# IYGB - MP2 PAPER Y - QUESTION 1

## METHOD A

$$3\cos^2 x - \cos x = 1.99375$$

USING THE DOUBLE ANGLE FORMULA FOR "$\cos 2\theta$" $\equiv 2\cos^2\theta - 1$

$$\Rightarrow 3\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right) - \cos x = 1.99375$$

$$\Rightarrow \tfrac{3}{2} + \tfrac{3}{2}\cos 2x - \cos x = 1.99375$$

$$\Rightarrow 3 + 3\cos 2x - 2\cos x = 1.99375 \times 2$$

USING A QUADRATIC APPROXIMATION FOR $\cos x$ & $\cos 2x$

$$\cos x \approx 1 - \frac{x^2}{2}$$

$$\cos 2x \approx 1 - \frac{(2x)^2}{2} \approx 1 - \frac{4x^2}{2} \approx 1 - 2x^2$$

HENCE WE OBTAIN

$$\Rightarrow 3 + 3(1 - 2x^2) - 2\left(1 - \frac{x^2}{2}\right) = 1.99375 \times 2$$

$$\Rightarrow 3 + 3 - 6x^2 - 2 + x^2 = 1.99375 \times 2$$

$$\Rightarrow 4 - 2 \times 1.99375 = 5x^2$$

$$\Rightarrow 5x^2 = 0.0125$$

$$\Rightarrow x^2 = 0.0025$$

$$\Rightarrow x = \pm 0.05$$

BOTH ARE O.K AS $\cos x$ IS EVEN

# IYGB - MP2 PAPER Y - QUESTION 1

## METHOD B

$$3\cos^2 x - \cos x = 1.99375$$
$$3\cos^2 x - \cos x - 1.99375 = 0$$

BY THE QUADRATIC FORMULA

$$\cos x = \frac{1 \pm \sqrt{1 - 4 \times 3 \times (-1.99375)}}{6}$$

$$\cos x = \frac{1 \pm \sqrt{24.925}}{6}$$

NOW USING A QUADRATIC APPROXIMATION FOR $\cos x$

$$1 - \frac{x^2}{2} = \frac{1 \pm \sqrt{24.925}}{6}$$

$$-\frac{x^2}{2} = -1 + \frac{1 \pm \sqrt{24.925}}{6}$$

$$x^2 = 2\left[1 - \frac{1 \pm \sqrt{24.925}}{6}\right]$$

$$x^2 = \begin{cases} 0.0025018778\ldots \\ 3.3308314\ldots \end{cases}$$

$$x = \begin{cases} \pm 0.050\ldots \\ \cancel{\pm 1.825\ldots} \end{cases}$$

[$x$ HAS TO BE "SMALL"]

# IYGB - MP2 PAPER Y - QUESTION 2

- START BY GENERATING TERMS FROM THE RECURRENCE RELATION

$$u_{n+1} = \frac{5u_n}{8u_n+1}$$

$$u_1 = \frac{1}{5}$$

$$u_2 = \frac{5\times\frac{1}{5}}{8\times\frac{1}{5}+1} = \frac{1}{\frac{8}{5}+1} = \frac{5}{8+5} = \frac{5}{13}$$

$$u_3 = \frac{5\times\frac{5}{13}}{8\times\frac{5}{13}+1} = \frac{25}{40+13} = \frac{25}{53}$$

- NOW FORM SOME EQUATIONS USING THE FIRST 3 TERMS

$$\boxed{u_n = \frac{a^{n-1}}{ka^{n-1}+c}}$$

- $u_1 = \frac{1}{5}$

$$\frac{1}{k+c} = \frac{1}{5}$$

$$k+c = 5$$

$$c = 5-k$$

- $u_2 = \frac{5}{13}$

$$\frac{a}{ka+c} = \frac{5}{13}$$

$$ka+c = \frac{13a}{5}$$

$$ka+5-k = \frac{13}{5}a$$

$$\boxed{k(a-1) = \frac{13}{5}a-5}$$

- $u_3 = \frac{25}{53}$

$$\frac{a^2}{ka^2+c} = \frac{25}{53}$$

$$ka^2+c = \frac{53}{25}a^2$$

$$ka^2+5-k = \frac{53}{25}a^2$$

$$\boxed{k(a^2-1) = \frac{53}{25}a^2-5}$$

- DIVIDING THE TWO EQUATIONS, NOTING $k \neq 0$, $a \neq 1$

$$\Rightarrow \frac{k(a^2-1)}{k(a-1)} = \frac{\frac{53}{25}a^2-5}{\frac{13}{5}a-5}$$

