IYGB - MP1 PAPER X - QUESTION 1

a) EXPANDING IN THE "USUAL" MANNER

$$\left(2+\tfrac{1}{4}x\right)^8 = \binom{8}{0}(2)^8\left(\tfrac{1}{4}x\right)^0 + \binom{8}{1}(2)^7\left(\tfrac{1}{4}x\right)^1 + \binom{8}{2}(2)^6\left(\tfrac{1}{4}x\right)^2 + \binom{8}{3}(2)^5\left(\tfrac{1}{4}x\right)^3 + \dots$$

$$\left(2+\tfrac{1}{4}x\right)^8 = (1\times256\times1) + \left(8\times128\times\tfrac{1}{4}x\right) + \left(28\times64\times\tfrac{1}{16}x^2\right) + \left(56\times32\times\tfrac{1}{64}x^3\right) + \dots$$

$$\left(2+\tfrac{1}{4}x\right)^8 = 256 + 256x + 112x^2 + 28x^3 + \dots$$

b) START BY FINDING THE VALUE OF x WHICH PRODUCES $\frac{81}{40}$

$$\Rightarrow 2+\tfrac{1}{4}x = \frac{81}{40}$$

$$\Rightarrow \tfrac{1}{4}x = \frac{1}{40}$$

$$\Rightarrow x = \frac{1}{10} = 0.1$$

SUBSTITUTE $x=\frac{1}{10}$ IN THE EXPANSION

$$\Rightarrow \left(2+\tfrac{1}{4}\times\tfrac{1}{10}\right)^8 = 256 + 256\times\tfrac{1}{10} + 112\times\left(\tfrac{1}{10}\right)^2 + 28\times\left(\tfrac{1}{10}\right)^3 + \dots$$

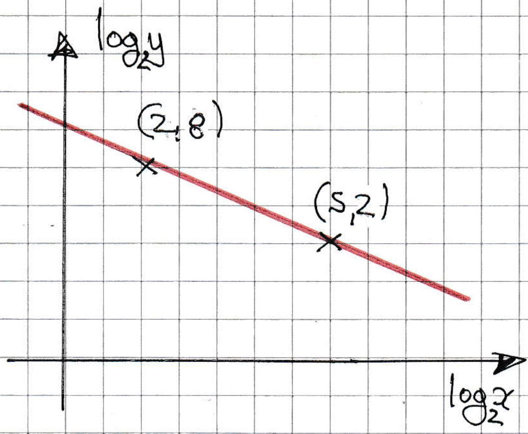
$$\Rightarrow \left(\frac{81}{40}\right)^8 = 256 + 25.6 + 1.12 + 0.028 + \dots$$

$$\Rightarrow \left(\frac{81}{40}\right)^8 = 282.748\dots$$

$$\therefore \left(\frac{81}{40}\right)^8 = 283 \quad (3\text{ sf})$$

IYGB – MP1 PAPER X – QUESTION 2

LOOKING AT THE GRAPH



- LET $Y = \log_2 y$ & $X = \log_2 x$
- GRADIENT $= \dfrac{Y_2 - Y_1}{X_2 - X_1} = \dfrac{2-8}{5-2} = -2$
- EQUATION

$$Y - Y_0 = m(X - X_0)$$
$$Y - 8 = -2(X - 2)$$
$$Y - 8 = -2X + 4$$
$$Y = 12 - 2X$$

REVERSING THE TRANSFORMATION

$$\log_2 y = 12 - 2\log_2 x$$
$$\log_2 y = 12\log_2 2 - \log_2 x^2$$
$$\log_2 y = \log_2 2^{12} - \log_2 x^2$$
$$\log_2 y = \log_2 4096 - \log_2 x^2$$
$$\log_2 y = \log_2 \left(\frac{4096}{x^2}\right)$$
$$y = \frac{4096}{x^2}$$

