IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper Y

Difficulty Rating: 4.5200/1.6216

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 12 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$3\cos^2 x - \cos x = 1.99375 .$$

It is given that the above trigonometric equation has a solution that is numerically small.

Use small angle approximations to find this solution.

No credit will be given for standard solution methods. **(7)**

Question 2

A sequence is defined for $n \geq 1$ by the recurrence relation

$$u_{n+1} = \frac{5u_n}{1+8u_n}, \quad u_1 = \frac{1}{5}.$$

Determine an expression for u_n, given that it is of the form

$$u_n = \frac{a^{n-1}}{c + ka^{n-1}},$$

where a, c and k are constants to be found. **(12)**

Question 3

A curve C has equation

$$y = x^{-x}, \quad x \in \mathbb{R}, \quad x > 0 .$$

Show that y is a solution of the equation

$$y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 - \frac{y^2}{x} .$$ **(8)**

Question 4

Solve the following trigonometric equation.

$$\arctan\left(\frac{x-5}{x-1}\right)+\arctan\left(\frac{x-4}{x-3}\right)=\frac{\pi}{4}, \ x\in\mathbb{R}. \qquad \textbf{(10)}$$

Question 5

The points $A(3,2,14)$, $B(0,1,13)$ and $C(5,6,8)$ are defined with respect to a fixed origin O.

The straight line L passes through A and it is parallel to the vector $\overrightarrow{BC}$.

The point D lies on L so that $ABCD$ is a parallelogram.

a) Find the coordinates of D. **(2)**

b) If instead $ABCD$ is an isosceles trapezium and the point D still lies on L, determine the new coordinates of D. **(10)**

Question 6

A curve C is given by the parametric equations

$$x=\cos t, \quad y=\cos 2t \ , \quad -\pi\le t\le\pi.$$

The point P lies on C, where $t=\frac{\pi}{3}$.

a) Show that an equation of the normal to C at P is

$$2x+4y+1=0. \qquad \textbf{(5)}$$

The normal at P meets C again at the point Q.

b) Determine, by showing a clear detailed method, the exact coordinates of Q. **(7)**

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Question 7

The sum to infinity of a geometric series is 2187.

The $(k-1)^{\text{th}}$ and k^{th} term of the same series are 96 and 64, respectively.

Determine the value of

$$\sum_{n=k+1}^{\infty} u_n ,$$

where u_n is the n^{th} term of the series. **(10)**

Question 8

A small forest with an area of 25 km^2 has caught fire.

Let A, in km^2, be the area of the forest destroyed by the fire, t hours after the fire was first noticed.

The rate at which the forest is destroyed is proportional to the difference between the total area of the forest squared, and the area of the forest destroyed squared.

When the fire was first noticed 7 km^2 of the forest had been destroyed and **at that instant** the rate at which the area of the forest was destroyed was 7.2 km^2 per hour.

a) Show clearly that

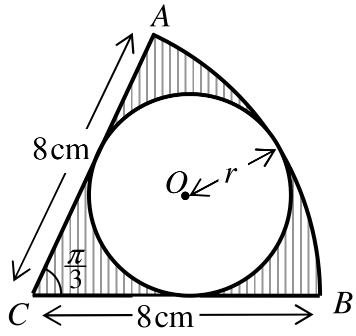
$$50\frac{dA}{dt} = \frac{5}{8}\left(625 - A^2\right).$$ **(4)**

b) Solve the differential equation to obtain

$$\frac{25+A}{25-A} = \frac{16}{9}e^{\frac{5}{8}t}.$$ **(9)**

c) Show further that 14 km^2 of the forest will be destroyed, approximately 66 minutes after the fire was first noticed. **(4)**

Question 9



The figure above shows a sector CAB of radius 8 cm, centred at C and subtending an angle of $\frac{\pi}{3}$ radians at C.

A circle centred at O and of radius r cm is inscribed in the sector.

Find in terms of π, the area of the shaded region in the figure. **(6)**

Question 10

$$f(x) = (1+ax)(1-3x)^{\frac{1}{3}} + \frac{b}{\left(1+\frac{1}{2}x\right)^2}, \quad |3x|<1, \ |ax|<1.$$

In the binomial expansion of $f(x)$ the coefficients of x^2 and x^3 are both zero.

Show clearly that the coefficient of x^4 is $-\frac{7}{6}$. **(9)**

Question 11

Liquid is pouring into a container at the constant rate of $12\pi \text{ cm}^3\text{s}^{-1}$.

The container is initially empty and when the height of the liquid in the container is h cm the volume of the liquid, V cm^3, is given by

$$V = \pi h(h+20).$$

Determine the rate at which the height of the liquid in the container is rising 8 seconds after the liquid started pouring in.

(10)

Question 12

By using the substitution $\sqrt{x} = \tan\theta$, or otherwise, find

$$\int \frac{(x+3)\sqrt{x}}{(x+1)^2}\,dx.$$

(12)
