IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper A

Difficulty Rating: 3.785/1.2641

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Relative to a fixed origin O, the point A has coordinates $(2,1,-3)$.

The point B is such so that $\overrightarrow{AB} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.

Determine the distance of B from O. **(4)**

Question 2

Given that x is measured in radians, use small angle approximations to simplify the following expression.

$$\frac{\cos 7x \ - \ 1}{x \sin x}$$ **(4)**

Question 3

The sum to infinity of a geometric series is 3 times as large as its first term and the third term of the same series is 40.

a) Find the value of the first term of the series. **(5)**

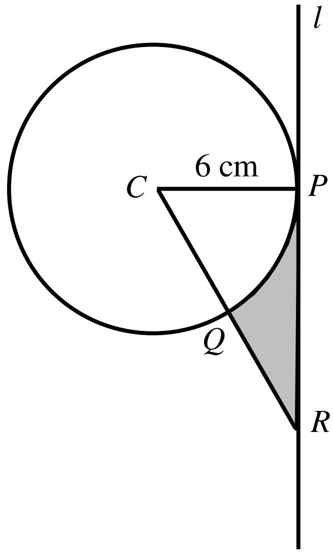
b) Determine the exact value of the sum of the first four terms of the series. **(2)**

Question 4

Prove by **contradiction** that for all real θ

$$\cos\theta + \sin\theta \le \sqrt{2}.$$ **(5)**

Question 5



The figure above shows a circle of radius 6 cm, centre at point C, and the straight line l which is a tangent to the circle at the point P.

The point R lies on l.

The straight line segment CR meets the circle at the point Q.

Given that the length of the arc QP is 2π cm, show that the area of the finite region bounded by PR, RQ and QP, shown shaded in the figure, is

$$6\left(3\sqrt{3}-\pi\right).$$

(9)

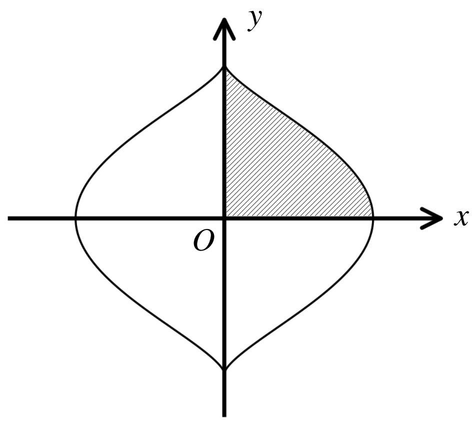
Question 6

The equation of a curve is given by the implicit relationship

$$\frac{x}{x+1}+\frac{y}{y+1}=x^2.$$

Show that at the point on the curve with coordinates $(1,1)$, the gradient is 7. **(8)**

Question 7



The figure above shows the curve C with parametric equations

$$x = \cos^3 \theta, \quad y = 12\sin\theta \ , \quad 0 \le \theta < 2\pi \ .$$

The finite region in the first quadrant, bounded by C and the coordinate axes is shown shaded in the figure above. The curve is symmetrical in both the x and in the y axis.

a) Show that the area of the shaded region is given by the integral

$$36\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta \ d\theta \ . \qquad (4)$$

b) Use trigonometric identities to show that

$$\cos^2\theta\sin^2\theta \equiv \frac{1}{8}(1-\cos 4\theta) \ . \qquad (4)$$

c) Hence find, in terms of π, the **total** area enclosed by C. **(3)**

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Question 8

A curve C has equation

$$y = \frac{1}{x^3 + 1}, \quad x \in \mathbb{R}, x \neq -1.$$

a) Determine an equation of the curve which is obtained by translating C by the vector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$. **(2)**

b) Describe fully a sequence of two transformations which map the graph of C onto the graph with equation

$$y = \frac{1}{x^3 - 1}, \quad x \in \mathbb{R}, x \neq 1.$$ **(2)**

Question 9

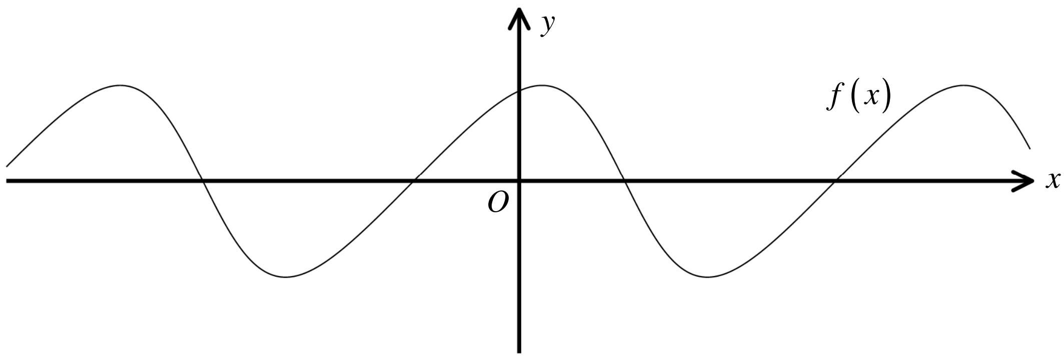
$$f(x) = 3x^2 - 18x + 21, \quad x \in \mathbb{R}, \quad x > 4.$$

a) Express $f(x)$ in the form $A(x+B)^2 + C$, where A, B and C are integers. **(2)**

b) Find a simplified expression for $f^{-1}(x)$, the inverse of $f(x)$. **(3)**

c) Determine the domain and range of $f^{-1}(x)$. **(3)**

Question 10



The figure above shows part of the graph of the curve with equation

$$f(x) = \frac{\cos x}{3 - \sin x}, \quad x \in \mathbb{R}.$$

Use differentiation to show that

$$-\tfrac{1}{4}\sqrt{2} \le f(x) \le \tfrac{1}{4}\sqrt{2}.$$

(9)

Question 11

Water is leaking out of a hole at the bottom of a tank.

Let the height of the water in the tank be y cm at time t minutes.

At any given time after the leaking started, the height of the water in the tank is decreasing at a rate proportional to the cube root of the height of the water in the tank.

When $t = 0$, $y = 125$ and when $t = 3$, $y = 64$.

By forming and solving a differential equation, find the value of y when $t = 7\frac{7}{12}$.

(10)

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Question 12

$$y = \frac{x^2}{2x+1}, \; x \neq -\frac{1}{2}$$

a) Calculate the two missing values of y in the following table. **(1)**

x	0	0.1	0.2	0.3	0.4	0.5
y	0	$\frac{1}{120}$	$\frac{1}{35}$			$\frac{1}{8}$

b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate, correct to 4 significant figures, for the following integral.

$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1} \, dx .$$ **(3)**

a) Use the substitution $u = 2x+1$ to find an exact simplified value for

$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1} \, dx .$$ **(7)**

b) Hence deduce, by referring to parts **(b) and (c)**, the approximate value of $\ln 2$ correct to 2 significant figures. **(2)**

Question 13

Solve the following trigonometric equation

$$\operatorname{cosec}^4\theta - \cot^4\theta = \frac{2}{3} + \sqrt{3}\cot\theta, \; 0 \leq \theta < 2\pi .$$ **(8)**

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