambda \end{pmatrix} = \begin{pmatrix} -1+8\mu \\ 4+\mu \end{pmatrix}$$

$$\Rightarrow \begin{array}{rcl} 2\lambda & = & -1 + 8\mu \\ -24\lambda & = & -32 - 8\mu \end{array}$$

$$\Rightarrow -22\lambda = -33$$

$$\Rightarrow \lambda = \frac{3}{2}$$

HENCE AS $\overrightarrow{OP} = \lambda \overrightarrow{OB}$

$$\overrightarrow{OP} = \frac{3}{2}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9/2 \end{pmatrix}$$

$$\therefore P\left(3, \frac{9}{2}\right)$$

IYGB - MP1 PAPER U - QUESTION 10

a) EXPANDING BY THE STANDARD BINOMIAL FORMULA

$$(6x-3)^8 = \binom{8}{0}(6x)^0(-3)^8 + \binom{8}{1}(6x)^1(-3)^7 + \binom{8}{2}(6x)^2(-3)^6 + \binom{8}{3}(6x)^3(-3)^5 + \dots$$

$$= (1 \times 1 \times 6561) + (8 \times 6x \times (-2187)) + (28 \times 36x^2 \times 729)$$

$$+ (56 \times 216x^3 \times (-243)) + \dots$$

$$= 6561 - 104976x + 734832x^2 - 2939328x^3 + \dots$$

b) PROCEED AS FOLLOWS

$$\frac{y+9}{3} = 6x+3$$

$$y+9 = 18x+9$$

$$18x = y$$

$$x = \frac{1}{18}y$$

USING PART (a)

$$\left[6\left(\frac{1}{18}y\right)-3\right]^8 = 6561 - \dots\dots - 2939328\left(\frac{1}{18}y\right)^3 + \dots$$

$$\left[\frac{y+9}{3}\right]^8 = 6561 - \dots\dots - 504y^3 + \dots$$

$$\therefore\ 504$$

c) WORK AS FOLLOWS

$$(\sqrt{2}z-1)^8(\sqrt{2}z+1)^8 = \left[(\sqrt{2}z-1)(\sqrt{2}z+1)\right]^8$$

$$= (2z^2-1)^8$$

$$= \frac{1}{3^8} \times 3^8 \times (2z^2-1)^8$$

IYGB - MP1 PAPER U - QUESTION 10

$$= \frac{1}{3^8} \times \left[3(2z^2-1)\right]^8$$

$$= \frac{1}{6561}\left[6z^2-3\right]^8$$

$$= \frac{1}{6561}\left[6561 - 104976(z^2) + 734832(z^2)^2 - 2939328(z^2)^3 + \dots\right]$$

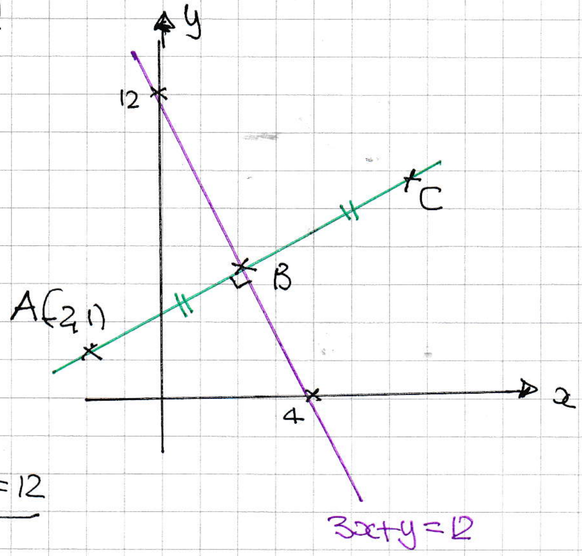
$$= \frac{1}{6561}\left[\dots - 2939328z^6 + \dots\right]$$

$$= \dots - 448z^6 + \dots$$

I.E -448

IYGB - MP1 PAPER U - QUESTION 11

a) • START WITH A DIAGRAM (NOT TO SCALE)



$$\Longrightarrow 3x + y = 12$$
$$\Longrightarrow y = -3x + 12$$

• GRADIENT OF ABC WILL BE $+\frac{1}{3}$

• EQUATION OF LINE ABC

$$y - 1 = \frac{1}{3}(x+2)$$

• SOLVING SIMULTANEOUSLY WITH $3x+y=12$

$$\left.\begin{array}{l} y-1 = \frac{1}{3}(x+2) \\ y = 12-3x \end{array}\right\} \Longrightarrow (12-3x)-1 = \frac{1}{3}(x+2)$$
$$\Longrightarrow 11-3x = \frac{1}{3}(x+2)$$
$$\Longrightarrow 33-9x = x+2$$
$$\Longrightarrow 31 = 10x$$
$$\Longrightarrow x = 3.1$$

