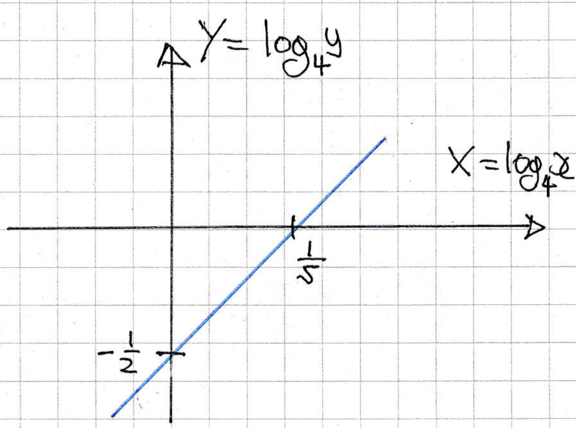
IYGB - MP1 PAPER W - QUESTION 1

STARTING FROM THE GRAPH



$$m = \frac{\Delta y}{\Delta x} = \frac{0 - \left(-\frac{1}{2}\right)}{\frac{1}{5} - 0} = \frac{\frac{1}{2}}{\frac{1}{5}} = \frac{5}{2}$$

$$\therefore \ Y = \frac{5}{2}X - \frac{1}{2}$$

$$2Y = 5X - 1$$

$$2\log_4 y = 5\log_4 x - 1$$

$$\log_4 y^2 = \log_4 x^5 - \log_4 4$$

$$\log_4 y^2 = \log_4\left(\frac{x^5}{4}\right)$$

$$y^2 = \frac{x^5}{4}$$

$$4y^2 = x^5$$

IYGB - MP1 PAPER W - QUESTION 2

a) $h = a + b\sin(30t)^\circ$

WHEN $t=2$, $h=9.5$ $\Longrightarrow$ $9.5 = a + b\sin 60^\circ$

WHEN $t=8$, $h=3.5$ $\Longrightarrow$ $3.5 = a + b\sin 240^\circ$

$$\Rightarrow \begin{cases} 9.5 = a + \frac{\sqrt{3}}{2}b \\ 3.5 = a - \frac{\sqrt{3}}{2}b \end{cases}$$

ADDING THE EQUATIONS YIELDS

$13 = 2a$

$a = 6.5$

FINALLY

$9.5 = 6.5 + \frac{\sqrt{3}}{2}b$

$3 = \frac{\sqrt{3}}{2}b$

$6 = \sqrt{3}b$

$b = 2\sqrt{3}$

b) USING THE FORMULA WITH $a = 6.5$ AND $b = 2\sqrt{3}$

$\Rightarrow h = 6.5 + 2\sqrt{3}\sin(30t)^\circ$

$\Rightarrow 5 = 6.5 + 2\sqrt{3}\sin(30t)^\circ$

$\Rightarrow -1.5 = 2\sqrt{3}\sin(30t)$

$\Rightarrow -\frac{\sqrt{3}}{4} = \sin(30t)$

$\arcsin\left(-\frac{\sqrt{3}}{4}\right) = -25.6589\ldots$

$$\Rightarrow \begin{cases} 30t = -25.6589\ldots \pm 360n \\ 30t = 205.6589\ldots \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\Rightarrow \begin{cases} t = -0.8553 \pm 12n \\ t = 6.8553 \pm 12n \end{cases}$$

