IYGB - MP2 PAPER C - QUESTION 1

a) MANIPULATE INTO THE FORM $k(1+ax)^{\frac{1}{2}}$ & EXPAND

$$f(x) = \sqrt{225+15x} = \sqrt{225\left(1+\frac{1}{15}x\right)} = \sqrt{225}\sqrt{1+\frac{1}{15}x}$$

$$= 15\left(1+\frac{1}{15}x\right)^{\frac{1}{2}} = 15\left[1+\frac{\frac{1}{2}}{1}\left(\frac{1}{15}x\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}\left(\frac{1}{15}x\right)^2+O(x^3)\right]$$

$$= 15\left[1+\frac{1}{30}x-\frac{1}{1800}x^2+O(x^3)\right]$$

$$= 15+\frac{1}{2}x-\frac{1}{120}x^2+O(x^3)$$

b) START FROM THE ENTIRE EXPRESSION (BOTH SIDES)

$$\sqrt{225+15x} \simeq 15+\frac{1}{2}x-\frac{1}{120}x^2$$

$$\sqrt{225+15} \simeq 15+\frac{1}{2}-\frac{1}{120}$$

$$\sqrt{240} \simeq \frac{1859}{120}$$

$$4\sqrt{15} \simeq \frac{1859}{120}$$

$$\sqrt{15} \simeq \frac{1859}{480}$$

AS REQUIRED

IYGB - MP2 PAPER C - QUESTION 2

GEOMETRIC IMPLIES RATIO BETWEEN SUCCESSIVE TERM REMAINS CONSTANT

EITHER $\frac{\frac{1}{2}}{1-5p} = \frac{4p-2}{\frac{1}{2}}$ **OR** $1-5p \xrightarrow{\times r} \frac{1}{2} \xrightarrow{\times r} 4p-2$

$\Rightarrow \frac{1}{4} = (1-5p)(4p-2)$

$\Rightarrow \frac{1}{4} = 4p-2-20p^2+10p$

$\Rightarrow \frac{1}{4} = -20p^2+14p-2$

$\Rightarrow 1 = -80p^2+56p-8$

$\Rightarrow 80p^2-56p+9=0$

$\Rightarrow (4p-1)(20p-9)=0$

$\Rightarrow p = \begin{cases} 1/4 \\ \frac{9}{20} \end{cases}$

$\left\{ \begin{array}{l} (1-5p)r = \frac{1}{2} \\ \frac{1}{2}r = 4p-2 \end{array} \right\}$

IYGB - MP2 PAPER C - QUESTION 3

a) SOLVING SIMULTANEOUSLY

$$\Rightarrow 3 - 2x = 2^x$$

$$\Rightarrow 2^x + 2x - 3 = 0$$

$$\Rightarrow f(x) = 2^x + 2x - 3$$

$$f(0) = 1 + 0 - 3 = -2 < 0$$

$$f(1) = 2 + 2 - 3 = 1 > 0$$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE INTERVAL $(0,1)$, THERE MUST BE AT LEAST ONE ROOT IN $(0,1)$

b) PREPARING FOR NEWTON-RAPHSON METHOD

$$f(x) = 2^x + 2x - 3$$

$$f'(x) = 2^x \ln 2 + 2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2^{x_n} + 2x_n - 3}{2^{x_n}\ln 2 + 2}$$

$$= \frac{x_n 2^{x_n}\ln 2 + \cancel{2x_n} - 2^{x_n} - \cancel{2x_n} + 3}{2^{x_n}\ln 2 + 2}$$

$$\therefore \quad x_{n+1} = \frac{3 + 2^{x_n}(x_n \ln 2 - 1)}{2^{x_n}\ln 2 + 2}$$

USING ABOVE FORMULA WITH $x_1 = 0.5$

$x_1 = 0.5$

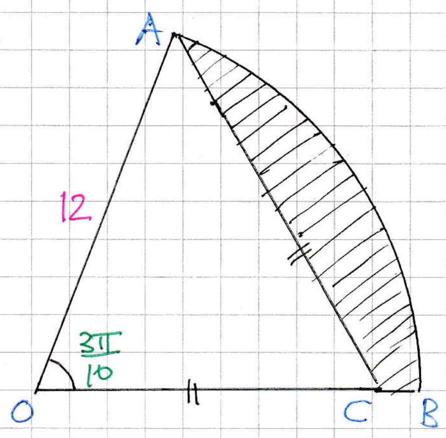
$x_2 = 0.6965556034\ldots$

$x_3 = 0.6921557681\ldots$

$x_4 = 0.6921533549\ldots$

$$\therefore \quad x \simeq 0.692 \quad (3 \text{ d.p.})$$

IYGB – MP2 PAPER C – QUESTION 4



a) ARC LENGTH $\overset{\frown}{AB}$, USING $L = r\theta^c$

$$L = r\theta^c = 12 \times \frac{3\pi}{10} = \frac{18\pi}{5}$$

$$\simeq 11.3 \text{ cm}$$

b) AREA OF SECTOR $\overset{\frown}{OAB}$

$$\frac{1}{2}r^2\theta^c = \frac{1}{2} \times 12^2 \times \frac{3\pi}{10} = \frac{108}{5}\pi \simeq 67.9 \text{ cm}^2$$