FINALLY WHEN $x = y$

$$y = \frac{4096}{y^2}$$
$$y^3 = 4096$$
$$y = 16$$

IYGB – MP1 PAPER X – QUESTION 3

MULTIPLY ACROSS & TIDY USING $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \frac{1-\cos\theta}{\sin\theta} = \sqrt{3}\sin\theta$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}\sin^2\theta$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}(1-\cos^2\theta)$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}-\sqrt{3}\cos^2\theta$$

$$\Rightarrow \sqrt{3}\cos^2\theta - \cos\theta + 1 - \sqrt{3} = 0$$

BY THE QUADRATIC FORMULA

$$\Rightarrow \cos\theta = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\times\sqrt{3}\times(1-\sqrt{3})}}{2\times\sqrt{3}}$$

$$\Rightarrow \cos\theta = \frac{1\pm\sqrt{1-4\sqrt{3}+12}}{2\sqrt{3}} = \begin{cases} 1 \\ -0.4226\ldots \end{cases}$$

SOLVING EACH CASE SEPARATELY

- $\cos\theta = 1$

  $\arccos(1) = 0^\circ$

  $\begin{cases}\theta = 0 \pm 360n \\ \theta = 360 \pm 360n\end{cases}$ $n = 0,1,2,3,\ldots$

- $\cos\theta = -0.4226\ldots$

  $\arccos(-0.4226\ldots) = 114.9988^\circ$

  $\begin{cases}\theta = 115.0 \pm 360n \\ \theta = 245.0 \pm 360n\end{cases}$ $n = 0,1,2,3,\ldots$

$\theta = 115^\circ$ IS THE ONLY SOLUTION IN RANGE

IYGB - MP1 PAPER X - QUESTION 4

a) LOOKING AT THE TWO FUNCTIONS

$$f(x) = \left(1 + \tfrac{1}{2}x\right)^4$$

$$f(6x) = \left[1 + \tfrac{1}{2}(6x)\right]^4 = (1+3x)^4 = g(x)$$

$$\therefore \quad g(x) = f(6x)$$

$\therefore$ HORIZONTAL STRETCH, BY SCALE FACTOR OF $\frac{1}{6}$

(OR STRETCH PARALLEL TO THE x AXIS, BY SCALE FACTOR OF $\frac{1}{6}$)

b) TRANSLATION BY THE VECTOR $\binom{2}{0}$ IS $f(x-2)$

$$f(x-2) = \left[1 + \tfrac{1}{2}(x-2)\right]^4$$

$$h(x) = \left[1 + \tfrac{1}{2}x - 1\right]^4$$

$$h(x) = \left(\tfrac{1}{2}x\right)^4$$

$$h(x) = \frac{1}{16}x^4$$

IYGB - MP1 PAPER X - QUESTION 5

REWRITE THE EQUATION "IN INDICES" & DIFFERENTIATE

$$\Longrightarrow y = x^2 - 6x\sqrt[3]{x} + 2$$

$$\Longrightarrow y = x^2 - 6x^1 x^{\frac{1}{3}} + 2$$

$$\Longrightarrow y = x^2 - 6x^{\frac{4}{3}} + 2$$

$$\Longrightarrow \frac{dy}{dx} = 2x - 8x^{\frac{1}{3}}$$

SOLVING FOR ZERO, SEEKING STATIONARY POINTS

$$\Longrightarrow 2x - 8x^{\frac{1}{3}} = 0$$

$$\Longrightarrow 2x = 8x^{\frac{1}{3}}$$

$$\Longrightarrow x = 4x^{\frac{1}{3}}$$

EITHER $x=0$ (BY INSPECTION) OR IF WE DIVIDE WE OBTAIN

$$\Longrightarrow x^{\frac{2}{3}} = 4$$

$$\Longrightarrow \left(\sqrt[3]{x}\right)^2 = 4$$

$$\Longrightarrow \sqrt[3]{x} = \begin{cases} 2 \\ -2 \end{cases}$$

$$\Longrightarrow x = \begin{cases} 8 \\ \cancel{-8} \end{cases} \qquad x \geqslant 0$$

FIND FIRST THE CORRESPONDING y CO-ORDINATES

$x=0$, $y=2$

$x=8$, $y = 8^2 - 6 \times 8^{\frac{4}{3}} + 2 = 64 - 6 \times 16 + 2 = 66 - 96 = -30$

$\therefore (0,2)$ & $(8,-30)$

IYGB - MP1 PAPER X - QUESTION 5

DETERMINING THE NATURE OF THESE POINTS BY USING THE SECOND DERIVATIVE TEST

$$\Rightarrow \frac{dy}{dx} = 2x - 8x^{\frac{1}{3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{8}{3}x^{-\frac{2}{3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{8}{3x^{\frac{2}{3}}}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=8} = 2 - \frac{8}{3\times 8^{\frac{2}{3}}} = 2 - \frac{8}{12} = \frac{4}{3} > 0$$