$$\Rightarrow \frac{\cancel{k}\cancel{(a-1)}(a+1)}{\cancel{k}\cancel{(a-1)}} = \frac{53a^2-125}{65a-125}$$

# IYGB - MP2 PAPER Y - QUESTION 2

$$\Rightarrow (a+1)(65a-125) = 53a^2-125$$

$$\Rightarrow 65a^2 - 125a + 65a - \cancel{125} = 53a^2 - \cancel{125}$$

$$\Rightarrow 12a^2 - 60a = 0$$

$$\Rightarrow 12a(a-5) = 0$$

$$\therefore \underline{a=5} \qquad a \neq 0$$

$$\Rightarrow k(a-1) = \frac{13}{5}a - 5$$

$$\Rightarrow 4k = 13 - 5$$

$$\Rightarrow \underline{k=2} \quad \& \quad \underline{c=3}$$

$$\therefore \quad u_n = \frac{5^{n-1}}{2(5^{n-1})+3}$$

# IYGB – MP2 PAPER Y – QUESTION 3

START BY TAKING LOGS IN BOTH SIDES

$$y = x^{-x}$$
$$\ln y = \ln x^{-x}$$
$$\ln y = -x\ln x$$
$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(-x\ln x)$$
$$\frac{1}{y}\frac{dy}{dx} = -1\times\ln x - x\times\frac{1}{x}$$
$$\frac{1}{y}\frac{dy}{dx} = -\ln x - 1$$
$$\boxed{\frac{dy}{dx} = -y(1+\ln x)}$$

DIFFERENTIATE WITH RESPECT TO $x$ AGAIN

$$\frac{d^2y}{dx^2} = -1\frac{dy}{dx}(1+\ln x) - y\left(0+\frac{1}{x}\right)$$
$$\frac{d^2y}{dx^2} = -\frac{dy}{dx}(1+\ln x) - \frac{y}{x}$$

NOW REARRANGING THE "BOXED" EXPRESSION AS $(1+\ln x) = -\frac{1}{y}\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = -\frac{dy}{dx}\left(-\frac{1}{y}\frac{dy}{dx}\right) - \frac{y}{x}$$
$$\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}$$
$$y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 - \frac{y^2}{x}$$

AS REQUIRED

# IYGB - MP2 PAPER Y - QUESTION 4

LET $\theta = \arctan\left(\frac{x-5}{x-1}\right)$ & $\phi = \arctan\left(\frac{x-4}{x-3}\right)$

$$\Rightarrow \quad \theta + \phi = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan(\theta+\phi) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$$

$$\Rightarrow \quad \frac{\frac{x-5}{x-1} + \frac{x-4}{x-3}}{1 - \frac{x-5}{x-1} \times \frac{x-4}{x-3}} = 1$$

$$\Rightarrow \quad \frac{x-5}{x-1} + \frac{x-4}{x-3} = 1 - \frac{(x-5)(x-4)}{(x-1)(x-3)}$$