$0.8553 \times 60 = 51.3\ldots$

$\therefore t = 6.8553$

$\therefore$ AT 06:51

IYGB - MP1 PAPER W - QUESTION 3

COMPLETING THE SQUARE IN x

$\Rightarrow f(x) = x^2 + 2kx - 15k^2$

$\Rightarrow f(x) = (x+k)^2 - k^2 - 15k^2$

$\Rightarrow f(x) = (x+k)^2 - 16k^2$

SOLVING THE EQUATION

$f(x) = 0$

$(x+k)^2 - 16k^2 = 0$

$(x+k)^2 - (4k)^2 = 0$

$(x+k-4k)(x+k+4k) = 0$

$(x-3k)(x+5k) = 0$

$x = \begin{cases} 3k \\ -5k \end{cases}$

OR

$f(x) = 0$

$(x+k)^2 - 16k^2 = 0$

$(x+k)^2 = 16k^2$

$x+k = \pm 4k$

$x = -k \pm 4k$

$x = \begin{cases} 3k \\ -5k \end{cases}$

IYGB – MP1 PAPER W – QUESTION 4

WRITE THE EXPRESSIONS IN TERMS OF C

$$f(x) = 4x^2 + a = 4x^2 - 2c$$
$$g(x) = x^2 + bx + a = x^2 - 3cx - 2c$$

AS $(x+c)$ IS A COMMON FACTOR, $f(-c) = g(-c) = 0$

$$4(-c)^2 - 2c = 0$$
$$4c^2 - 2c = 0$$
$$2c(2c-1) = 0$$
$$c = \begin{cases} 0 \\ \frac{1}{2} \end{cases}$$

$$(-c)^2 - 3c(-c) - 2c = 0$$
$$c^2 + 3c^2 - 2c = 0$$
$$4c^2 - 2c = 0$$
$$2c(2c-1) = 0$$
$$c = \begin{cases} 0 \\ \frac{1}{2} \end{cases}$$

IF $c=0$ THEN $a = b = 0$ & THE EXPRESSIONS ARE TRIVIAL, SINCE $f(x) = 4x^2$ & $g(x) = x^2$

$$\therefore\ c = \frac{1}{2} \implies a = -1$$
$$\implies b = -\frac{3}{2}$$

FINALLY

$$f(x) = 4x^2 - 1$$
$$f(x) = (2x-1)(2x+1)$$
$$f(x) = 4\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$$

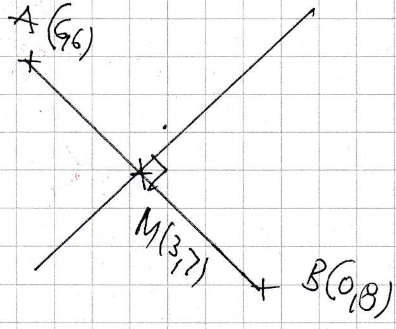
$$g(x) = x^2 - \frac{3}{2}x - 1$$
$$g(x) = \left(x + \frac{1}{2}\right)(x-2)$$

IYGB – MP1 PAPER W – QUESTION 5

a) OBTAIN GRADIENT & MIDPOINT OF AB

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{8-6}{0-6} = \frac{2}{-6} = -\frac{1}{3}$$