$\therefore$ $(8,-30)$ IS A LOCAL MINIMUM

$$\left.\frac{d^2y}{dx^2}\right|_{x=0} = 2 - \frac{8}{3\times 0^{\frac{2}{3}}} = 2 - \frac{8}{0}$$

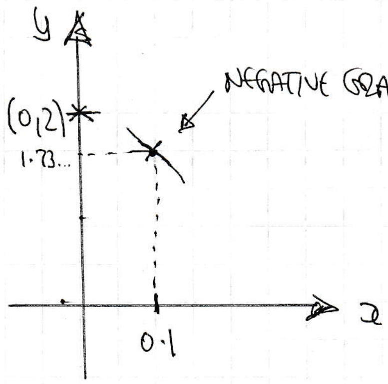
$\Leftarrow$ WE CANNOT USE THIS TEST WITHOUT KNOWLEDGE OF LIMITING TECHNIQUES

CHECKING EITHER THE GRADIENT OR THE VALUE OF y TO THE "RIGHT" OF $x=0$ (AS $x \geq 0$)

- $\left.\frac{dy}{dx}\right|_{x=0.1} \approx -3.51\ldots$

OR

- $y\big|_{x=0.1} \simeq 1.73\ldots$



$\therefore$ $(0,2)$ IS A LOCAL MAX

IYGB - MP1 PAPER X - QUESTION 6

$$f(x) \equiv 2x^2 + (4k+3)x + (2k-1)(k+2), \quad x \in \mathbb{R}$$

a) CALCULATE THE DISCRIMINANT OF THE QUADRATIC

$$\Delta = b^2 - 4ac = (4k+3)^2 - 4 \times 2 \times (2k-1)(k+2)$$
$$= 16k^2 + 24k + 9 - 8(2k^2 + 3k - 2)$$
$$= \cancel{16k^2} + \cancel{24k} + 9 - \cancel{16k^2} - \cancel{24k} + 16$$
$$= 25$$

b) THE EQUATION $f(x) = 0$, HAS TWO DISTINCT SOLUTIONS WHICH CAN BE FOUND BY THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(4k+3) \pm \sqrt{25}}{2 \times 2} = \frac{-4k - 3 \pm 5}{4}$$

THUS WE HAVE TWO POSSIBILITIES

- $x = \dfrac{-4k+2}{4}$

  $x = \dfrac{-2k+1}{2}$

  $2x = -2k+1$

  $2x + 2k - 1 = 0$

- $x = \dfrac{-4k-8}{4}$

  $x = -k-2$

  $x + k + 2 = 0$

$$\therefore \ f(x) = (2x+2k-1)(x+k+2)$$

IYGB - MP1 PAPER X - QUESTION 7

a) START WITH THE EQUATION OF THE CIRCLE

$\Rightarrow (x-6)^2+(y-2)^2=4^2$ CENTRE $(6,2)$, RADIUS 4

$\Rightarrow (x-6)^2+(y-2)^2=16$

$\Rightarrow (6+2\sqrt{2}-6)^2+(y-2)^2=16$

$\Rightarrow (2\sqrt{2})^2+(y-2)^2=16$

$\Rightarrow 8+(y-2)^2=16$

$\Rightarrow (y-2)^2=8$

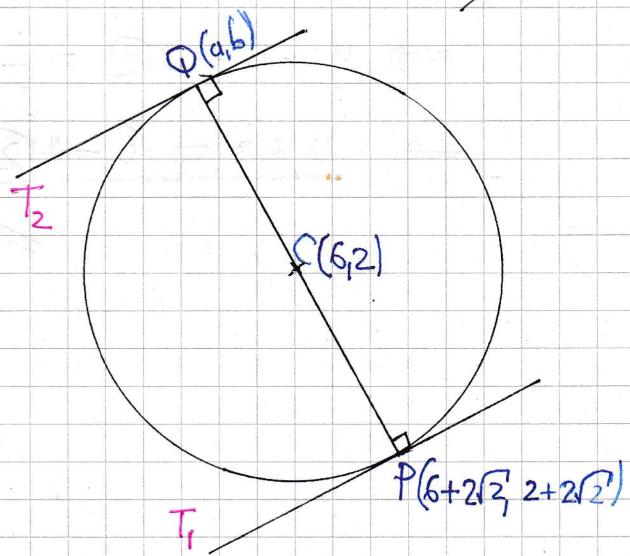
$\Rightarrow y-2 = \begin{cases} \sqrt{8} \\ -\sqrt{8} \end{cases}$