MULTIPLY THROUGH BY $(x-1)(x-3)$

$$(x-5)(x-3) + (x-4)(x-1) = (x-1)(x-3) - (x-5)(x-4)$$

$$x^2 - 8x + 15 + x^2 - 5x + 4 = x^2 - 4x + 3 - (x^2 - 9x + 20)$$

$$2x^2 - 13x + 19 = 5x - 17$$

$$2x^2 - 18x + 36 = 0$$

$$x^2 - 9x + 18 = 0$$

$$(x-3)(x-6) = 0$$

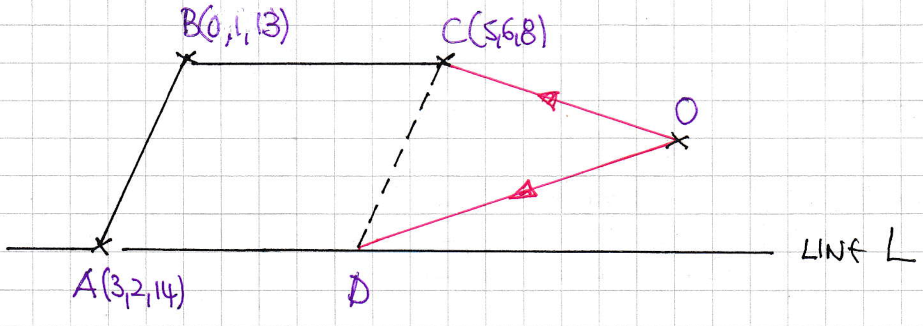
$$x = \begin{cases} 3 \\ 6 \end{cases}$$ BOTH ARE FINE

$$\arctan\frac{1}{5} + \arctan\frac{2}{3} = \pi/4$$

$$\arctan(-1) + \arctan(\infty) = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

# IYGB - MP2 PAPER Y - QUESTION 5

a) STARTING WITH A DIAGRAM




$$\vec{OD} = \vec{OC} + \vec{CD}$$
$$= \vec{OC} + \vec{BA}$$
$$= \underline{c} + (\underline{a} - \underline{b})$$
$$= \begin{pmatrix}5\\6\\8\end{pmatrix} + \begin{pmatrix}3\\2\\14\end{pmatrix} - \begin{pmatrix}0\\1\\13\end{pmatrix}$$
$$= \begin{pmatrix}8\\7\\9\end{pmatrix}$$

$\therefore D(8,7,9)$ //

**ALTERNATIVE BY INSPECTION**

"B to A" $\quad 0 \mapsto +3$

$1 \mapsto +1$

$13 \mapsto +1$

THEREFORE

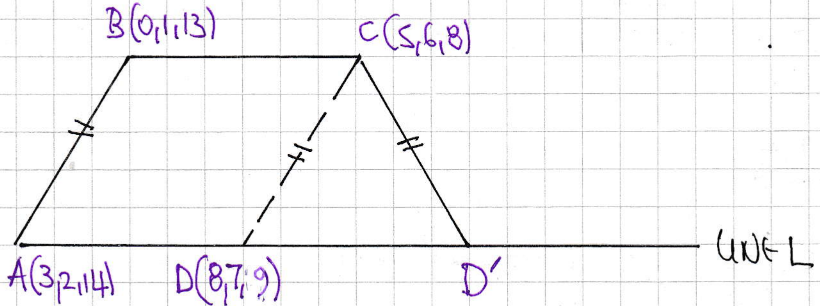
"C to D" $\quad 5 \xmapsto{+3} 8$

$6 \xmapsto{+1} 7$

$8 \xmapsto{+1} 9$

$\therefore D(8,7,9)$

b) REDRAWING THE DIAGRAM




- $\vec{AD} = \underline{d} - \underline{a}$

$$= \begin{pmatrix}8\\7\\9\end{pmatrix} - \begin{pmatrix}3\\2\\14\end{pmatrix}$$
$$= \begin{pmatrix}5\\5\\-5\end{pmatrix}$$

IYGB - MP2 PAPER Y - QUESTION 5

SCALE THE VECTOR $\begin{pmatrix}5\\5\\-5\end{pmatrix}$ TO $\begin{pmatrix}1\\1\\-1\end{pmatrix}$

- $\vec{AD'} = k\begin{pmatrix}1\\1\\-1\end{pmatrix}$

- $|\vec{AB}| = |\underline{b} - \underline{a}| = \left|\begin{pmatrix}0\\1\\13\end{pmatrix} - \begin{pmatrix}3\\2\\14\end{pmatrix}\right| = \left|\begin{matrix}-3\\-1\\-1\end{matrix}\right| = \sqrt{9+1+1} = \sqrt{11}$

