IYGB – MP2 PAPER V – QUESTION 1

MANIPULATE AS FOLLOWS

$$\sum_{r=13}^{30}\left[(-2)^r-4r-78\right] = \sum_{r=1}^{30}\left[(-2)^r-4r-78\right] - \sum_{r=1}^{12}\left[(-2)^r-4r-78\right]$$

SPLIT EACH SUMMATION INTO ARITHMETIC & GEOMETRIC PARTS

$$= \left[\sum_{r=1}^{30}(-2)^r - \sum_{r=1}^{30}(4r+78)\right] - \left[\sum_{r=1}^{12}(-2)^r - \sum_{r=1}^{12}(4r+78)\right]$$

G.P

$-2+4-8+16-\dots$

$$S_n = \frac{a(r^n-1)}{r-1}$$

A.P.

$82+86+90+\dots$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

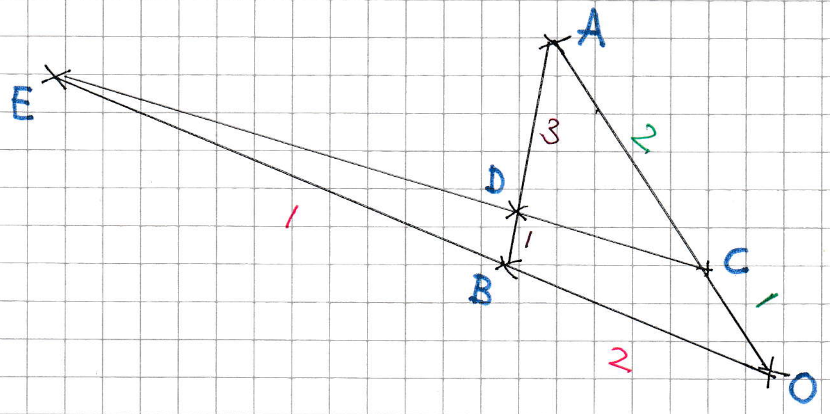
$$= \left[\frac{-2\left[(-2)^{30}-1\right]}{-2-1} - \frac{30}{2}\left[164+29\times4\right]\right] - \left[\frac{-2\left[(-2)^{12}-1\right]}{-2-1} - \frac{12}{2}\left[164+11\times4\right]\right]$$

$$= (715827882 - 4200) - (2730 - 1248)$$

$$= 715{,}822{,}200$$

IYGB - MP2 PAPER U - QUESTION 2

a) START BY FINDING THE POSITION VECTORS OF C, D & E



A(−6, 27, −9)
B(4, 6, −6)

- $\vec{OE} = 3\vec{OB} = 3(4,6,-6) = (12,18,-18)$ IE $E(12,18,-18)$
- $\vec{OC} = \frac{1}{3}\vec{OA} = \frac{1}{3}(-6,27,-9) = (-2,9,-3)$ IE $C(-2,9,-3)$
- $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \frac{1}{4}\vec{BA}$

$$= \vec{OB} + \frac{1}{4}(\vec{BO} + \vec{OA}) = \vec{OB} + \frac{1}{4}\vec{BO} + \frac{1}{4}\vec{OA}$$

$$= \vec{OB} - \frac{1}{4}\vec{OB} + \frac{1}{4}\vec{OA} = \frac{3}{4}\vec{OB} + \frac{1}{4}\vec{OA}$$

$$= \frac{3}{4}(4,6,-6) + \frac{1}{4}(-6,27,-9) = \left(\frac{3}{2}, \frac{45}{4}, -\frac{27}{4}\right) \quad \text{IE } D\left(\frac{3}{2}, \frac{45}{4}, -\frac{27}{4}\right)$$

b) DETERMINE THE VECTORS $\vec{CD}$ & $\vec{DE}$

$$\vec{CD} = \underline{d} - \underline{c} = \left(\frac{3}{2}, \frac{45}{4}, -\frac{27}{4}\right) - (-2,9,-3) = \left(\frac{7}{2}, \frac{9}{4}, -\frac{15}{4}\right)$$

$$\vec{DE} = \underline{e} - \underline{d} = (12,18,-18) - \left(\frac{3}{2}, \frac{45}{4}, -\frac{27}{4}\right) = \left(\frac{21}{2}, \frac{27}{4}, -\frac{45}{4}\right)$$

$$\vec{CD} = \frac{1}{4}(14,9,-15)$$

$$\vec{DE} = \frac{3}{4}(14,9,-15)$$

AS BOTH $\vec{CD}$ & $\vec{DE}$ ARE IN THE SAME DIRECTION & SHARE THE POINT D, IMPLIES THAT C, D & E ARE COLLINEAR

$$|\vec{CD}| : |\vec{DE}|$$
$$\frac{1}{4} : \frac{3}{4}$$
$$1 : 3$$

IYGB – MP2 PAPER V – QUESTION 2

c) SIMILARLY COMPARE $\vec{CB}$ & $\vec{AE}$

$$\begin{cases} \vec{CB} = \underline{b} - \underline{c} = (4,6,-6) - (-2,9,-3) = (6,-3,-3) \\ \vec{AE} = \underline{e} - \underline{a} = (12,18,-18) - (-6,27,-9) = (18,-9,-9) \end{cases}$$