$\Rightarrow y = \begin{cases} 2+\sqrt{8} \\ \cancel{2-\sqrt{8}} \end{cases}$

$\therefore k = 2+\sqrt{8}$

$k = 2+2\sqrt{2}$, $k>0$

b) LOOKING AT THE DIAGRAM



$\dfrac{a+6+2\sqrt{2}}{2}=6$ $\qquad$ $\dfrac{b+2+2\sqrt{2}}{2}=2$

$a+6+2\sqrt{2}=12$ $\qquad$ $b+2+2\sqrt{2}=4$

$a = 6-2\sqrt{2}$ $\qquad$ $b=2-2\sqrt{2}$

$\therefore Q(6-2\sqrt{2}, 2-2\sqrt{2})$

GRADIENT OF PC

$m=\dfrac{\Delta y}{\Delta x} = \dfrac{2+2\sqrt{2}-2}{6+2\sqrt{2}-6} = \dfrac{2\sqrt{2}}{2\sqrt{2}} = 1$

$\therefore$ GRADIENT OF T_1 IS THE NEGATIVE RECIPROCAL, IE -1

IYGB - MP1 PAPER X - QUESTION 7

HENCE WE HAVE THE EQUATION OF T_1

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - (2 + 2\sqrt{2}) = -1(x - (6 + 2\sqrt{2}))$$

$$\Rightarrow y - 2 - 2\sqrt{2} = -(x - 6 - 2\sqrt{2})$$

$$\Rightarrow y - 2 - 2\sqrt{2} = -x + 6 + 2\sqrt{2}$$

$$\Rightarrow y + x = 8 + 4\sqrt{2}$$

AND SIMILARLY THE EQUATION OF T_2

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - (2 - 2\sqrt{2}) = -(x - (6 - 2\sqrt{2}))$$

$$\Rightarrow y - 2 + 2\sqrt{2} = -(x - 6 + 2\sqrt{2})$$

$$\Rightarrow y - 2 + 2\sqrt{2} = -x + 6 - 2\sqrt{2}$$

$$\Rightarrow y + x = 8 - 4\sqrt{2}$$

IYGB - MP1 PAPER X - QUESTION 8

PROCEED AS FOLLOWS

$$a^3 + 5a = a^3 - a + 6a$$
$$= a(a^2-1) + 6a$$
$$= a(a-1)(a+1) + 6a$$
$$= (a-1)a(a+1) + 6a$$

NOW $(a-1)a(a+1)$ REPRESENTS 3 CONSECUTIVE INTEGERS

- AT LEAST ONE OF THESE IS EVEN (DIVISIBLE BY 2)
- ONE OF THEM IS A MULTIPLE OF 3

HENCE THE EXPRESSION $(a-1)a(a+1)$ IS DIVISIBLE BY 6

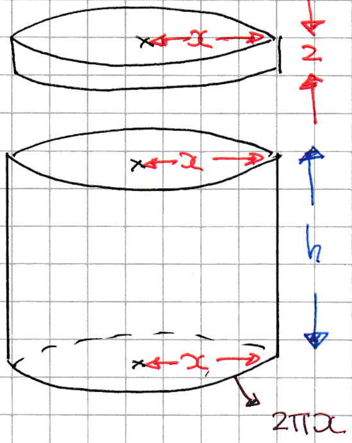
FINALLY WE HAVE

$$a^3 + 5a = \dots (a-1)a(a+1) + 6a$$
$$= 6b + 6a \quad \text{, FOR SOME INTEGER } b$$
$$= 6(b+a)$$