$$M_{AB}\left(\frac{6+0}{2}, \frac{6+8}{2}\right) = M(3,7)$$



EQUATION OF PERPENDICULAR BISECTOR

$$y - y_0 = m(x - x_0)$$

$$y - 7 = +3(x-3)$$

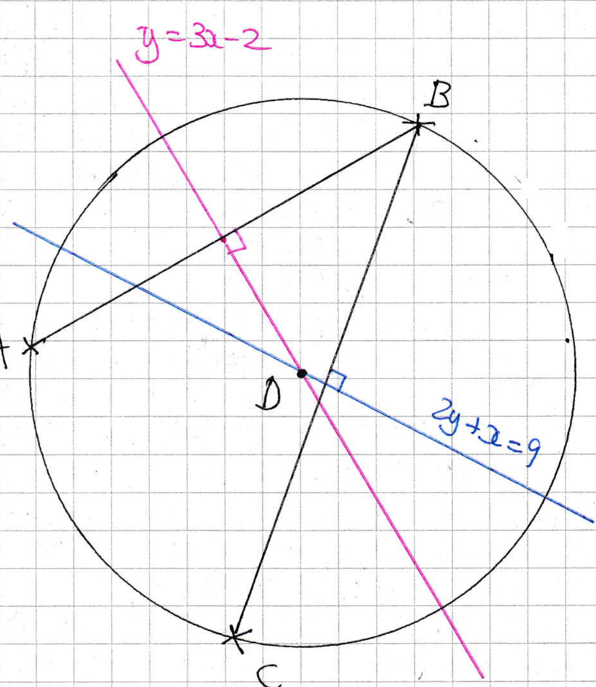
$$y - 7 = 3x - 9$$

$$y = 3x - 2$$

b) REPEAT THE PROCESS FOR B & C

$$m_{BC} = \frac{\Delta y}{\Delta x} = \frac{2-8}{-2-0} = \frac{-6}{-2} = 3$$

$$M_{BC}\left(\frac{0-2}{2}, \frac{8+2}{2}\right) = M(-1,5)$$



PERPENDICULAR BISECTOR OF BC

$$y - y_0 = m(x - x_0)$$

$$y - 5 = -\frac{1}{3}(x+1)$$

$$3y - 15 = -x - 1$$

$$3y + x = 14$$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} 3y + x = 14 \\ y = 3x - 2 \end{array}\right\} \Rightarrow 3(3x-2) + x = 14$$

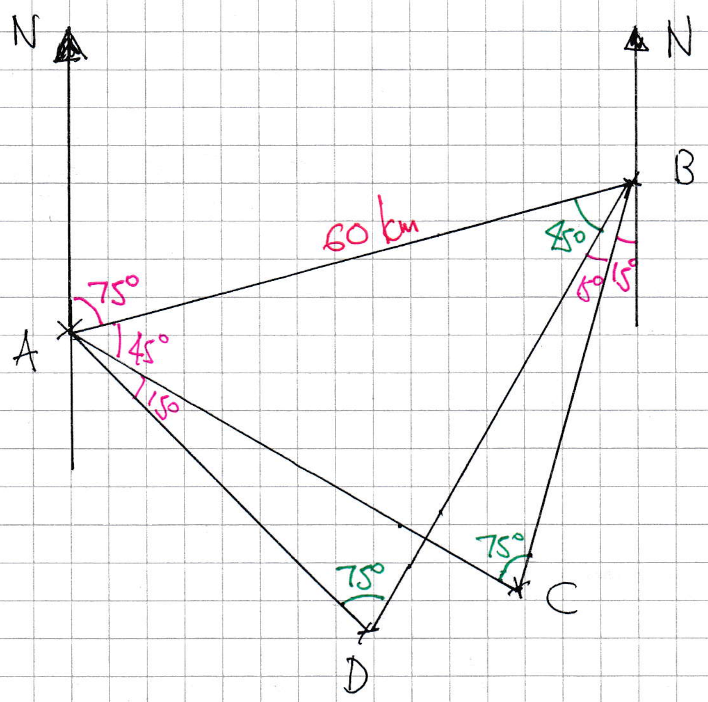
$$\Rightarrow 9x - 6 + x = 14$$

$$10x = 20$$

$$x = 2 \quad \& \quad y = 4 \qquad \therefore D(2,4)$$

IYGB - MP1 PAPER N - QUESTION 6

a) START WITH A DETAILED DIAGRAM



BY SIMPLE GEOMETRY

- $A\hat{B}D = 75° - 30° = 45°$
- $A\hat{C}B = 180 - (45° + 60°) = 75°$
- $A\hat{D}B = 180 - (60° + 45°) = 75°$

$\overset{\triangle}{ABD}$ & $\overset{\triangle}{ABC}$ HAVE ONE COMMON SIDE & ALL ANGLES THE SAME $(75°, 60°, 45°)$