$$\vec{CB} = (6,-3,-3) = 3(2,-1,-1)$$
$$\vec{AE} = (18,-9,-9) = 9(2,-1,-1)$$

AS $\vec{CB}$ & $\vec{AE}$ ARE IN THE SAME DIRECTION, CB IS PARALLEL TO AE

$$|\vec{CB}| : |\vec{AE}|$$
$$3 : 9$$
$$1 : 3$$

IYGB - MP2 PAPER U - QUESTION 3

PROCEED AS FOLLOWS

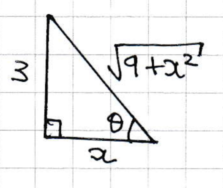
$$\Longrightarrow \underbrace{2\arctan\left(\frac{3}{x}\right)}_{\theta} = \underbrace{\arcsin\left(\frac{6x}{25}\right)}_{\phi}$$

$\theta = \arctan\frac{3}{x}$

$\tan\theta = \frac{3}{x}$

$\phi = \arcsin\left(\frac{6x}{25}\right)$

$\sin\phi = \frac{6x}{25}$



$\sin\theta = \frac{3}{\sqrt{9+x^2}}$

$\cos\theta = \frac{x}{\sqrt{9+x^2}}$

& $\sin\phi = \frac{6x}{25}$

$$\Longrightarrow 2\theta = \phi$$

$$\Longrightarrow \sin 2\theta = \sin\phi$$

$$\Longrightarrow 2\sin\theta\cos\theta = \sin\phi$$

USING THE VALUES FROM ABOVE

$$\Longrightarrow 2\left(\frac{3}{\sqrt{9+x^2}}\right)\left(\frac{x}{\sqrt{9+x^2}}\right) = \frac{6x}{25}$$

