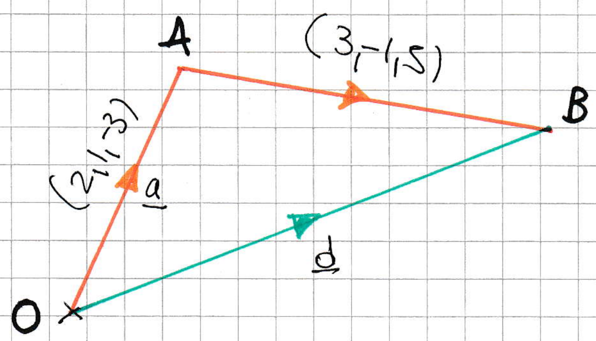
IYGB - MP2 PAPER A - QUESTION 1

- STARTING WITH A DIAGRAM



- FIND THE CO-ORDINATES OF B

$$\Rightarrow \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$\Rightarrow \underline{b} = (2,1,-3) + (3,-1,5)$$

$$\Rightarrow \underline{b} = (5,0,2)$$

- FINALLY THE DISTANCE OB CAN BE FOUND

$$\Rightarrow |\overrightarrow{OB}| = |5,0,2|$$

$$\Rightarrow |\underline{b}| = \sqrt{5^2+0^2+2^2}$$

$$\Rightarrow |\underline{b}| = \sqrt{29} \approx 5.39$$

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USING THE APPROXIMATIONS FOR SMALL θ IN RADIANS

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$$\Longrightarrow \frac{\cos 7x - 1}{x\sin x} \approx \frac{\left(1 - \frac{1}{2}(7x)^2\right) - 1}{x(x)}$$

$$\approx \frac{-\frac{49}{2}x^2}{x^2}$$

$$\approx -\frac{49}{2}$$

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a) SETTING UP TWO EQUATIONS

$$\Rightarrow S_\infty = 3 \times a$$
$$\Rightarrow \frac{a}{1-r} = 3a$$
$$\Rightarrow \frac{1}{1-r} = 3$$
$$\Rightarrow \frac{1}{3} = 1-r$$
$$\Rightarrow r = \frac{2}{3}$$

$$\Rightarrow u_3 = 40$$
$$\Rightarrow ar^2 = 40$$
$$\Rightarrow a \times \left(\frac{2}{3}\right)^2 = 40$$
$$\Rightarrow \frac{4}{9}a = 40$$
$$\Rightarrow 4a = 360$$
$$\Rightarrow a = 90$$

b) USING THE SUMMATION FORMULA FOR A G.P.

$$\Rightarrow S_n = \frac{a(1-r^n)}{1-r}$$
$$\Rightarrow S_4 = \frac{90\left(1-\left(\frac{2}{3}\right)^4\right)}{1-\frac{2}{3}}$$
$$\Rightarrow S_4 = \frac{90\left(1-\frac{16}{81}\right)}{\frac{1}{3}}$$
$$\Rightarrow S_4 = \frac{650}{3} = 216\frac{2}{3}$$

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START BY ASSUMING THE CONVERSE, IE SUPPOSE THAT

$$\Rightarrow \cos\theta + \sin\theta > \sqrt{2}$$

$$\Rightarrow (\cos\theta + \sin\theta)^2 > 2$$

$$\Rightarrow \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta > 2$$

$$\Rightarrow 1 + \sin 2\theta > 2$$

$$\Rightarrow \sin 2\theta > 1$$

$\cos^2\theta + \sin^2\theta \equiv 1$

$\sin 2\theta \equiv 2\sin\theta\cos\theta$

BUT THIS IS A CONTRADICTION AS $\sin 2\theta \leq 1$

$\therefore$ ASSERTION $\cos\theta + \sin\theta > \sqrt{2} \Rightarrow \cos\theta + \sin\theta \leq \sqrt{2}$ //

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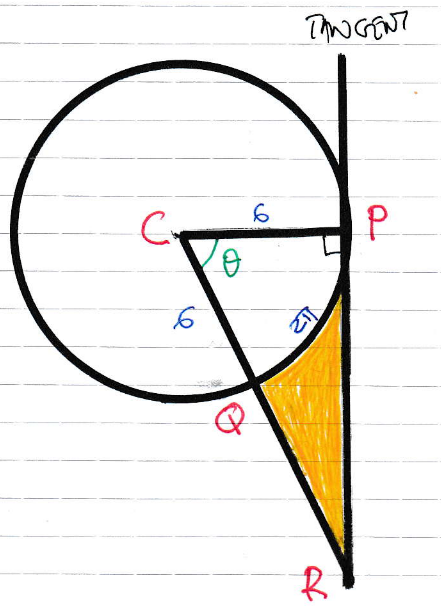