$\therefore$ CONGRUENT

I) LOOKING AT $A\hat{B}C$ BY THE SINE RULE

$$\frac{|AB|}{\sin 75} = \frac{|BC|}{\sin 45} \Longrightarrow \frac{60}{\sin 75} = \frac{|BC|}{\sin 45}$$

$$\Longrightarrow |BC| = \frac{60 \sin 45}{\sin 75}$$

$$\Longrightarrow |BC| = 60(\sqrt{3}-1) \simeq 43.9 \text{ km}$$

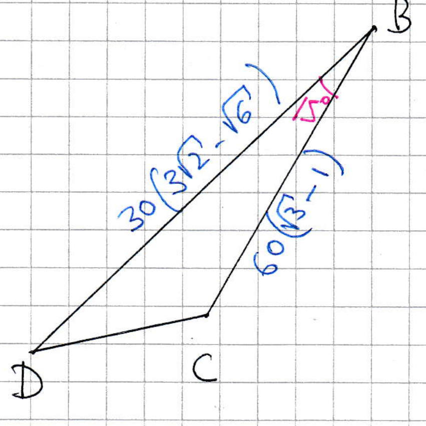
II) LOOKING AT $A\hat{B}D$ BY THE SINE RULE

$$\frac{|BD|}{\sin 60°} = \frac{|AB|}{\sin 75} \Longrightarrow |BD| = \frac{|AB| \sin 60°}{\sin 75°}$$

$$\Longrightarrow |BD| = \frac{60 \sin 60°}{\sin 75°} = 30(3\sqrt{2}-\sqrt{6}) \simeq 53.8 \text{ km}$$

IYGB – MP1 PAPER W – QUESTION 6

iii) DRAWING A DIAGRAM



BY THE COSINE RULE

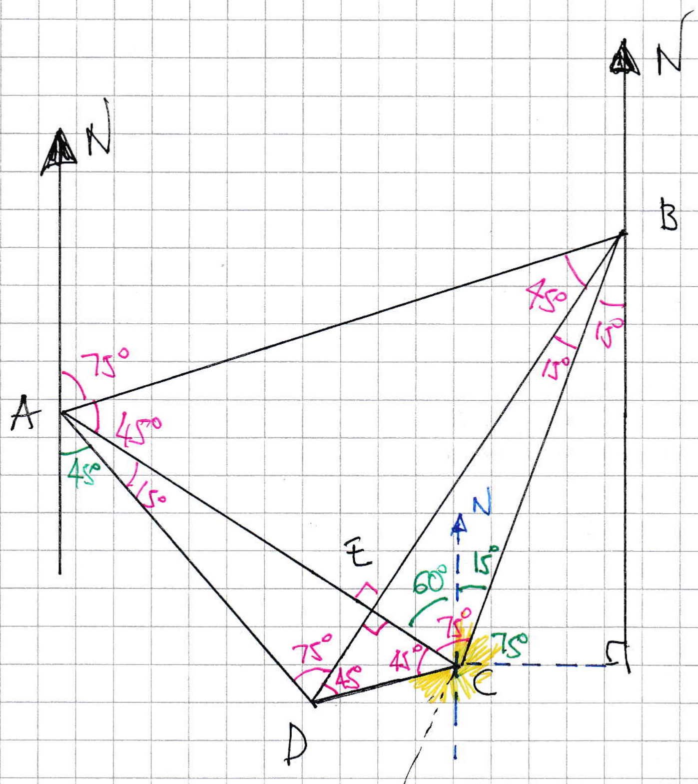
$$|DC|^2 = |DB|^2 + |BC|^2 - 2|DB||BC|\cos 15°$$

$$|DC|^2 = [53.7945...]^2 + [43.9230]^2 - 2(53.794..)(43.923)\cos 15$$

$$|DC|^2 = 258.515...$$

$$|DC| \simeq 16.1 \text{ km}$$

b) LOOKING AT AN ANGLE DIAGRAM



- TRIANGLE $D\hat{E}C$ IS RIGHT ANGLED & ISOSCELES
- BEARING IN YELLOW IS

$360 - (60° + 45°)$

$255°$

IYGB - MP1 PAPER N - QUESTION 7

C: $y = 4x^2 - 6x + 3$ • L: $2x - 4y + 3 = 0$

- START BY SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\Rightarrow 2x - 4[4x^2 - 6x + 3] + 3 = 0$$
$$\Rightarrow 2x - 16x^2 + 24x - 12 + 3 = 0$$
$$\Rightarrow 0 = 16x^2 + 26x + 9$$
$$\Rightarrow 0 = (2x - 1)(8x - 9)$$
$$\Rightarrow x = \begin{cases} \frac{1}{2} \\ \frac{9}{8} \end{cases}$$