$$\Longrightarrow \frac{6x}{9+x^2} = \frac{6x}{25}$$

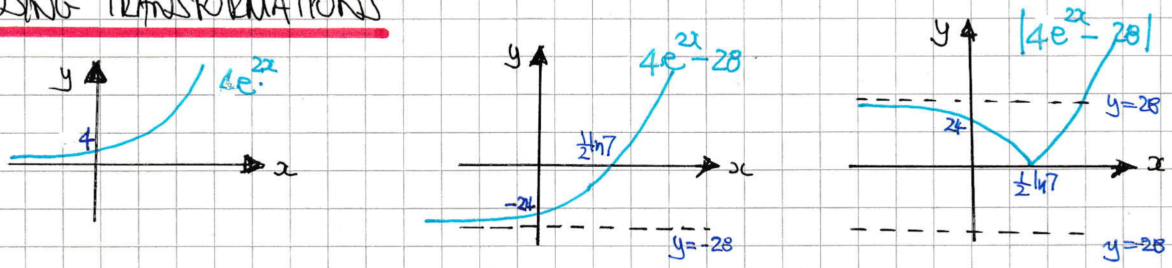
$$\Longrightarrow 9+x^2 = 25$$

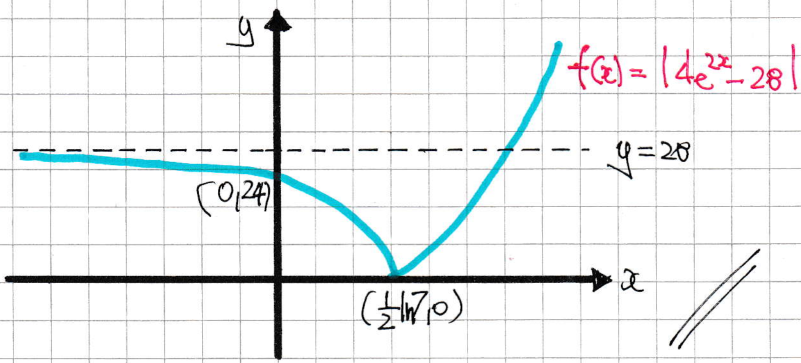
$$\Longrightarrow x^2 = 16$$

$$\Longrightarrow x = \pm 4$$

IYGB - MP2 PAPER V - QUESTION 4

a) USING TRANSFORMATIONS



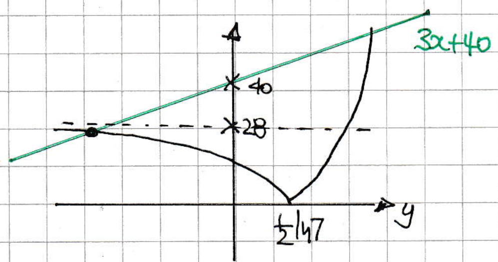


b) PROCEED AS FOLLOWS

$\Rightarrow \quad f(x) - 40 = 3x$

$\Rightarrow \quad f(x) = 3x + 40$

$\Rightarrow \quad |4e^{2x} - 28| = 3x + 40$



LOOKING AT THE DIAGRAM OPPOSITE

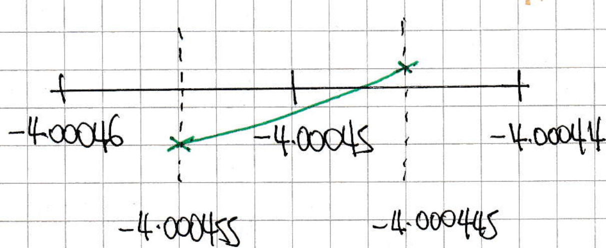
$\Rightarrow \quad 28 - 4e^{2x} = 3x + 40$

$\Rightarrow \quad 0 = 4e^{2x} + 3x + 12$

LET $g(x) = 4e^{2x} + 3x + 12$

$g(-4.000455) = -0.000024 < 0$

$g(-4.000445) = +0.000006 > 0$



$\Rightarrow -4.000455 < \alpha < -4.000445$

$\alpha \simeq -4.00045$

IYGB - MP2 PAPER V - QUESTION 4

c) FOR THE POSITIVE ROOT

$$\Rightarrow f(x) - 40 = 3x$$
$$\Rightarrow f(x) = 3x + 40$$
$$\Rightarrow |4e^{2x} - 28| = 3x + 40$$
$$\Rightarrow 4e^{2x} - 28 = 3x + 40$$
$$\Rightarrow 4e^{2x} - 3x - 68 = 0$$

LET $h(x) = 4e^{2x} - 3x - 68$

x	1	2
$h(x)$	$-41.44\ldots$	$144.39\ldots$

$\therefore$ ROOT BETWEEN 1 & 2

TRY A SIMPLE REARRANGEMENT

$$\Rightarrow 4e^{2x} - 3x - 68 = 0$$
$$\Rightarrow 4e^{2x} = 3x + 68$$
$$\Rightarrow e^{2x} = \frac{3x+68}{4}$$
$$\Rightarrow 2x = \ln\left(\frac{3x+68}{4}\right)$$
$$\Rightarrow x = \frac{1}{2}\ln\left(\frac{3x+68}{4}\right)$$

USING $x_{n+1} = \frac{1}{2}\ln\left(\frac{3x_n+68}{4}\right)$

$$\Rightarrow x_1 = 1.5$$
$$\Rightarrow x_2 = 1.448646\ldots$$
$$\Rightarrow x_3 = 1.447582\ldots$$
$$\Rightarrow x_4 = 1.44760\ldots$$
$$\Rightarrow x_5 = 1.447599\ldots$$
$$\Rightarrow x_6 = 1.447599\ldots$$

$\therefore b = 1.44760$ 5 d.p.

IYGB - MP2 PAPER V - QUESTION 5

a) EXPAND IN TERMS OF k & n, UP TO x^3

$$(1+kx)^n = 1 + \frac{n}{1}(kx)^1 + \frac{n(n-1)}{1\times 2}(kx)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(kx)^3 + O(x^4)$$

$$(1+kx)^n = 1 + \underbrace{nk}_{\text{NEGATIVE}}x + \underbrace{\tfrac{1}{2}n(n-1)k^2}_{12}x^2 + \underbrace{\tfrac{1}{6}n(n-1)(n-2)k^3}_{32}x^3 + O(x^4)$$

FORMING & SOLVING EQUATIONS

$$\left.\begin{aligned}\tfrac{1}{2}n(n-1)k^2 &= 12\\ \tfrac{1}{6}n(n-1)(n-2)k^3 &= 32\end{aligned}\right\} \Rightarrow \begin{aligned}n(n-1)k^2 &= 24\\ n(n-1)(n-2)k^3 &= 192\end{aligned}$$