INDEED DIVISIBLE BY 6

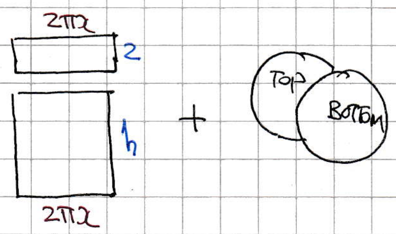
IYGB - MP1 PAPER X - QUESTION 9

a)



CONSTRAINT ON SURFACE AREA

$\Longrightarrow \quad A = 190\pi$

$\Longrightarrow$ 

$= 190\pi$

$\Longrightarrow \quad \cancel{2\pi}x(h+2) + 2(\cancel{\pi}x^2) = 190\cancel{\pi}$

$\Longrightarrow \quad 2x(h+2) + 2x^2 = 190$

$\Longrightarrow \quad x(h+2) + x^2 = 95$

$\Longrightarrow \quad xh + 2x + x^2 = 95$

$\Longrightarrow \quad \boxed{xh = 95 - 2x - x^2}$

VOLUME OF THE JAR

$\Longrightarrow \quad V = \pi r^2 h$

$\Longrightarrow \quad V = \pi x^2 h$

$\Longrightarrow \quad V = \pi x(xh)$

$\Longrightarrow \quad V = \pi x(95 - 2x - x^2)$

$\Longrightarrow \quad V = \pi(95x - 2x^2 - x^3)$

AS REQUIRED

IYGB – MP1 PAPER X – QUESTION 9

b) DIFFERENTIATE & SOLVE FOR ZERO

$$\Rightarrow V = \pi(95x - 2x^2 - x^3)$$

$$\Rightarrow \frac{dV}{dx} = \pi(95 - 4x - 3x^2)$$

$$\Rightarrow 0 = \cancel{\pi}(95 - 4x - 3x^2)$$

$$\Rightarrow 3x^2 - 4x - 95 = 0$$

$$\Rightarrow (3x + 19)(x - 5) = 0$$

$$\Rightarrow x = \begin{cases} 5 \\ \cancel{-\frac{19}{3}} \end{cases}$$

c) USING THE 2ND DERIVATIVE TEST

$$\frac{dV}{dx} = \pi(95 - 4x - 3x^2)$$

$$\frac{d^2V}{dx^2} = \pi(-4 - 6x)$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=5} = -34\pi < 0$$

INDEED A MAXIMUM

d) $V = \pi(95x - 2x^2 - x^3)$

$$V_{MAX} = \pi(95 \times 5 - 2 \times 5^2 - 5^3)$$

$$V_{MAX} = 300\pi \simeq 943 \text{ cm}^3$$

IYGB – MP1 PAPER X – QUESTION 10

USING THE RULES OF LOGARITHMS

- $2\log_2 x - \log_2 y = 1$

$\Rightarrow \log_2 x^2 - \log_2 y = 1 \times \log_2 2$

$\Rightarrow \log_2\left(\frac{x^2}{y}\right) = \log_2 2$

$\Rightarrow \frac{x^2}{y} = 2$

$\Rightarrow x^2 = 2y$

- $\log_2(4x\sqrt{y}) = 1$

$\Rightarrow \log_2(4x\sqrt{y}) = 1 \times \log_2 2$

$\Rightarrow \log_2(4x\sqrt{y}) = \log_2 2$

$\Rightarrow 4x\sqrt{y} = 2$

$\Rightarrow 16x^2 y = 4$

$\Rightarrow x^2 = \frac{1}{4y}$

$2y = \frac{1}{4y}$

$\Rightarrow y^2 = \frac{1}{8}$

$\Rightarrow y = +\sqrt{\frac{1}{8}}$ (OTHERWISE $\log_2 y$ IS NOT DEFINED)

$\Rightarrow y = +8^{-\frac{1}{2}}$

$\Rightarrow y = (2^3)^{-\frac{1}{2}}$

$\Rightarrow y = 2^{-\frac{3}{2}}$

NOW WE CAN OBTAIN x

$\Rightarrow x^2 = 2y$

$\Rightarrow x^2 = 2 \times 2^{-\frac{3}{2}}$

$\Rightarrow x^2 = 2^{-\frac{1}{2}}$

$\Rightarrow x = +\sqrt{2^{-\frac{1}{2}}}$ (OTHERWISE $\log_2 x$ IS NOT DEFINED)