LET THE CO-ORDINATES OF $D'$ BE $(x,y,z)$

- $\vec{CD'} = \underline{d}' - \underline{c} = \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}5\\6\\8\end{pmatrix} = \begin{pmatrix}x-5\\y-6\\z-8\end{pmatrix}$

- $|\vec{CD'}| = \left|\begin{matrix}x-5\\y-6\\z-8\end{matrix}\right| = \sqrt{(x-5)^2+(y-6)^2+(z-8)^2} = \sqrt{11}$

$$\therefore \boxed{(x-5)^2+(y-6)^2+(z-8)^2 = 11}$$

BUT $\vec{AD'} = k\begin{pmatrix}1\\1\\-1\end{pmatrix}$ AND $\vec{AD'} = \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}3\\2\\14\end{pmatrix} = \begin{pmatrix}x-3\\y-2\\z-14\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}x-3\\y-2\\z-14\end{pmatrix} = \begin{pmatrix}k\\k\\-k\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}k+3\\k+2\\-k+14\end{pmatrix}$$

THUS WE NOW HAVE

$$\Rightarrow (x-5)^2+(y-6)^2+(z-8)^2 = 11$$

$$\Rightarrow (k+3-5)^2+(k+2-6)^2+(-k+14-8)^2 = 11$$

$$\Rightarrow (k-2)^2+(k-4)^2+(6-k)^2 = 11$$

# IYGB - MP2 PAPER Y - QUESTION 5

$$\Longrightarrow \left.\begin{matrix} k^2-4k+4 \\ k^2-8k+16 \\ k^2-12k+36 \end{matrix}\right\} = 11$$

$$\Longrightarrow 3k^2-24k+56 = 11$$

$$\Longrightarrow 3k^2-24k+45 = 0$$

$$\Longrightarrow k^2-8k+15 = 0$$

$$\Longrightarrow (k-5)(k-3) = 0$$

$$\Longrightarrow k = \begin{cases} 3 \\ 5 \end{cases}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} \begin{pmatrix} 3+3 \\ 3+2 \\ -3+14 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 11 \end{pmatrix} \leftarrow \text{POINT } D' \\ \begin{pmatrix} 5+3 \\ 5+2 \\ -5+14 \end{pmatrix} = \cancel{\begin{pmatrix} 8 \\ 7 \\ 9 \end{pmatrix}} \quad \text{POINT } D \end{cases}$$

$$\therefore D(6,5,11)$$

# IYGB - MP2 PAPER Y - QUESTION 6

a) OBTAIN THE GRADIENT PARAMETRICALLY

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{-\sin t} = \frac{-4\sin t\cos t}{-\sin t} = 4\cos t$$

AT $t = \pi/3$ $\quad P\left(\cos\frac{\pi}{3}, \cos\frac{2\pi}{3}\right)$ $\quad \frac{dy}{dx} = 4\cos\frac{\pi}{3}$

$P\left(\frac{1}{2}, -\frac{1}{2}\right)$ $\quad m = 2$

EQUATION OF A NORMAL AT

$$y + \frac{1}{2} = -\frac{1}{2}\left(x - \frac{1}{2}\right)$$

$$y + \frac{1}{2} = -\frac{1}{2}x + \frac{1}{4}$$

$$4y + 2 = -2x + 1$$

$$2x + 4y + 1 = 0$$

AS REQUIRED

b) SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$$\Rightarrow 2x + 4y + 1 = 0$$

$$\Rightarrow 2\cos t + 4\cos 2t + 1 = 0$$

$$\Rightarrow 2\cos t + 4(2\cos^2 t - 1) + 1 = 0$$

$$\Rightarrow 2\cos t + 8\cos^2 t - 3 = 0$$

$$\Rightarrow 8\cos^2 t + 2\cos t - 3 = 0$$

$$\Rightarrow (4\cos t + 3)(2\cos t - 1) = 0$$

$$\Rightarrow \cos t = \begin{cases} \frac{1}{2} & \leftarrow \text{POINT OF NORMALITY } P \\ -\frac{3}{4} & \leftarrow \text{POINT } Q \end{cases}$$