$$\& \quad y = 12-3(3.1) = 12-9.3 = 2.7$$

THUS SO FAR $B(3.1, 2.7)$

• NOW THE REQUIRED POINT C MUST BE SUCH SO THAT B IS THE MIDPOINT OF AC

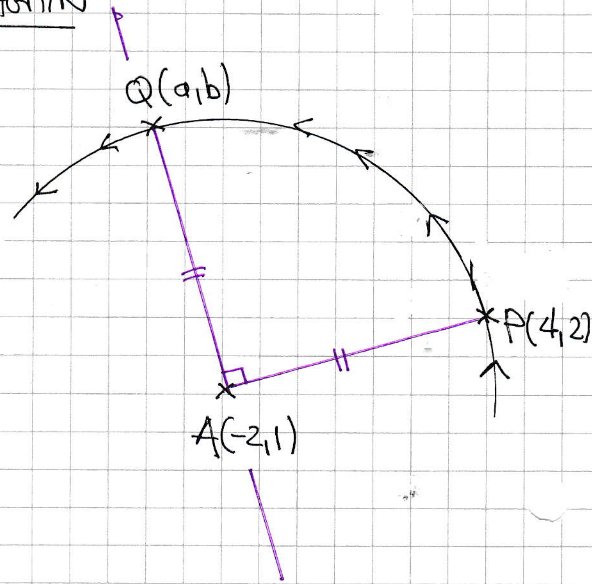
$$\underset{A}{\begin{pmatrix} -2 \\ 1 \end{pmatrix}} \xrightarrow[+1.7]{+5.1} \underset{B}{\begin{pmatrix} 3.1 \\ 2.7 \end{pmatrix}} \xrightarrow[+1.7]{+5.1} \underset{C}{\begin{pmatrix} 8.2 \\ 4.4 \end{pmatrix}}$$

$\therefore$ $C(8.2, 4.4)$

IYGB - MP1 PAPER U - QUESTION 11

b)

- STARTING WITH A DIAGRAM - AGAIN



- GRADIENT AP $= \frac{2-1}{4+2} = \frac{1}{6}$

- THUS GRADIENT AQ $= -6$

- $|AP| = \sqrt{(2-1)^2+(4+2)^2}$

  $|AP| = \sqrt{1+36}$

  $|AP| = \sqrt{37}$

- THE EQUATION OF THE LINE THROUGH A & Q IS

$$\Rightarrow y-1 = -6(x+2)$$

- THIS MUST BE SATISFIED BY Q(a,b)

$$\Rightarrow b-1 = -6(a+2)$$

- ALSO THE DISTANCE $|AQ| = \sqrt{37}$

$$\Rightarrow \sqrt{(a+2)^2+(b-1)^2} = \sqrt{37}$$

$$\Rightarrow (a+2)^2+(b-1)^2 = 37$$

$$\Rightarrow (a+2)^2+[-6(a+2)]^2 = 37$$

$$\Rightarrow (a+2)^2+36(a+2)^2 = 37$$

$$\Rightarrow 37(2+a)^2 = 37$$

$$\Rightarrow (a+2)^2 = 1$$

$$\Rightarrow a+2 = \begin{cases} 1 \\ -1 \end{cases}$$

$$\Rightarrow a = \begin{cases} -1 \\ -3 \end{cases} \qquad b = \begin{cases} -5 \\ 7 \end{cases} \qquad \text{ie} \quad \begin{matrix} \cancel{(-1,-5)} \\ (-3,7) \end{matrix}$$

$\therefore$ $Q(-3,7)$

IYGB – MP1 PAPER V – QUESTION 12

LOOKING AT THE FIRST EQUATION

$$\int_1^2 kx^2 + a \, dx = 11 \implies \left[\tfrac{1}{3}kx^3 + ax\right]_1^2 = 11$$

$$\implies \left(\tfrac{8}{3}k + 2a\right) - \left(\tfrac{1}{3}k + a\right) = 11$$

$$\implies \tfrac{7}{3}k + a = 11$$

LOOKING AT THE SECOND EQUATION

$$\int_1^k \frac{6}{x^2} \, dx = a \implies \left[-\frac{6}{x}\right]_1^k = a$$

$$\implies -\frac{6}{k} + 6 = a$$

SOLVING SIMULTANEOUSLY BY SUBSTITUTION

$$\implies \tfrac{7}{3}k + \left(-\tfrac{6}{k} + 6\right) = 11$$

$$\implies \tfrac{7}{3}k - \tfrac{6}{k} + 6 = 11 \qquad \times 3k$$

$$\implies 7k^2 - 18 + 18k = 33k$$

$$\implies 7k^2 - 15k - 18 = 0$$

$$\implies (7k+6)(k-3) = 0$$

$$k = \begin{cases} 3 \\ -\frac{6}{7} \end{cases}$$