IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper C

Difficulty Rating: 3.565/1.1499

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 11 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$f(x) = \sqrt{225 + 15x}, \quad |x| < 15.$$

a) Expand $f(x)$ as an infinite series, up and including the term in x^2. **(4)**

b) By substituting $x = 1$ in the expansion of $f(x)$, show that

$$\sqrt{15} \approx \frac{1859}{480}.$$ **(3)**

Question 2

Three consecutive terms in geometric progression are given in sequential order as

$$(1 - 5p), \quad \frac{1}{2} \quad \text{and} \quad (4p - 2),$$

where p is a constant.

Show that one possible value of p is $\frac{1}{4}$ and find the other. **(5)**

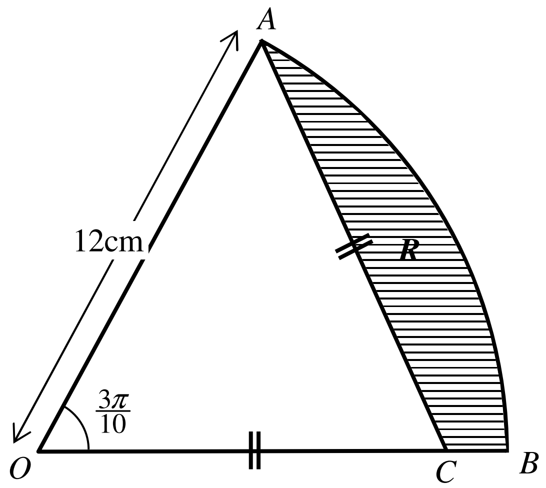
Question 3

The curve with equation $y = 2^x$ intersects the straight line with equation $y = 3 - 2x$ at the point P, whose x coordinate is α.

a) Show that $0 < \alpha < 1$. **(3)**

b) Starting with $x = 0.5$, use the Newton Raphson method to find the value of α, correct to 3 decimal places. **(6)**

Question 4



The figure above shows a circular arc OAB of radius 12 cm, subtending an angle of $\frac{3\pi}{10}$ radians at O.

Find to three significant figures …

a) … the length of the arc AB. **(2)**

b) … the area of the sector OAB. **(2)**

The point C lies on OB so that $OC = AC$.

The region R, shown shaded in the figure, is bounded by the arc AB and the straight lines AC and BC.

c) Determine, to three significant figures, the perimeter and area of R. **(5)**

Question 5

Use the method of **proof by contradiction** to show that if x then

$$\left| x + \frac{1}{x} \right| \geq 2.$$

(5)

Question 6

The variable points $A(2t,t,2)$ and $B(t,4,1)$, where t is a scalar variable, are referred relative to a fixed origin O.

a) Show that

$$\left|\overrightarrow{AB}\right| = \sqrt{2t^2 - 8t + 17}\,. \qquad (3)$$

b) Hence find the shortest distance between A and B, as t varies. **(5)**

Question 7

A curve C is given by the parametric equations

$$x = \frac{3t-2}{t-1}, \quad y = \frac{t^2 - 2t + 2}{t-1}, \quad t \in \mathbb{R}, \quad t \neq 1.$$

a) Show clearly that

$$\frac{dy}{dx} = 2t - t^2. \qquad (7)$$

The point $P\left(1, -\frac{5}{2}\right)$ lies on C.

b) Show that the equation of the tangent to C at the point P is

$$3x - 4y - 13 = 0. \qquad (4)$$

Question 8

$$y = \frac{e^{2x}}{e^x + 1}, \quad x \in \mathbb{R}$$

a) Calculate the missing values of x and y in the following table.

x	$\ln 2$	x_2	x_3	x_4	$\ln 8$
y	1.333	y_2	y_3	y_4	7.111

(3)

b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_{\ln 2}^{\ln 8} \frac{e^{2x}}{e^x + 1} \, dx.$$

(3)

c) Use the substitution $u = e^x + 1$ to find an exact simplified value for

$$\int_{\ln 2}^{\ln 8} \frac{e^{2x}}{e^x + 1} \, dx.$$

(6)

Question 9

The shape of a weather balloon remains spherical at all times.

It is filled with a special type of gas and is floating at very high altitude.

The rate at which the volume of the balloon is decreasing is directly proportional to the square of the surface area of the balloon at that instant.

Let r m be the radius of the balloon, t hours since $r = 5$.

a) By relating the volume, the surface area and the radius of the weather balloon show that

$$\frac{dr}{dt} = -kr^2,$$

where k is a positive constant. **(4)**

$\left[\text{volume of a sphere of radius } r \text{ is given by } \tfrac{4}{3}\pi r^3\right]$

$\left[\text{surface area of a sphere of radius } r \text{ is given by } 4\pi r^2\right]$

When $t = 10$, $r = 4.8$.

b) Determine the value of t when $r = 4$. **(7)**

Question 10

$$y = \cot x, \ 0 < x < \tfrac{\pi}{2}.$$

Show, with detailed workings, that

a) $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. **(3)**

b) $\dfrac{d^2y}{dx^2} = 2y\left(y^2 + 1\right)$. **(5)**

Question 11

$$f(x) = \sqrt{3}\sin x + \cos x,\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R > 0$, $0 < \alpha < 90°$. **(3)**

b) State the maximum value of $f(x)$ and find the smallest positive value of x for which this maximum occurs. **(3)**

The temperature of the water T °C in a tropical fish tank is modelled by the equation

$$T = 32 + \sqrt{3}\sin(15t)° + \cos(15t)°,\quad 0 \le t < 24,$$

where t is the time in hours measured since midnight.

c) State the maximum temperature of the water in the tank and the time when this maximum temperature occurs. **(3)**

d) Show that the temperature of the water in the tank reaches 30.5 °C at 13:14 and at 18:46.

[*You may not verify the answers in this part*] **(6)**