START BY FINDING THE ANGLE θ

"$L = r\theta^c$"

$2\pi = 6\theta$

$\theta = \frac{\pi}{3}$

BY TRIGONOMETRY ON $\hat{PCR}$

$\frac{|PR|}{|CP|} = \tan\theta$

$\frac{|PR|}{6} = \tan\frac{\pi}{3}$

$|PR| = 6\tan\frac{\pi}{3}$

$|PR| = 6\sqrt{3}$

THE AREA OF THE TRIANGLE IS

$= \frac{1}{2}|CP||PR|$

$= \frac{1}{2}\times 6 \times 6\sqrt{3}$

$= 18\sqrt{3}$

THE AREA OF THE SECTOR CPQ

$AREA = \frac{1}{2}r^2\theta^c$

$AREA = \frac{1}{2}\times 6^2 \times \frac{\pi}{3}$

$AREA = 6\pi$

THE REQUIRED AREA IS

$18\sqrt{3} - 6\pi$

$= 6\left[3\sqrt{3} - \pi\right]$

AS REQUIRED

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TIDY THE EQUATION FIRST

$$\Rightarrow \frac{x}{x+1} + \frac{y}{y+1} = x^2$$

$\times (x+1)(y+1)$

$$\Rightarrow x(y+1) + y(x+1) = x^2(x+1)(y+1)$$

$$\Rightarrow xy + x + xy + y = x^2(xy + x + y + 1)$$

$$\Rightarrow 2xy + x + y = x^3y + x^3 + x^2y + x^2$$

$$\Rightarrow 2xy + y - x^3y - x^2y = x^3 + x^2 - x$$

$$\Rightarrow y(2x + 1 - x^3 - x^2) = x^3 + x^2 - x$$

DIFFERENTIATE WITH RESPECT TO x

$$\frac{dy}{dx}(2x+1-x^3-x^2) + y(2+0-3x^2-2x) = 3x^2+2x-1$$

AT (1,1) WE OBTAIN

$$\left.\frac{dy}{dx}\right|_{(1,1)}(\cancel{2}+1-\cancel{1}-\cancel{1}) + 1(2-3-2) = 3+2-1$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} + (-3) = 4$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = 7$$

AS REQUIRED

ALTERNATIVE IS TO DIFFERENTIATE W.R.T x STRAIGHT AWAY

$$\frac{(x+1)\times 1 - x(1)}{(x+1)^2} + \frac{(y+1)\frac{dy}{dx} - y(1)\frac{dy}{dx}}{(y+1)^2} = 2x$$

$$\frac{1}{(x+1)^2} + \frac{dy}{dx}\left(\frac{1}{(y+1)^2}\right) = 2x \quad \text{ETC}$$

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MULTIPLY THROUGH AND TIDY

$$\Rightarrow \frac{x}{x+1} + \frac{y}{y+1} = x^2$$

$$\Rightarrow x(y+1) + y(x+1) = x^2(x+1)(y+1)$$

$$\Rightarrow xy + x + xy + y = x^2(xy + x + y + 1)$$

$$\Rightarrow 2xy + x + y = x^3y + x^3 + x^2y + x^2$$

DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}(2xy) + \frac{d}{dx}(x) + \frac{d}{dx}(y) = \frac{d}{dx}(x^3y) + \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2y) + \frac{d}{dx}(x^2)$$

$$\Rightarrow 2y + 2x\frac{dy}{dx} + 1 + \frac{dy}{dx} = 3x^2y + x^3\frac{dy}{dx} + 3x^2 + 2xy + x^2\frac{dy}{dx} + 2x$$