DIVIDING THE EQUATIONS YIELDS

$$\Rightarrow \frac{n(n-1)(n-2)k^3}{n(n-1)k^2} = \frac{192}{24} \qquad n\neq 0,\ k\neq 0,\ n\neq 1$$

$$\Rightarrow k(n-2) = 8$$

$$\Rightarrow k = \frac{8}{n-2}$$

SUBSTITUTE INTO $n(n-1)k^2 = 24$

$$\Rightarrow n(n-1)\left(\frac{8}{n-2}\right)^2 = 24$$

$$\Rightarrow \frac{64n(n-1)}{(n-2)^2} = 24$$

$$\Rightarrow 64n^2 - 64n = 24(n-2)^2$$

$$\Rightarrow 64n^2 - 64n = 24(n^2-4n+4)$$

$$\Rightarrow 64n^2 - 64n = 24n^2 - 96n + 96$$

$$\Rightarrow 40n^2 + 32n - 96 = 0$$

$$\Rightarrow 5n^2 + 4n - 12 = 0$$

IYGB - MP2 PAPER V - QUESTION 5

$\Rightarrow \quad (5n-6)(n+2)$

$\Rightarrow \quad n = \begin{cases} 6/5 \\ -2 \end{cases}$

$\& \quad k = \dfrac{8}{n-2} = \begin{cases} \dfrac{8}{\frac{6}{5}-2} = -10 \\ \dfrac{8}{-4} = -2 \end{cases}$

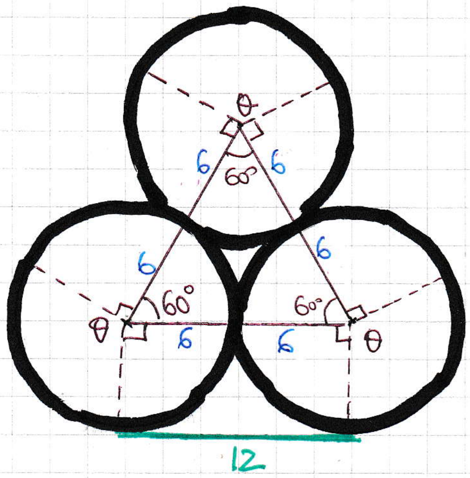
EITHER $\quad n = \frac{6}{5}, \; k = -10$

OR $\quad \cancel{n=-2, \; k=-2}$

$nk<0$, GIVEN

IYGB - MP2 PAPER V - QUESTION 6

LOOKING AT THE DIAGRAM BELOW AND WORKING IN RADIANS



- $\theta = 2\pi - \left(\frac{\pi}{2} \times 2\right) - \frac{\pi}{3}$

  $\theta = \frac{2\pi}{3}$

- ARC LENGTH "$r\theta^c$"

  $6 \times \frac{2\pi}{3} = 4\pi$

- NOT IN CONTACT WITH THE CIRCLES (ONE SECTION) IS 12

TOTAL LENGTH IS $(4\pi + 12) \times 3 = 12\pi + 36$

$= 12(\pi + 3)$ // AS REQUIRED

IYGB - MP2 PAPER V - QUESTION 7

DIFFERENTIATE NOTING THAT $\frac{d}{dx}(\ln x) = \frac{1}{x}$ & $\frac{d}{dx}(\tan x) = \sec^2 x$

$$y = \ln\left[\tan\left(x+\frac{\pi}{4}\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{\tan\left(x+\frac{\pi}{4}\right)} \times \sec^2\left(x+\frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = \frac{\cos(x+\pi/4)}{\sin(x+\pi/4)} \times \frac{1}{\cos^2\left(x+\frac{\pi}{4}\right)}$$

$$\frac{dy}{dx} = \frac{1}{\sin\left(x+\frac{\pi}{4}\right)\cos\left(x+\frac{\pi}{4}\right)}$$

EXPAND USING THE COMPOUND ANGLES

$$\frac{dy}{dx} = \frac{1}{\left[\sin x\cos\frac{\pi}{4} + \cos x\cos\frac{\pi}{4}\right]\left[\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}\right]}$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x\right)\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)}$$

$\cos\frac{\pi}{4} = \sqrt{2}/2$
$\sin\frac{\pi}{4} = \sqrt{2}/2$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x}$$

$$\frac{dy}{dx} = \frac{2}{\cos^2 x - \sin^2 x}$$

$$\frac{dy}{dx} = \frac{2}{\cos 2x}$$

$$\frac{dy}{dx} = 2\sec 2x$$

AS REQUIRED

IYGB – MP2 PAPER V – QUESTION 8

START BY DIFFERENTIATING THE FIRST EQUATION IMPLICITLY

$$\Rightarrow x^2 + 2xy + 2y^2 = 10$$

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) + \frac{d}{dx}(2y^2) = \frac{d}{dx}(10)$$

$$\Rightarrow 2x + 2y + 2x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$$

$$\Rightarrow (2x+4y)\frac{dy}{dx} = -2x-2y$$

$$\Rightarrow (x+2y)\frac{dy}{dx} = -x-y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x+y}{x+2y}$$

DIFFERENTIATING THE SECOND EQUATION W.R.T t

$$\Rightarrow y^2 = 4t$$

$$\Rightarrow \frac{d}{dt}(y^2) = \frac{d}{dt}(4t)$$

$$\Rightarrow 2y\frac{dy}{dt} = 4$$

$$\Rightarrow \frac{dy}{dt} = \frac{2}{y}$$

NEXT GET AN EXPRESSION FOR $\frac{dx}{dt}$

$$\Rightarrow \frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{x+2y}{x+y} \times \frac{2}{y}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{2(x+2y)}{xy+y^2}$$