IYGB - MP2 PAPER Y - QUESTION 6

FINALLY TO FIND THE CO-ORDINATES OF Q

$Q(\cos t, \cos 2t)$

$Q(\cos t, 2\cos^2 t - 1)$

BUT $\cos t = -\frac{3}{4}$

$Q\left(-\frac{3}{4}, 2\left(-\frac{3}{4}\right)^2 - 1\right)$

$Q\left(-\frac{3}{4}, 2\left(\frac{9}{16}\right) - 1\right)$

$Q\left(-\frac{3}{4}, \frac{1}{8}\right)$

# IYGB - MP2 PAPER Y - QUESTION 7

$u_{k-1} = 96$ $\quad$ $u_k = 64$ $\quad$ $S_\infty = 2187$

$ar^{k-2} = 96$ $\quad$ $ar^{k-1} = 64$ $\quad$ $\frac{a}{1-r} = 2187$

- FROM THE FIRST TWO RELATIONSHIPS

$$r = \frac{u_k}{u_{k-1}} = \frac{64}{96} = \frac{2}{3}$$

- FROM THE THIRD RELATIONSHIP

$$\frac{a}{1-\frac{2}{3}} = 2187$$

$$\frac{a}{\frac{1}{3}} = 2187$$

$$a = 729$$

- NEXT WE HAVE

$$u_k = 64$$

$$ar^{k-1} = 64$$

$$729 \times \left(\frac{2}{3}\right)^{k-1} = 64$$

$$\left(\frac{2}{3}\right)^{k-1} = \frac{64}{729}$$

BY INSPECTION, TRIAL & IMPROVEMENT (AS $k$ IS A POSITIVE INTEGER) OR LOGARITHMS

$$\left(\frac{2}{3}\right)^{k-1} = \left(\frac{2}{3}\right)^6$$

$$k = 7$$

# IYGB - MP2 PAPER Y - QUESTION 7

- FINALLY WE HAVE

$$\sum_{n=k+1}^{\infty} u_n = \sum_{n=1}^{\infty} u_n - \sum_{n=1}^{k} u_n$$

$$= S_{\infty} - S_k$$

$$= 2187 - \frac{a(1-r^k)}{1-r}$$

$$= 2187 - \frac{729\left(1-\left(\frac{2}{3}\right)^7\right)}{1-\frac{2}{3}}$$

$$= 2187 - 2059$$

$$= 128$$

# IYGB - MP2 PAPER Y - QUESTION 8

a) FORMING THE DIFFERENTIAL EQUATION

A = AREA OF FOREST DESTROYED ($km^2$)

t = TIME (IN HOURS)

$$t=0 \quad , \quad A=7 \quad , \quad \left.\frac{dA}{dt}\right|_{\substack{t=0\\A=7}} = 7.2$$

$$\frac{dA}{dt} = +k\left(25^2 - A^2\right)$$

← DIFFERENCE BETWEEN...

RATE

AREA OF THE FOREST BEING DESTROYED IS INCREASING

PROPORTIONAL

AREA OF FOREST SQUARED

AREA OF THE FOREST DESTROYED, SQUARED

APPLY THE CONDITION $\left.\frac{dA}{dt}\right|_{A=7} = 7.2$

$$\Rightarrow 7.2 = k\left(25^2 - 7^2\right)$$

$$\Rightarrow 7.2 = 576k$$

$$\Rightarrow k = \frac{1}{80}$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{80}\left(625 - A^2\right)$$