AT $x=1$ & $y=1$

$$\Rightarrow 2 + 2\frac{dy}{dx} + 1 + \frac{dy}{dx} = 3 + \frac{dy}{dx} + 3 + 2 + \frac{dy}{dx} + 2$$

$$\Rightarrow 3 + 3\frac{dy}{dx} = 2\frac{dy}{dx} + 10$$

$$\Rightarrow \frac{dy}{dx} = 7$$

AS REQUIRED

ALTERNATIVE WITHOUT ANY INITIAL TIDY UP

$$\Rightarrow \frac{d}{dx}\left[\frac{x}{x+1}\right] + \frac{d}{dx}\left[\frac{y}{y+1}\right] = \frac{d}{dx}\left[x^2\right]$$

$$\Rightarrow \frac{d}{dx}\left[\frac{(x+1)-1}{(x+1)}\right] + \frac{d}{dx}\left[\frac{(y+1)-1}{(y+1)}\right] = 2x$$

$$\Rightarrow \frac{d}{dx}\left[1 - \frac{1}{x+1}\right] + \frac{d}{dx}\left[1 - \frac{1}{y+1}\right] = 2x$$

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$$\Rightarrow \frac{d}{dx}\left[1-(x+1)^{-1}\right]+\frac{d}{dx}\left[1-(y+1)^{-1}\right]=2x$$

$$\Rightarrow 0+(x+1)^{-2}+0+(y+1)^{-2}\times\frac{dy}{dx}=2x$$

$$\Rightarrow \frac{1}{(x+1)^2}+\frac{1}{(y+1)^2}\frac{dy}{dx}=2x$$

AT $x=1,\ y=1$

$$\Rightarrow \frac{1}{4}+\frac{1}{4}\frac{dy}{dx}=2$$

$$\Rightarrow 1+\frac{dy}{dx}=8$$

$$\Rightarrow \frac{dy}{dx}=7$$

AS REQUIRED

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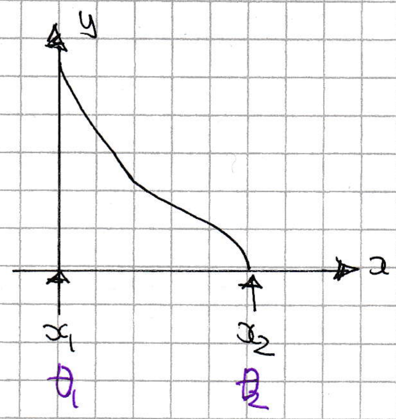
a) LOOKING AT THE DIAGRAM



TO FIND θ_1 SET $x=0$

$0 = \cos^3\theta$

$\cos\theta = 0$

$\theta = \begin{cases} \pi/2 \leftarrow \theta_1 \\ \cancel{3\pi/2} \end{cases}$ THIS PRODUCES $y=-12$

TO FIND θ_2, SET $y=0$

$12\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0, \cancel{\pi}$ THIS PRODUCES $x=-1$

θ_2 is 0

SETTING UP AN INTEGRAL

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{\theta_1}^{\theta_2} y(\theta)\frac{dx}{d\theta}\,d\theta = \int_{\pi/2}^{0} 12\sin\theta\left(-3\cos^2\theta\sin\theta\right)d\theta$$

$$= \int_{\pi/2}^{0} -36\cos^2\theta\sin^2\theta\,d\theta = +36\int_{0}^{\frac{\pi}{2}} \cos^2\theta\sin^2\theta\,d\theta = 36\int_{0}^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$$

AS REQUIRED

b) USING $\cos 2\theta \equiv 2\cos^2\theta - 1$ & $\cos 2\theta \equiv 1 - 2\sin^2\theta$

$$\cos^2\theta\sin^2\theta = \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right)\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta\right)$$

DIFFERENCE OF SQUARES

$$= \tfrac{1}{4} - \tfrac{1}{4}\cos^2 2\theta$$

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NOW REAPPLY THE IDENTITY $\cos 2\theta = 2\cos^2\theta - 1$ AS $\cos 4\theta = 2\cos^2 2\theta - 1$

$$= \frac{1}{4} - \frac{1}{4}\cos^2 2\theta = \frac{1}{4} - \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 4\theta\right) = \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4\theta$$

$$= \frac{1}{8} - \frac{1}{8}\cos 4\theta = \frac{1}{8}(1 - \cos 4\theta)$$

AS REQUIRED

c) INTEGRATE, EVALUATE AND MULTIPLY BY 4 TO FIND THE TOTAL AREA

$$\text{AREA} = 4\int_0^{\frac{\pi}{2}} 36\sin^2\theta\cos^2\theta \, d\theta = 4\int_0^{\frac{\pi}{2}} 36 \times \frac{1}{8}(1 - \cos 4\theta)\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 18 - 18\cos 4\theta \, d\theta = \left[18\theta - \frac{9}{2}\cos 4\theta\right]_0^{\pi/2}$$

$$= \left(9\pi - \cancel{\frac{9}{2}}\right) - \left(0 - \cancel{\frac{9}{2}}\right) = 9\pi$$