IYGB - MP2 PAPER V - QUESTION 8

NOW WHEN $t=\frac{1}{4}$

$$y^2 = 4t \implies y^2 = 1$$
$$\implies y = \begin{cases} 1 \\ -1 \end{cases}$$

THUS WE NOW HAVE BY USING THE "FIRST" EQUATION

$y = 1$:
$$x^2+2x+2=10$$
$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$
$$x = \begin{cases} -4 \\ 2 \end{cases}$$

$y = -1$:
$$x^2-2x+2=10$$
$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$
$$x = \begin{cases} 4 \\ -2 \end{cases}$$

COLLECTING ALL RESULTS

$(-4,1)$: $\frac{dx}{dt} = -\frac{2(-4+2)}{-4+1} = -\frac{-4}{-3} = -\frac{4}{3}$

$(2,1)$: $\frac{dx}{dt} = -\frac{2(2+2)}{2+1} = -\frac{8}{3}$

$(4,-1)$: $\frac{dx}{dt} = -\frac{2(4-2)}{-4+1} = -\frac{4}{-3} = \frac{4}{3}$

$(-2,-1)$: $\frac{dx}{dt} = -\frac{2(-2-2)}{2+1} = \frac{8}{3}$

IYGB – MP2 PAPER V – QUESTION 9

USING $1+\tan^2\theta \equiv \sec^2\theta$

$$\Rightarrow \sec^2 x - (1+\sqrt{3})\tan x + \sqrt{3} = 1$$

$$\Rightarrow \tan^2 x + 1 - (1+\sqrt{3})\tan x + \sqrt{3} = 1$$

$$\Rightarrow \tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3} = 0$$

BY THE QUADRATIC FORMULA

$$\Rightarrow \tan x = \frac{(1+\sqrt{3}) \pm \sqrt{[(1+\sqrt{3})]^2 - 4\times 1\times\sqrt{3}}}{2\times 1}$$

$$\Rightarrow \tan x = \frac{1+\sqrt{3} \pm \sqrt{1+2\sqrt{3}+3-4\sqrt{3}}}{2}$$

$$\Rightarrow \tan x = \frac{1+\sqrt{3} \pm \sqrt{1-2\sqrt{3}+3}}{2}$$

$$\Rightarrow \tan x = \frac{1+\sqrt{3} \pm \sqrt{(\sqrt{3}-1)^2}}{2}$$

$$\Rightarrow \tan x = \begin{cases} \dfrac{1+\sqrt{3}+(\sqrt{3}-1)}{2} \\[2ex] \dfrac{1+\sqrt{3}-(\sqrt{3}-1)}{2} \end{cases}$$

$$\Rightarrow \tan x = \begin{cases} \dfrac{2\sqrt{3}}{2} \\[2ex] \dfrac{2}{2} \end{cases}$$

$$\Rightarrow \tan x = \begin{cases} \sqrt{3} \\ 1 \end{cases}$$

IYGB - MP2 PAPER V - QUESTION 10

FIND THE EQUATION OF A TANGENT AT A GENERAL POINT "$\theta = \theta$"

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2\sin 2\theta}{4\cos\theta} = \frac{-4\sin\theta\cos\theta}{4\cos\theta} = -\sin\theta$$

EQUATION OF A GENERAL TANGENT

$$y - \cos 2\theta = -\sin\theta(x - 4\sin\theta)$$

USING $(3,0)$ WE OBTAIN

$$0 - \cos 2\theta = -\sin\theta(3 - 4\sin\theta)$$

$$-\cos 2\theta = -3\sin\theta + 4\sin^2\theta$$

$$-(1 - 2\sin^2\theta) = -3\sin\theta + 4\sin^2\theta$$

$$-1 + 2\sin^2\theta = -3\sin\theta + 4\sin^2\theta$$

$$0 = 2\sin^2\theta - 3\sin\theta + 1$$

$$(2\sin\theta - 1)(\sin\theta - 1) = 0$$

$$\sin\theta = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

THUS WE HAVE

$$(4\sin\theta, \cos 2\theta) = (4\sin\theta, 1 - 2\sin^2\theta) = \begin{cases} (4, 1 - 2\times 1^2) \\ (2, 1 - 2(\frac{1}{2})^2) \end{cases}$$

$$= \begin{cases} (4, -1) \\ (2, \frac{1}{2}) \end{cases}$$

$\therefore$ $(4,-1)$ & $(2, \frac{1}{2})$

IYGB - MP2 PAPER V - QUESTION 11

FORMING A DIFFERENTIAL EQUATION

- A = AREA COVERED BY MOULD (m^2)
- t = TIME (WEEKS)