×50

$$\Rightarrow 50\frac{dA}{dt} = \frac{5}{8}\left(625 - A^2\right)$$

// AS REQUIRED

# IYGB - MP2 PAPER Y - QUESTION 8

b) SEPARATING VARIABLES

$$\Rightarrow 50\,dA = \frac{5}{8}(625 - A^2)\,dt$$

$$\Rightarrow \frac{50}{625 - A^2}\,dA = \frac{5}{8}\,dt$$

$$\Rightarrow \int \frac{50}{(25+A)(25-A)}\,dA = \int \frac{5}{8}\,dt$$

OBTAIN THE PARTIAL FRACTIONS

$$\frac{50}{(25+A)(25-A)} \equiv \frac{P}{25+A} + \frac{Q}{25-A}$$

$$50 \equiv P(25 - A) + Q(25+A)$$

- IF $A = 25$
  $50 = 50Q$
  $Q = 1$
- IF $A = -25$
  $50 = 50P$
  $P = 1$

RETURNING TO THE INTEGRAL

$$\Rightarrow \int \frac{1}{25+A} + \frac{1}{25-A}\,dA = \int \frac{5}{8}\,dt$$

$$\Rightarrow \ln|25+A| - \ln|25-A| = \frac{5}{8}t + C$$

$$\Rightarrow \ln\left|\frac{25+A}{25-A}\right| = \frac{5}{8}t + C$$

$$\Rightarrow \frac{25+A}{25-A} = e^{\frac{5}{8}t + C}$$

$$\Rightarrow \frac{25+A}{25-A} = e^{\frac{5}{8}t} \times e^{C}$$

$$\Rightarrow \frac{25+A}{25-A} = Be^{\frac{5}{8}t} \qquad (B = e^{C})$$

# IYGB – MP2 PAPER Y – QUESTION 8

APPLY THE CONDITION $t=0 \quad A=7$

$$\Longrightarrow \frac{25+7}{25-7} = B$$

$$\Longrightarrow B = \frac{32}{18}$$

$$\Longrightarrow B = \frac{16}{9}$$

$$\Longrightarrow \frac{25+A}{25-A} = \frac{16}{9}e^{\frac{5}{8}t}$$

AS REQUIRED

c) FINALLY WHEN $A=14$

$$\Longrightarrow \frac{25+14}{25-14} = \frac{16}{9}e^{\frac{5}{8}t}$$

$$\Longrightarrow \frac{39}{11} = \frac{16}{9}e^{\frac{5}{8}t}$$

$$\Longrightarrow \frac{351}{176} = e^{\frac{5}{8}t}$$

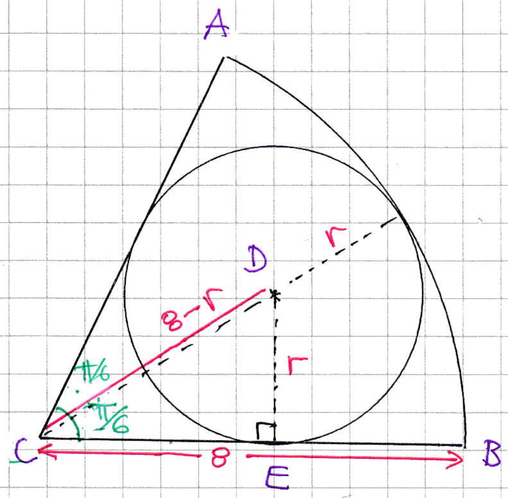
$$\Longrightarrow \frac{5}{8}t = 0.6903022\ldots$$

$$\Longrightarrow t \approx 1.1044\ldots \text{ HOURS}$$

$$\Longrightarrow t \simeq 66.269\ldots \text{ MINUTES}$$

IE APPROX 66 MINUTES

# IYGB - MP2 PAPER Y - QUESTION 9




LOOKING AT THE RIGHT ANGLED TRIANGLE $\overset{\triangle}{CDE}$

$$\Rightarrow \frac{r}{8-r} = \sin\frac{\pi}{6}$$

$$\Rightarrow \frac{r}{8-r} = \frac{1}{2}$$

$$\Rightarrow 2r = 8 - r$$

$$\Rightarrow 3r = 8$$

$$\Rightarrow r = \frac{8}{3}$$

AREA OF SECTOR, USING $\frac{1}{2}r^2\theta^c$ GIVES

$$\text{AREA OF SECTOR} = \frac{1}{2} \times 8^2 \times \frac{\pi}{3} = \frac{32\pi}{3}$$

AREA OF CIRCLE, IS $\pi r^2$

$$\text{AREA OF CIRCLE} = \pi \times \left(\frac{8}{3}\right)^2 = \frac{64\pi}{9}$$