$\Rightarrow x = +(2^{-\frac{1}{2}})^{\frac{1}{2}}$

$\Rightarrow x = 2^{-\frac{1}{4}}$

IYGB – MP1 PAPER X – QUESTION 10

ALTERNATIVE METHOD

$2\log_2 x - \log_2 y = 1$

$\log_2(4x\sqrt{y}) = 1$

$\log_2 4 + \log_2 x + \log_2 \sqrt{y} = 1$

$\log_2 2^2 + \log_2 x + \log_2 y^{\frac{1}{2}} = 1$

$2\cancel{\log_2 2} + \log_2 x + \frac{1}{2}\log_2 y = 1$

$2 + \log_2 x + \frac{1}{2}\log_2 y = 1$

$4 + 2\log_2 x + \log_2 y = 2$

$2\log_2 x + \log_2 y = -2$

NOW LET $X = \log_2 x$ AND $Y = \log_2 y$

$2X - Y = 1$ & $2X + Y = -2$

ADDING YIELDS

$4X = -1$

$X = -\frac{1}{4}$

$\log_2 x = -\frac{1}{4}$

$x = 2^{-\frac{1}{4}}$

SUBTRACTING YIELDS

$2Y = -3$

$Y = -\frac{3}{2}$

$\log_2 y = -\frac{3}{2}$

$y = 2^{-\frac{3}{2}}$

IYGB - MP1 PAPER X - QUESTION 11

OBTAIN THE x CO-ORDINATES OF A & B

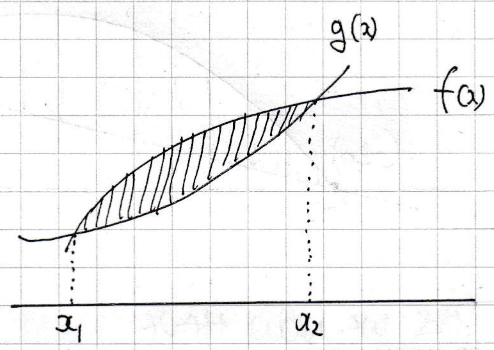
$$\left.\begin{array}{l} y = 3x-6 \\ y = 4x - x^2 \end{array}\right\} \Rightarrow 3x - 6 = 4x - x^2$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$\Rightarrow x = \begin{cases} -2 \\ 3 \end{cases}$$

THE REQUIRED AREA IS GIVEN BY



$$\int_{x_1}^{x_2} f(x) - g(x)\, dx$$

$$= \int_{-2}^{3} (4x - x^2) - (3x-6)\, dx$$

$$= \int_{-2}^{3} 4x - x^2 - 3x + 6\, dx$$

$$= \int_{-2}^{3} 6 + x - x^2\, dx$$

$$= \left[6x + \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 \right]_{-2}^{3}$$

$$= \left(18 + \tfrac{9}{2} - 9\right) - \left(-12 + 2 + \tfrac{8}{3}\right)$$

$$= \frac{27}{2} - \left(-\frac{22}{3}\right)$$

$$= \frac{125}{6}$$

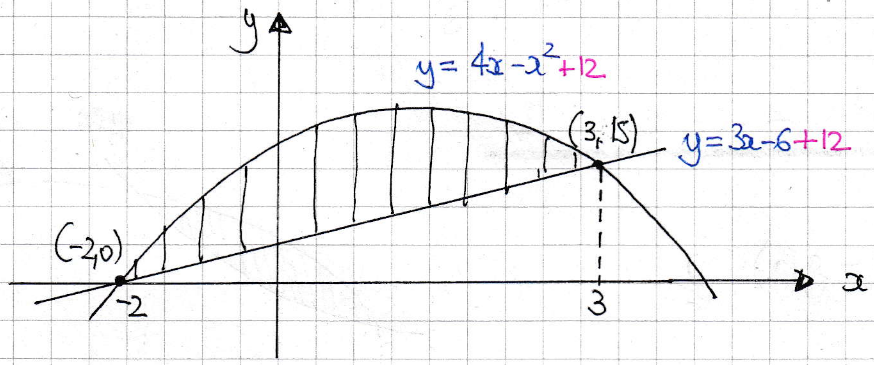
AS REQUIRED

IYGB – MP1 PAPER X – QUESTION 11

ALTERNATIVE APPROACH BY TRANSFORMATIONS – FIND THE x CO-ORDINATES OF A & B AS BEFORE.