$$\frac{dA}{dt} = +kA(20-A)$$

RATE

INCREASE/SPREAD

PROPORTIONAL

AREA COVERED BY MOULD

AREA NOT YET COVERED

SOLVING BY SEPARATING VARIABLES

$$\Longrightarrow dA = kA(20-A)\,dt$$

$$\Longrightarrow \frac{1}{A(20-A)}\,dA = k\,dt$$

$$\Longrightarrow \int \frac{1}{A(20-A)}\,dA = \int k\,dt$$

PROCEED BY PARTIAL FRACTIONS

$$\frac{1}{A(20-A)} \equiv \frac{P}{A} + \frac{Q}{20-A}$$

$$\boxed{1 \equiv P(20-A) + QA}$$

- IF $A=0$

$20P = 1$

$\underline{P = \frac{1}{20}}$

- IF $A=20$

$20Q = 1$

$\underline{Q = \frac{1}{20}}$

IYGB - MP2 PAPER V - QUESTION 11

RETURNING TO THE INTEGRAL

$$\Longrightarrow \int \frac{\frac{1}{20}}{A} + \frac{\frac{1}{20}}{20-A} \, dA = \int k \, dt$$

$$\Longrightarrow \int \frac{1}{A} + \frac{1}{20-A} \, dA = \int 20k \, dt$$

$$\Longrightarrow \ln A - \ln|20-A| = 20kt + C$$

$$\Longrightarrow \ln\left|\frac{A}{20-A}\right| = 20kt + C$$

$$\Longrightarrow \frac{A}{20-A} = e^{20kt+C}$$

$$\Longrightarrow \frac{A}{20-A} = e^{20kt} \times e^{C}$$

$$\Longrightarrow \frac{A}{20-A} = B e^{\alpha t} \qquad (\alpha = 20k \,, \; B = e^{C})$$

APPLY CONDITION $t=0$, $A=2$

$$\Longrightarrow \frac{2}{20-2} = Be^{0}$$

$$\Longrightarrow B = \frac{1}{9}$$

$$\Longrightarrow \frac{A}{20-A} = \frac{1}{9}e^{\alpha t}$$

$$\Longrightarrow \frac{9A}{20-A} = e^{\alpha t}$$

APPLY CONDITION $t=2$, $A=4$

$$\Longrightarrow \frac{36}{20-4} = e^{2\alpha}$$

$$\Longrightarrow \frac{9}{4} = e^{2\alpha}$$

$$\Longrightarrow 2\alpha = \ln\frac{9}{4}$$

IYGB - MP2 PAPER V - QUESTION 11

$$\Longrightarrow \quad \alpha = \frac{1}{2}\ln\frac{9}{4}$$

$$\Longrightarrow \quad \alpha = \ln\left(\frac{9}{4}\right)^{\frac{1}{2}}$$

$$\Longrightarrow \quad \alpha = \ln\frac{3}{2}$$

REWRITE & TIDY THE SOLUTION

$$\Longrightarrow \quad \frac{9A}{20-A} = e^{\alpha t}$$

$$\Longrightarrow \quad \frac{9A}{20-A} = e^{(\ln\frac{3}{2})t}$$

$$\Longrightarrow \quad \frac{9A}{20-A} = \left(\frac{3}{2}\right)^{t}$$

$$\Longrightarrow \quad (20-A)\left(\frac{3}{2}\right)^{t} = 9A$$

$$\Longrightarrow \quad 20\left(\frac{3}{2}\right)^{t} - A\left(\frac{3}{2}\right)^{t} = 9A$$

$$\Longrightarrow \quad 20\left(\frac{3}{2}\right)^{t} = 9A + A\left(\frac{3}{2}\right)^{t}$$

$$\Longrightarrow \quad A\left[9 + \left(\frac{3}{2}\right)^{t}\right] = 20\left(\frac{3}{2}\right)^{t}$$

$$\Longrightarrow \quad A = \frac{20\left(\frac{3}{2}\right)^{t}}{\left(\frac{3}{2}\right)^{t} + 9}$$

$$\Longrightarrow \quad A = \frac{20\left(\frac{3}{2}\right)^{t}\left(\frac{2}{3}\right)^{t}}{\left(\frac{3}{2}\right)^{t}\left(\frac{2}{3}\right)^{t} + 9\left(\frac{2}{3}\right)^{t}}$$

$$\Longrightarrow \quad A = \frac{20}{1 + 9\left(\frac{2}{3}\right)^{t}}$$

// AS REQUIRED

IYGB - MP2 PAPER V - QUESTION 12

a) START BY OBTAINING THE EQUATION OF THE TANGENT AT $\left(p, \sqrt{e^{2p}+1}\right)$

$$\Rightarrow y = (e^{2x}+1)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(e^{2x}+1)^{-\frac{1}{2}}(2e^{2x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{2x}}{(e^{2x}+1)^{\frac{1}{2}}}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_p = \frac{e^{2p}}{(e^{2p}+1)^{\frac{1}{2}}}$$

$$\Rightarrow y - (e^{2p}+1)^{\frac{1}{2}} = \frac{e^{2p}}{(e^{2p}+1)^{\frac{1}{2}}}(x-p)$$