REQUIRED AREA $= \frac{32\pi}{3} - \frac{64\pi}{9} = \frac{32\pi}{9}$ //

# IYGB - MP2 PAPER Y - QUESTION 10

WORKING IN SECTIONS UP TO $x^4$

$$(1+ax)(1-3x)^{\frac{1}{3}} = (1+ax)\left[1+\frac{\frac{1}{3}}{1}(-3x)^1+\frac{\frac{1}{3}(-\frac{2}{3})}{1\times 2}(-3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{1\times 2\times 3}(-3x)^3+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})(-\frac{8}{3})}{1\times 2\times 3\times 4}(-3x)^4+O(x^5)\right]$$

$$= (1+ax)\left[1-x-x^2-\tfrac{5}{3}x^3-\tfrac{10}{3}x^4+O(x^5)\right]$$

$$= \begin{array}{l} 1-x-x^2-\tfrac{5}{3}x^3-\tfrac{10}{3}x^4+O(x^5) \\ \quad\; ax-ax^2-ax^3-\tfrac{5}{3}ax^4+O(x^5) \end{array}$$

$$= 1+(a-1)x+(-a-1)x^2+\left(-a-\tfrac{5}{3}\right)x^3+\left(-\tfrac{5}{3}a-\tfrac{10}{3}\right)x^4+O(x^5)$$

SIMILARLY WITH THE SECOND TERM

$$b(1+\tfrac{1}{2}x)^{-2} = b\left[1+\frac{-2}{1}(\tfrac{1}{2}x)^1+\frac{-2(-3)}{1\times 2}(\tfrac{1}{2}x)^2+\frac{-2(-3)(-4)}{1\times 2\times 3}(\tfrac{1}{2}x)^3+\frac{-2(-3)(-4)(-5)}{1\times 2\times 3\times 4}(\tfrac{1}{2}x)^4+O(x^5)\right]$$

$$= b\left[1-x+\tfrac{3}{4}x^2-\tfrac{1}{2}x^3+\tfrac{5}{16}x^4+O(x^5)\right]$$

$$= b-bx+\tfrac{3}{4}bx^2-\tfrac{1}{2}bx^3+\tfrac{5}{16}bx^4+O(x^5)$$

COMBINING EXPRESSION & LOOKING AT THE COEFFICIENTS OF $x^2, x^3$ AND $x^4$

- $-a-1+\frac{3}{4}b=0$
- $-a-\frac{5}{3}-\frac{1}{2}b=0$

} SUBTRACTING EQNS

$$\tfrac{2}{3}+\tfrac{5}{4}b=0$$

$$\tfrac{5}{4}b=-\tfrac{2}{3}$$

$$b=-\tfrac{8}{15}$$

# IYGB - MP2 PAPER Y - QUESTION 10

$\Rightarrow -a-1+\frac{3}{4}b=0$

$\Rightarrow -a-1+\frac{3}{4}\left(-\frac{8}{15}\right)=0$

$\Rightarrow -a-1-\frac{2}{5}=0$

$\Rightarrow -\frac{7}{5}=a$

$\Rightarrow a=-\frac{7}{5}$

FINALLY THE COEFFICIENT OF $x^4$

- $-\frac{5}{3}a-\frac{10}{3}-\frac{5}{6}b = -\frac{5}{3}\left(-\frac{7}{5}\right)-\frac{10}{3}+\frac{5}{6}\left(-\frac{8}{15}\right)$

$= \frac{7}{3}-\frac{10}{3}-\frac{4}{9}$

$= -1-\frac{4}{9}$

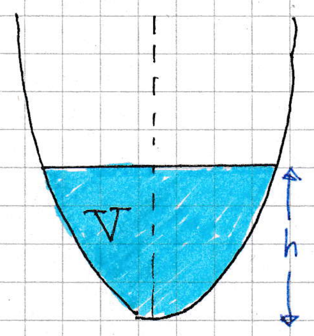
$= -\frac{13}{9}$

AS REQUIRED

# IYGB - MP2 PAPER Y - QUESTION 11

WORK AS FOLLOWS

$$\frac{dV}{dt} = 12\pi \quad \longleftarrow \text{GIVEN (FROM UNITS)}$$




$$V = \pi h(h+20)$$
$$V = \pi(h^2+20h)$$

$$\frac{dV}{dh} = \pi(2h+20)$$
$$\frac{dV}{dh} = 2\pi(h+10)$$

WE REQUIRE $\frac{dh}{dt}$ AT A CERTAIN INSTANT

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{1}{2\pi(h+20)} \times 12\pi$$