MP2 – IYGB – PAPER A – QUESTION 8

a) TRANSLATION BY THE VECTOR $\binom{3}{1}$ IMPLIES $f(x-3)+1$

$$\therefore \quad y = \frac{1}{(x-3)^3+1} + 1$$

NEED NOT SIMPLIFIED

b) THIS IS DIFFICULT TO SEE BUT THIS IS A DOUBLE REFLECTION

- REPLACE x BY $-x$ $\Longrightarrow$ $y = \dfrac{1}{(-x)^3+1} = \dfrac{1}{-x^3+1}$

  $f(-x)$

- MULTIPLY THE EXPRESSION BY -1 $\Longrightarrow$ $y = -\left(\dfrac{1}{-x^3+1}\right) = \dfrac{1}{x^3-1}$

  $-f(x)$

$\therefore$ REFLECTION ABOUT THE x AXIS, FOLLOWED BY REFLECTION ABOUT THE y AXIS — IN ANY ORDER

NOTE THAT A DOUBLE REFLECTION SUCH AS THE ONE ABOVE IS A ROTATION ABOUT THE ORIGIN BY 180°, BUT THIS ANSWER DOES NOT QUALIFY AS IT ASKS FOR "A SEQUENCE OF 2 TRANSFORMATIONS"

IYGB – MP2 PAPER A – QUESTION 9

a) COMPLETING THE SQUARE

$$\Rightarrow f(x) = 3x^2 - 18x + 21$$
$$\Rightarrow \frac{1}{3}f(x) = x^2 - 6x + 7$$
$$\Rightarrow \frac{1}{3}f(x) = (x-3)^2 - 9 + 7$$
$$\Rightarrow \frac{1}{3}f(x) = (x-3)^2 - 2$$
$$\Rightarrow f(x) = 3(x-3)^2 - 6$$

b) USING PART (a)

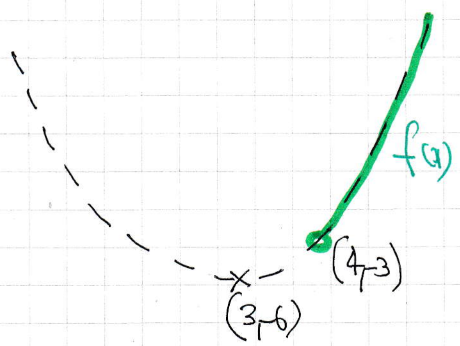
$$\Rightarrow y = 3(x-3)^2 - 6$$
$$\Rightarrow y + 6 = 3(x-3)^2$$
$$\Rightarrow \frac{y+6}{3} = (x-3)^2$$
$$\Rightarrow x - 3 = \pm\sqrt{\frac{y+6}{3}}$$

BUT $x > 4$ SO LHS IS POSITIVE

$$\Rightarrow x - 3 = +\sqrt{\frac{y+6}{3}}$$
$$\Rightarrow x = 3 + \sqrt{\frac{y+6}{3}}$$

$$\therefore f^{-1}(x) = 3 + \sqrt{\frac{x+6}{3}}$$

c) SKETCHING $f(x)$ TO SEE ITS RANGE



	f	f^{-1}
D	$x > 4$	$x > -3$
R	$f(x) > -3$	$f^{-1}(x) > 4$

$\therefore$ DOMAIN : $x > -3$

RANGE : $f^{-1}(x) > 4$

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DIFFERENTIATE VIA QUOTIENT RULE & TIDY

$$f'(x) = \frac{(3-\sin x)(-\sin x) - \cos x(-\cos x)}{(3-\sin x)^2}$$

$$= \frac{-3\sin x + \sin^2 x + \cos^2 x}{(3-\sin x)^2} = \frac{1-3\sin x}{(3-\sin x)^2}$$