NOW IT IS GIVEN THAT THE ABOVE TANGENT PASSES THROUGH $(0,0)$

$$\Rightarrow -(e^{2p}+1)^{\frac{1}{2}} = \frac{e^{2p}}{(e^{2p}+1)^{\frac{1}{2}}}(-p)$$

$$\Rightarrow -(e^{2p}+1)^{\frac{1}{2}} = \frac{-pe^{2p}}{(e^{2p}+1)^{\frac{1}{2}}}$$

$$\Rightarrow e^{2p}+1 = pe^{2p}$$

$$\Rightarrow 1 = pe^{2p} - e^{2p}$$

$$\Rightarrow 1 = e^{2p}(p-1)$$

OR WRITTEN IN x, AS $x=p$

$$\Rightarrow (x-1)e^{2x} = 1$$

AS REQUIRED

IYGB - MP2 PAPER V - QUESTION 12

b) REWRITE THE EQUATION AS A FUNCTION

$$f(x) = (x-1)e^{2x} - 1$$

$$f(1) = -1 \quad < 0$$

$$f(2) = e^4 - 1 > 0$$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN $[1,2]$

THERE IS AT LEAST ONE SOLUTION IN THE INTERVAL

c) $$f'(x) = 1 \times e^{2x} + 2(x-1)e^{2x} = e^{2x}[1 + 2x - 2] = (2x-1)e^{2x}$$

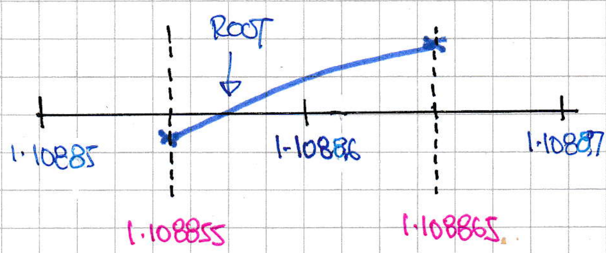
$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$\alpha = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{e^2} = 1.1$$

d) USING THE GRAPH OF THE ABOVE FUNCTION

$$f(1.108855) = -0.000029.. < 0$$

$$f(1.108865) = 0.000083 > 0$$



AS THERE IS A CHANGE OF SIGN IN THE ABOVE INTERVAL

$$1.108855 < \text{ROOT} < 1.108865$$

$\therefore$ ROOT $\simeq$ 1.10886 CORRECT TO 5 d.p.