- OBTAIN THE GRADIENT OF THE LINE

$$\Rightarrow 2x - 4y + 3 = 0$$
$$\Rightarrow 2x + 3 = 4y$$
$$\Rightarrow y = \underbrace{\tfrac{1}{2}}_{m = \frac{1}{2}}x + \frac{3}{4}$$

- EVALUATE THE GRADIENT AT THE INTERSECTION POINTS OF THE CURVE AND THE LINE

$$\frac{dy}{dx} = 8x - 6$$
$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = 8\left(\tfrac{1}{2}\right) - 6 = -2 \quad \longleftarrow \text{GRADIENT OF TANGENT AT } x = \tfrac{1}{2}$$

$\therefore$ L IS A NORMAL TO C AT $x = \frac{1}{2}$

IYGB - MP1 PAPER N - QUESTION 8

PROCEED AS FOLLOWS, SINCE $6125 = 49 \times 125$

$$6125^{\frac{1}{4}} + 5^{\frac{5}{4}} = (49 \times 125)^{\frac{1}{4}} + 5^{\frac{5}{4}}$$

$$= 49^{\frac{1}{4}} \times 125^{\frac{1}{4}} + 5^{\frac{5}{4}}$$

$$= (7^2)^{\frac{1}{4}} \times (5^3)^{\frac{1}{4}} + 5^{\frac{5}{4}}$$

$$= 7^{\frac{1}{2}} \times 5^{\frac{3}{4}} + 5^{\frac{5}{4}}$$

$$= \left[\left(7^{\frac{1}{2}} \times 5^{\frac{3}{4}} + 5^{\frac{5}{4}}\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\left(7^{\frac{1}{2}} \times 5^{\frac{3}{4}}\right)^2 + 2\left(7^{\frac{1}{2}} \times 5^{\frac{3}{4}}\right)\left(5^{\frac{5}{4}}\right) + \left(5^{\frac{5}{4}}\right)^2\right]^{\frac{1}{2}}$$

$$= \left[7 \times 5^{\frac{3}{2}} + 2 \times 7^{\frac{1}{2}} \times 5^2 + 5^{\frac{5}{2}}\right]^{\frac{1}{2}}$$

$$= \left(7 \times 5\sqrt{5} + 2 \times \sqrt{7} \times 25 + 5 \times 5 \times \sqrt{5}\right)^{\frac{1}{2}}$$

$$= \left(35\sqrt{5} + 50\sqrt{7} + 25\sqrt{5}\right)^{\frac{1}{2}}$$

$$= \left(60\sqrt{5} + 50\sqrt{7}\right)^{\frac{1}{2}}$$

$$= \left[10\left(6\sqrt{5} + 5\sqrt{7}\right)\right]^{\frac{1}{2}}$$

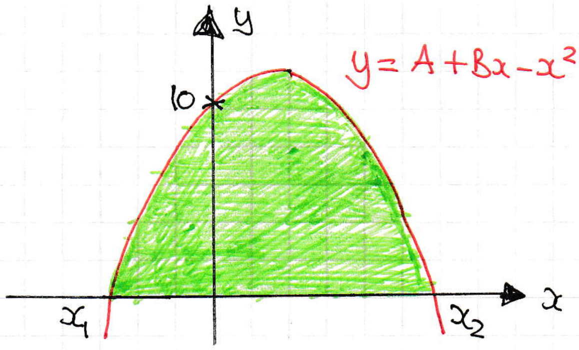
$$= \sqrt{10}\left(6\sqrt{5} + 5\sqrt{7}\right)^{\frac{1}{2}}$$