$$x = \left\langle\begin{matrix} -2 \\ 3 \end{matrix}\right. \qquad \text{USING } y = 3x - 6 \qquad y = \left\langle\begin{matrix} -12 \\ 3 \end{matrix}\right.$$

TRANSLATE BOTH OBJECTS "UP" BY 12 UNITS

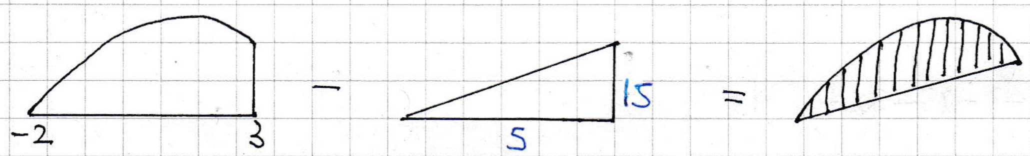


$(-2, -12)$ & $(3, 3)$

$\downarrow \qquad \downarrow$

$(-2, 0)$ & $(3, 15)$

THUS WE NOW HAVE



$\frac{1}{2} \times 5 \times 15 = \frac{75}{2}$

$$\int_{-2}^{3} 4x - x^2 + 12 \, dx = \left[2x^2 - \tfrac{1}{3}x^3 + 12x\right]_{-2}^{3} = (18 - 9 + 36) - \left(8 + \tfrac{8}{3} - 24\right)$$

$$= 45 - \left(-\tfrac{40}{3}\right)$$

$$= \frac{175}{3}$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\frac{175}{3} - \frac{75}{2} = \frac{125}{6}$$

AS BEFORE

IYGB - MP1 PAPER X - QUESTION 12

FIND THE INTERSECTIONS OF l_1 & l_2 WITH $y=-1$

l_1: $y = 3x$

$-1 = 3x$

$x = -\frac{1}{3}$

$\therefore A\left(-\frac{1}{3}, -1\right)$

l_2: $3x + 2y = 13$

$3x + 2(-1) = 13$

$3x = 15$

$x = 5$

$\therefore B(5,-1)$

FIND THE CO-ORDINATES OF P

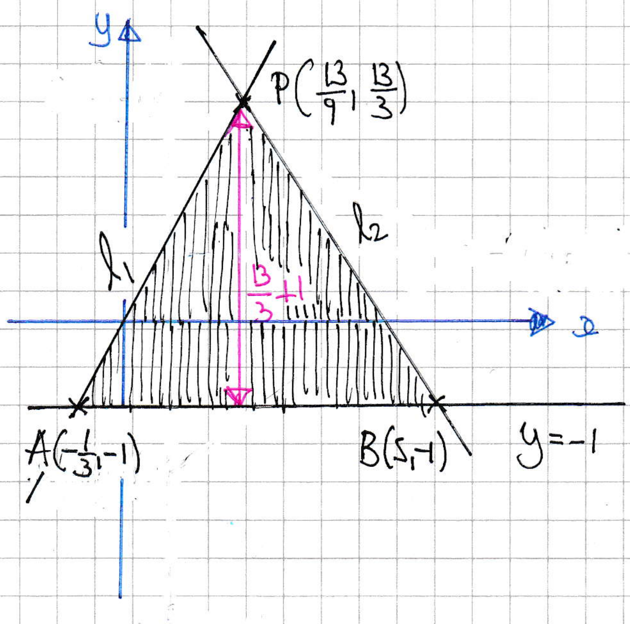
$$\left.\begin{array}{l} y = 3x \\ 3x + 2y = 13 \end{array}\right\} \Rightarrow 3x + 2(3x) = 13$$

$9x = 13$

$x = \frac{13}{9}$ & $y = \frac{13}{3}$

$\therefore P\left(\frac{13}{9}, \frac{13}{3}\right)$

LOOKING AT A DIAGRAM



AREA = $\frac{1}{2}$ × BASE × HEIGHT

$= \frac{1}{2} \times \left(5+\frac{1}{3}\right) \times \left(\frac{13}{3}+1\right)$

$= \frac{1}{2} \times \frac{16}{3} \times \frac{16}{3}$

$= \frac{128}{9}$ // AS REQUIRED