SOLVING FOR ZERO

$$1 - 3\sin x = 0$$

$$\sin x = \frac{1}{3} \leftarrow \text{"STATIONARY VALUE"}$$

USING $\sin^2\theta + \cos^2\theta \equiv 1$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos x = \pm\sqrt{1-\sin^2 x}$$

$$\Rightarrow \cos x = \pm\sqrt{1-\left(\tfrac{1}{3}\right)^2}$$

$$\Rightarrow \cos x = \pm\sqrt{\tfrac{8}{9}}$$

$$\Rightarrow \cos x = \pm\tfrac{2}{3}\sqrt{2}$$

FINALLY USING $\sin x = \frac{1}{3}$ WITH $\cos x = \pm\frac{2}{3}\sqrt{2}$

$$\frac{\cos x}{3-\sin x} = \frac{+\frac{2}{3}\sqrt{2}}{3-\frac{1}{3}} = \frac{2\sqrt{2}}{9-1} = \frac{1}{4}\sqrt{2}$$

$$= \frac{-\frac{2}{3}\sqrt{2}}{3-\frac{1}{3}} = \frac{-2\sqrt{2}}{9-1} = -\frac{1}{4}\sqrt{2}$$

$$\therefore \; -\tfrac{1}{4}\sqrt{2} \le f(x) \le \tfrac{1}{4}\sqrt{2}$$

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FORMING A DIFFERENTIAL EQUATION

$$\frac{dy}{dt} = -k\,y^{\frac{1}{3}}$$

RATE

LEAKING

PROPORTIONAL

CUBE ROOT OF THE HEIGHT

y = HEIGHT (cm)
t = TIME (min)

SOLVING BY SEPARATING VARIABLES

$$\Longrightarrow dy = -ky^{\frac{1}{3}}\,dt$$

$$\Longrightarrow \frac{1}{y^{\frac{1}{3}}}\,dy = -k\,dt$$

$$\Longrightarrow \int y^{-\frac{1}{3}}\,dy = \int -k\,dt$$

$$\Longrightarrow \frac{3}{2}y^{\frac{2}{3}} = -kt + C$$

$$\Longrightarrow \boxed{y^{\frac{2}{3}} = At + B} \qquad \left(A = -\frac{2k}{3},\ B = \frac{2C}{3}\right)$$

APPLY THE CONDITIONS GIVEN

$t=0,\ y=125 \Longrightarrow 125^{\frac{2}{3}} = B$

$\Longrightarrow B = 25$

$\Longrightarrow y^{\frac{2}{3}} = At + 25$

$t=3,\ y=64 \Longrightarrow 64^{\frac{2}{3}} = A\times 3 + 25$

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$$\Longrightarrow 16 = 3A + 25$$

$$\Longrightarrow -9 = 3A$$

$$\Longrightarrow A = -3$$

FINALLY USING THE FORMULA OBTAINED

$$\Longrightarrow y^{\frac{2}{3}} = 25 - 3t$$

$$\Longrightarrow y^{\frac{2}{3}} = 25 - 3\left(7 + \frac{7}{12}\right)$$

$$\Longrightarrow y^{\frac{2}{3}} = \frac{9}{4}$$

$$\Longrightarrow \left(y^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(\frac{9}{4}\right)^{\frac{3}{2}}$$

$$\Longrightarrow y = \frac{27}{8} = 3.375$$

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a) FILL IN THE TABLE

x	0	0.1	0.2	0.3	0.4	0.5
y	0	$\frac{1}{120}$	$\frac{1}{35}$	$\frac{9}{160}$	$\frac{4}{45}$	$\frac{1}{8}$

b) APPROXIMATING BY THE TRAPEZIUM RULE

$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx \approx \text{"}\frac{\text{THICKNESS}}{2}\left[\text{FIRST + LAST + 2 × REST}\right]\text{"}$$

$$\approx \frac{0.1}{2}\left[0+\frac{1}{8}+2\left[\frac{1}{120}+\frac{1}{35}+\frac{9}{160}+\frac{4}{45}\right]\right]$$