AS REQUIRED

IYGB - MP1 PAPER N - QUESTION 9

- STARTING WITH A SKETCH
- BY INSPECTION $A = 10$
- WRITING IN f NOTATION

$$f(x) = 10 + Bx - x^2$$



- "WHEN THE CURVE IS REFLECTED IN THE y AXIS IT IS THE SAME AS TRANSLATING THE CURVE BY 3 UNITS TO THE LEFT"

$$\Rightarrow f(-x) = f(x+3)$$

$$\Rightarrow \cancel{10} + B(-x) - (-x)^2 = \cancel{10} + B(x+3) - (x+3)^2$$

$$\Rightarrow -Bx - \cancel{x^2} = Bx + 3B - \cancel{x^2} - 6x - 9$$

$$\Rightarrow 0 = 2Bx - 6x + 3B - 9$$

$$\Rightarrow 0 = 2(B-3)x + 3(B-3)$$

$$\therefore B = 3$$

- HENCE WE HAVE

$$f(x) = 10 + 3x - x^2$$

$$-f(x) = x^2 - 3x - 10$$

$$-f(x) = (x+2)(x-5)$$

$$f(x) = (x+2)(5-x)$$

- FINALLY THE AREA CAN BE FOUND

$$\text{AREA} = \int_{-2}^{5} 10 + 3x - x^2 \, dx = \left[10x + \tfrac{3}{2}x^2 - \tfrac{1}{3}x^3 \right]_{-2}^{5}$$

$$= \left(50 + \tfrac{75}{2} - \tfrac{125}{3}\right) - \left(-20 + 6 + \tfrac{8}{3}\right) = \frac{343}{6}$$

IYGB - MP1 PAPER W - QUESTION 10

- IF WE ARE LOOKING FOR STATIONARY POINTS THEN THE INTERSECTION OF THE CURVE & THE HORIZONTAL LINE $y=k$ MUST PRODUCE REPEATED ROOTS