$$\frac{dh}{dt} = \frac{12\pi}{2\pi(h+10)}$$

$$\frac{dh}{dt} = \frac{6}{h+10}$$

WE REQUIRE $\left.\frac{dh}{dt}\right|_{t=8}$

$\frac{dV}{dt} = 12\pi$ $cm^3$ per sec

IN 8 seconds

$V = 8 \times 12\pi = 96\pi$

BUT $V = \pi(h^2+20h)$

$\Rightarrow 96\pi = \pi(h^2+20h)$

$\Rightarrow 96 = h^2+20h$

$\Rightarrow h^2+20h-96=0$

$\Rightarrow (h-4)(h+24)=0$

$\Rightarrow h = 4,\ \cancel{-24}$

$$\therefore \left.\frac{dh}{dt}\right|_{t=8} = \left.\frac{dh}{dt}\right|_{h=4} = \frac{6}{4+10}$$

$$= \frac{6}{14}$$

$$= \frac{3}{7} \simeq 0.429 \ cms^{-1}$$

# IYGB - MP2 PAPER Y - QUESTION 12

USING THE SUBSTITUTION GIVEN

$$\sqrt{x} = \tan\theta \qquad \left[\text{IE} \quad \theta = \arctan\sqrt{x}\right]$$

$$x = \tan^2\theta$$

$$dx = 2\sec^2\theta\tan\theta\,d\theta$$

TRANSFORMING THE INTEGRAL

$$\int \frac{(x+3)\sqrt{x}}{(x+1)^2}\,dx = \int \frac{(3+\tan^2\theta)\tan\theta}{(1+\tan^2\theta)^2} \times 2\sec^2\theta\tan\theta\,d\theta$$

$$= \int \frac{2\sec^2\theta\tan^2\theta(3+\tan^2\theta)}{(\sec^2\theta)^2}\,d\theta$$

$$= \int \frac{2\tan^2\theta(3+\tan^2\theta)}{\sec^2\theta}\,d\theta$$

SWITCHING ALL INTO $\sec\theta$

$$= \int \frac{2(\sec^2\theta-1)(3+\sec^2\theta-1)}{\sec^2\theta}\,d\theta$$

$$= \int \frac{2(\sec^2\theta-1)(\sec^2\theta+2)}{\sec^2\theta}\,d\theta$$

$$= \int \frac{2\sec^4\theta+2\sec^2\theta-4}{\sec^2\theta}\,d\theta$$

$$= \int 2\sec^2\theta + 2 - \frac{4}{\sec^2\theta}\,d\theta$$

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$$= \int 2\sec^2\theta + 2 - 4\cos^2\theta \, d\theta$$

$$= \int 2\sec^2\theta + 2 - 4\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right) d\theta$$

$$= \int 2\sec^2\theta + \cancel{2} - \cancel{2} - 2\cos 2\theta \, d\theta$$

$$= 2\tan\theta - \sin 2\theta + C$$

$$= 2\tan\theta - 2\sin\theta\cos\theta + C$$

$$= 2\tan\theta - \frac{2\sin\theta\cos\theta}{\cos^2\theta} \times \cos^2\theta + C$$

$$= 2\tan\theta - 2\tan\theta \times \frac{1}{\sec^2\theta} + C$$

$$= 2\tan\theta - \frac{2\tan\theta}{1+\tan^2\theta} + C$$

$$= 2\sqrt{x} - \frac{2\sqrt{x}}{1+x} + C$$

$$= 2\sqrt{x}\left[1 - \frac{1}{x+1}\right] + C$$

$$= 2\sqrt{x}\left[\frac{x+1-1}{x+1}\right] + C$$

$$= 2\sqrt{x}\left(\frac{x}{x+1}\right) + C$$

$$= \frac{2x^{3/2}}{x+1} + C$$