IYGB - MP2 PAPER U - QUESTION 13

a) SOLVING $y=0$ YIELDS

$\Rightarrow 0 = 3 - \ln 4x$

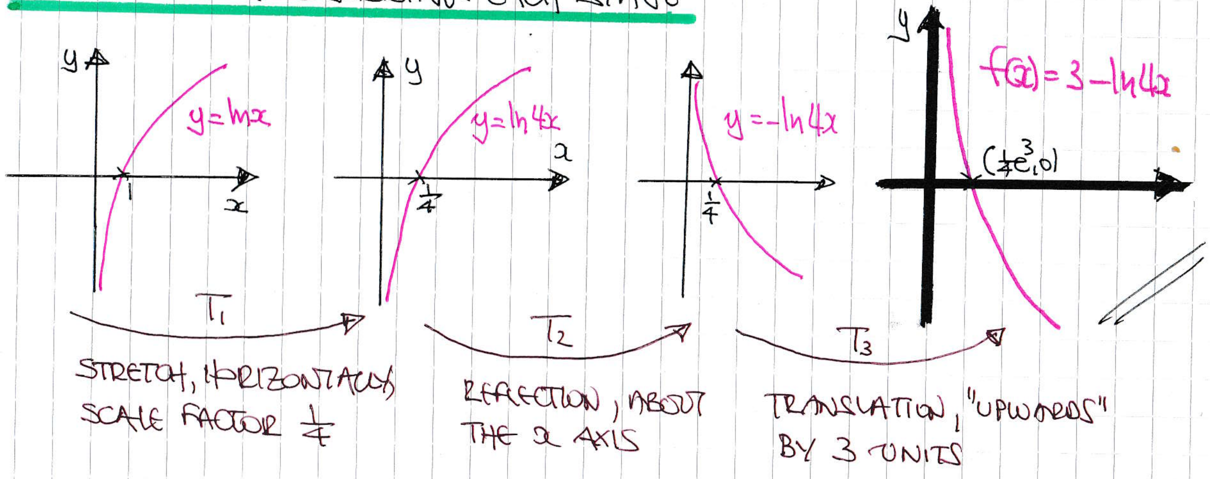
$\Rightarrow \ln 4x = 3$

$\Rightarrow 4x = e^3$

$\Rightarrow x = \frac{1}{4}e^3$

$\therefore \left(\frac{1}{4}e^3, 0\right)$

b) SKETCHING & DESCRIBING EACH STAGE



c) FINALLY THE COMPOSITION

$$f(g(x)) = f(e^{5-x})$$
$$= 3 - \ln(4e^{5-x})$$
$$= 3 - [\ln 4 + \ln e^{5-x}]$$
$$= 3 - [\ln 4 + (5-x)]$$
$$= 3 - \ln 4 - 5 + x$$
$$= x - 2 - \ln 4$$
$$= x - 2 - 2\ln 2$$

I.E $k=2$

IYGB – MP2 PAPER V – QUESTION 14

a) I) SQUARING & TRANSPOSING

$$\sqrt{5-4x-x^2} = (1-x)u$$

$$5-4x-x^2 = (1-x)^2u^2$$

$$-(x^2+4x-5) = u^2(x-1)^2$$

$$-(x-1)(x+5) = u^2(x-1)^2$$

NOTING THAT $x \neq 1, x \neq -5$ AND $(a-b)^2 \equiv (b-a)^2$

$$-(x+5) = u^2(x-1)$$

$$-x-5 = u^2x - u^2$$

$$u^2-5 = u^2x + x$$

$$u^2-5 = x(u^2+1)$$

$$x = \frac{u^2-5}{u^2+1}$$

AS REQUIRED

II) DIFFERENTIATE THE QUOTIENT WITH RESPECT TO u

$$\frac{dx}{du} = \frac{(u^2+1)(2u)-(u^2-5)(2u)}{(u^2+1)^2} = \frac{\cancel{2u^3}+2u-\cancel{2u^3}+10u}{(u^2+1)^2}$$

$$\frac{dx}{du} = \frac{12u}{(u^2+1)^2}$$

$$dx = \frac{12u}{(u^2+1)^2}\,du$$

AS REQUIRED

IYGB - MP2 PAPER V - QUESTION 14

b) USING THE RESULTS FROM PART (a)

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \int \frac{x}{(1-x)^3u^3} \times \frac{12u}{(1+u^2)^2}\,du$$

$$= \int \frac{\frac{u^2-5}{u^2+1}}{\left(1-\frac{u^2-5}{u^2+1}\right)^3 u^3} \times \frac{12u}{(1+u^2)^2}\,du$$

$$= \int \frac{1}{\left[\frac{\cancel{u^2}+1-\cancel{u^2}+5}{u^2+1}\times u\right]^3} \times \frac{u^2-5}{u^2+1} \times \frac{12u}{(u^2+1)^2}\,du$$

$$= \int \frac{1}{\left(\frac{6u}{u^2+1}\right)^3} \times \frac{12u(u^2-5)}{(u^2+1)^3}\,du$$

$$= \int \frac{\cancel{(u^2+1)^3}}{216u^3} \times \frac{12u(u^2-5)}{\cancel{(u^2+1)^3}}\,du$$

$$= \int \frac{12u(u^2-5)}{216u^3}\,du$$

$$= \int \frac{u^2-5}{18u^2}\,du$$

AS REQUIRED

c) SPLIT THE FRACTION AND INTEGRATE

$$= \int \frac{1}{18} - \frac{5}{18}u^{-2}\,du$$

$$= \frac{1}{18}u + \frac{5}{18}u^{-1} + C$$

IYGB - MP2 PAPER V - QUESTION 14

$$= \frac{1}{18}u + \frac{5}{18u} + C$$

$$= \frac{1}{18u}\left[u^2 + 5\right] + C$$

NOW FROM PART (a)

$$u^2(x-1) = -(x+5)$$

$$u^2 = -\frac{x+5}{x-1}$$

$$u^2 = \frac{x+5}{1-x}$$

TIDYING UP FINALLY

$$= \dots \frac{1}{18} \times \frac{\sqrt{1-x}}{\sqrt{x+5}} \times \left[\frac{x+5}{1-x} + 5\right] + C$$

$$= \dots \frac{1}{18} \times \frac{\sqrt{1-x}}{\sqrt{x+5}} \times \frac{x+5+5-5x}{1-x} + C$$

$$= \frac{1}{18} \times \frac{\sqrt{1-x}}{\sqrt{x+5}} \times \frac{10-4x}{1-x} + C$$

$$= \frac{1}{18} \times \frac{1}{\sqrt{x+5}} \times \frac{10-4x}{\sqrt{1-x}} + C$$

$$= \frac{1}{18} \times \frac{2(5-2x)}{\sqrt{(x+5)(1-x)}} + C$$

$$= \frac{5-2x}{9\sqrt{5-4x-x^2}} + C$$