$$y = 1 - \frac{3x}{x^2-2x+4} \quad \& \quad y=k$$

$$\Rightarrow k = 1 - \frac{3x}{x^2-2x+4}$$

$$\Rightarrow \frac{3x}{x^2-2x+4} = 1-k$$

$$\Rightarrow (1-k)x^2 - 2(1-k)x + 4(1-k) = 3x$$

$$\Rightarrow (1-k)x^2 + (2k-2)x + (4-4k) = 3x$$

$$\Rightarrow (1-k)x^2 + (2k-5)x + (4-4k) = 0$$

- LOOKING FOR REPEATED ROOTS, SO $b^2-4ac=0$

$$\Rightarrow (2k-5)^2 - 4(1-k)(4-4k) = 0$$

$$\Rightarrow (2k-5)^2 - 4(k-1)(4k-4) = 0$$

$$\Rightarrow 4k^2-20k+25-4(4k^2-8k+4)=0$$

$$\Rightarrow 4k^2-20k+25-16k^2+32k-16=0$$

$$\Rightarrow -12k^2+12k+9=0$$

$$\Rightarrow 12k^2-12k-9=0$$

$$\Rightarrow 4k^2-4k-3=0$$

IYGB - MP1 PAPER W - QUESTION 10

$$\Rightarrow (2k-3)(2k+1)=0$$

$$\Rightarrow k = \begin{cases} -\frac{1}{2} \\ \frac{3}{2} \end{cases}$$

THESE ARE THE y CO-ORDINATES

- FINALLY LOOKING AT THE EQUATION $(1-k)x^2+(2k-5)x+(4-4k)=0$

IF $k=-\frac{1}{2}$

$\Rightarrow \frac{3}{2}x^2-6x+6=0$

$\Rightarrow 3x^2-12x+12=0$

$\Rightarrow x^2-4x+4=0$

$\Rightarrow (x-2)^2=0$

$\Rightarrow x=2$

$\therefore \left(2,-\frac{1}{2}\right)$

IF $k=\frac{3}{2}$

$\Rightarrow -\frac{1}{2}x^2-2x-2=0$

$\Rightarrow \frac{1}{2}x^2+2x+2=0$

$\Rightarrow x^2+4x+4=0$

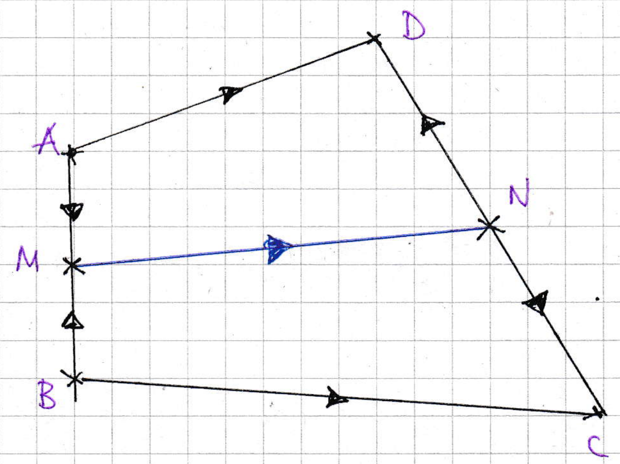
$\Rightarrow (x+2)^2$

$\Rightarrow x=-2$

$\therefore \left(-2,\frac{3}{2}\right)$

IYGB - MP1 PAPER W - QUESTION 11

STARTING WITH A DIAGRAM AND APPROACHING THE PROBLEM AS FOLLOWS...



$\vec{AD} = \vec{AM} + \vec{MN} + \vec{ND}$

$\vec{BC} = \vec{BM} + \vec{MN} + \vec{NC}$

ADDING THE EQUATIONS

$\vec{AD} + \vec{BC} = \vec{AM} + \vec{BM} + 2\vec{MN} + \vec{ND} + \vec{NC}$

BUT AS "M" & "N" ARE MIDPOINTS

$\vec{AM} + \vec{BM} = \vec{AM} - \vec{MB} = \vec{AM} - \vec{AM} =$ "ZERO VECTOR"

AND SIMILARLY

$\vec{ND} + \vec{NC} =$ "ZERO VECTOR"

HENCE WE NOW HAVE

$\Rightarrow \vec{AD} + \vec{BC} = 2\vec{MN}$

$\Rightarrow (\lambda^2 - 6\lambda + 10)\vec{MN} = 2\vec{MN}$

$\Rightarrow \lambda^2 - 6\lambda + 10 = 2$

$\Rightarrow \lambda^2 - 6\lambda + 8 = 0$

$\Rightarrow (\lambda - 4)(\lambda - 2) = 0$

$\Rightarrow \lambda = \begin{cases} 2 \\ 4 \end{cases}$ //

IYGB – MP1 PAPER W – QUESTION 12

REWRITE THIS AS A QUADRATIC IN e^x

$$\Rightarrow \quad e^x + e^{1-x} = e+1$$

$$\Rightarrow \quad e^x + \frac{e}{e^x} = e+1$$

$$\Rightarrow \quad (e^x)^2 + e = (e+1)e^x$$

$$\Rightarrow \quad e^{2x} - (e+1)e^x + e = 0$$

BY THE QUADRATIC FORMULA OR COMPLETING THE SQUARE

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 - \left(\frac{e+1}{2}\right)^2 + e = 0$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 - \frac{e^2+2e+1}{4} + e = 0$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 = \frac{e^2+2e+1}{4} - e$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 = \frac{e^2 - 2e+1-4e}{4}$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 = \frac{e^2-2e+1}{4}$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 = \frac{(e-1)^2}{4}$$

$$\Rightarrow \quad \left[e^x - \frac{e+1}{2}\right]^2 = \pm \frac{e-1}{2}$$

$$\Rightarrow \quad e^x = \frac{e+1}{2} \pm \frac{e-1}{2}$$

$$\Rightarrow \quad e^x = \begin{cases} e \\ 1 \end{cases}$$

$$\Rightarrow \quad x = \begin{cases} 1 \\ 0 \end{cases}$$