$$\approx \frac{493}{20160}$$

$$\approx 0.02445 \quad \text{4 s.f.}$$

c) BY THE SUBSTITUTION GIVEN WE HAVE

- $u = 2x+1 \implies 2x = u-1$
$$\implies \boxed{4x^2 = u^2-2u+1}$$

- $\frac{du}{dx} = 2$
$$du = 2\,dx$$
$$\boxed{dx = \frac{1}{2}du}$$

- $x=0 \longmapsto u=1$
$$x=\tfrac{1}{2} \longmapsto u=2$$

TRANSFORMING THE INTEGRAL

$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx = \frac{1}{4}\int_0^{\frac{1}{2}} \frac{4x^2}{u}\left(\frac{1}{2}du\right)$$

$$= \frac{1}{8}\int_1^2 \frac{u^2-2u+1}{u}\,du$$

$$= \frac{1}{8}\int_1^2 \frac{u^2}{u}-\frac{2u}{u}+\frac{1}{u}\,du$$

$$= \frac{1}{8}\int_1^2 u-2+\frac{1}{u}\,du$$

$$= \frac{1}{8}\left[\frac{1}{2}u^2-2u+\ln|u|\right]_1^2$$

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$$\ldots = \frac{1}{8}\left[\left(\cancel{2}-\cancel{4}+\ln 2\right)-\left(\frac{1}{2}-\cancel{2}+\cancel{\ln 1}\right)\right]$$

$$= \frac{1}{8}\left(\ln 2 - \frac{1}{2}\right)$$

$$= \frac{1}{16}\left(-1 + 2\ln 2\right)$$

d) FROM PART (b) $\displaystyle\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx \approx 0.02445$

FROM PART (c) $\displaystyle\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx = \frac{1}{16}(-1+2\ln 2)$

$$\Rightarrow \frac{1}{16}(-1+2\ln 2) \simeq 0.02445$$

$$\Rightarrow -1 + 2\ln 2 \approx 0.912$$

$$\Rightarrow 2\ln 2 \approx 1.3912$$

$$\Rightarrow \ln 2 \simeq 0.70$$

2 sf.

IYGB – MP2 PAPER A – QUESTION 13

USING THE IDENTITY $1+\cot^2\theta \equiv \operatorname{cosec}^2\theta \iff \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$

$$\Rightarrow \operatorname{cosec}^4\theta - \cot^4\theta = \frac{2}{3} + \sqrt{3}\cot\theta$$

DIFFERENCE OF SQUARES

$$\Rightarrow \underbrace{(\operatorname{cosec}^2\theta - \cot^2\theta)}_{=1}(\operatorname{cosec}^2\theta + \cot^2\theta) = \frac{2}{3} + \sqrt{3}\cot\theta$$

$$\Rightarrow \operatorname{cosec}^2\theta + \cot^2\theta = \frac{2}{3} + \sqrt{3}\cot\theta$$

$$\Rightarrow (1+\cot^2\theta) + \cot^2\theta = \frac{2}{3} + \sqrt{3}\cot\theta$$

$$\Rightarrow 2\cot^2\theta + 1 = \frac{2}{3} + \sqrt{3}\cot\theta$$

$\times 3$

$$\Rightarrow 6\cot^2\theta + 3 = 2 + 3\sqrt{3}\cot\theta$$

$$\Rightarrow 6\cot^2\theta - 3\sqrt{3}\cot\theta + 1 = 0$$

QUADRATIC FORMULA YIELDS

$$\cot\theta = \frac{3\sqrt{3} \pm \sqrt{27-4\times6\times1}}{2\times6} = \frac{3\sqrt{3}\pm\sqrt{3}}{12} = \begin{cases} \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3} \\ \frac{2\sqrt{3}}{12} = \frac{\sqrt{3}}{6} \end{cases}$$

$$\therefore \tan\theta = \begin{cases} \frac{3}{\sqrt{3}} = \sqrt{3} \\ \frac{6}{\sqrt{3}} = 2\sqrt{3} \end{cases}$$

INVERTING & SETTING A SOLUTION IN RADIANS

- $\tan\theta = \sqrt{3}$
- $\theta = \frac{\pi}{3} \pm n\pi \quad n = 0,1,2,3\ldots$

- $\tan\theta = 2\sqrt{3}$
- $\theta = 1.28976^c \pm n\pi \quad n = 0,1,2,3,\ldots$

$\theta_1 = 1.05^c \left(\frac{\pi}{3}\right)$

$\theta_2 = 4.19^c \left(\frac{4\pi}{3}\right)$

$\theta_3 = 1.29^c$

$\theta_4 = 4.43^c$