c) PROCEED AS FOLLOWS

$$O\hat{A}C = \frac{3\pi}{10}$$

$$O\hat{C}A = \pi - 2 \times \frac{3\pi}{10} = \frac{2\pi}{5}$$

BY THE SINE RULE ON $\overset{\triangle}{OAC}$

$$\frac{|OC|}{\sin(OAC)} = \frac{|OA|}{\sin(OCA)} \implies \frac{|OC|}{\sin\frac{3\pi}{10}} = \frac{12}{\sin\frac{2\pi}{5}}$$

$$\implies |OC| = \frac{12\sin\frac{3\pi}{10}}{\sin\frac{2\pi}{5}}$$

$$\implies |OC| = 10.2078\ldots \text{ cm}$$

$\therefore$ PERIMETER $= |CA| + |CB| + |AB| = |CA| + \big[|OB| - |OC|\big] + |AB|$

$$= \cancel{10.2078\ldots} + 12 - \cancel{10.2078\ldots} + 11.3 = 23.3 \text{ cm}$$

$\therefore$ AREA $=$ (AREA OF THE SECTOR OAB) $-$ (AREA OF TRIANGLE OAC)

$$= \frac{108}{5}\pi - \frac{1}{2}|OA||OC|\sin\frac{3\pi}{10}$$

$$= \frac{108}{5}\pi - \frac{1}{2} \times 12 \times 10.2078 \times \sin\frac{3\pi}{10}$$

$$\simeq 18.3 \text{ cm}^2$$

IYGB - MP2 PAPER C - QUESTION 5

NEED TO PROVE THAT FOR ALL REAL NUMBERS $\left|x+\frac{1}{x}\right| \geqslant 2$

SUPPOSE THE OPPOSITE

$$\left|x+\frac{1}{x}\right| < 2$$

SQUARING BOTH SIDES

$$\left|x+\frac{1}{x}\right|^2 < 4$$

$$\left(x+\frac{1}{x}\right)^2 < 4$$

$$x^2+2+\frac{1}{x^2} < 4$$

$$x^2-2+\frac{1}{x^2} < 0$$

$$\left(x-\frac{1}{x}\right)^2 < 0$$

BUT THIS IS A CONTRADICTION AS NO REAL QUANTITY SQUARED CAN BE NEGATIVE, AND THEREFORE THE ORIGINAL ASSUMPTION IS FALSE

$$\therefore \left|x+\frac{1}{x}\right| \geqslant 2 \quad //$$

IYGB – MP2 PAPER C – QUESTION 6

$A(2t, t, 2) \qquad B(t, 4, 1)$

a) $|\overrightarrow{AB}| = |\underline{b} - \underline{a}| = |(t,4,1) - (2t,t,2)| = |-t, 4-t, -1|$

$= \sqrt{(-t)^2 + (4-t)^2 + (-1)^2} = \sqrt{t^2 + 16 - 8t + t^2 + 1}$

$= \sqrt{2t^2 - 8t + 17}$ AS REQUIRED

b) BY COMPLETING THE SQUARE (OR CALCULUS)

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2t^2 - 8t + 17}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2\left(t^2 - 4t + \frac{17}{2}\right)}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2\left[(t-2)^2 - 4 + \frac{17}{2}\right]}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2(t-2)^2 - 8 + 17}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2(t-2)^2 + 9}$

HENCE $|\overrightarrow{AB}|_{MIN} = 3$ $\leftarrow \sqrt{9}$

WHICH OCCURS WHEN $t=2$

IYGB - MP2 PAPER C - QUESTION 7

a) CARRY OUT INDIVIDUAL DIFFERENTIATIONS BY QUOTIENT RULE

$$x = \frac{3t-2}{t-1} \qquad\qquad y = \frac{t^2-2t+2}{t-1}$$

$$\frac{dx}{dt} = \frac{(t-1)\times 3 - (3t-2)\times 1}{(t-1)^2} \qquad\qquad \frac{dy}{dt} = \frac{(t-1)(2t-2) - (t^2-2t+2)\times 1}{(t-1)^2}$$

$$\frac{dx}{dt} = \frac{3t-3-3t+2}{(t-1)^2} \qquad\qquad \frac{dy}{dt} = \frac{2t^2-4t+2-t^2+2t-2}{(t-1)^2}$$

$$\frac{dx}{dt} = -\frac{1}{(t-1)^2} \qquad\qquad \frac{dy}{dt} = \frac{t^2-2t}{(t-1)^2}$$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{t^2-2t}{(t-1)^2}}{-\frac{1}{(t-1)^2}} = -(t^2-2t) = 2t-t^2$

AS REQUIRED

b) USING $x=1$ INTO $x = \frac{3t-2}{t-1}$

$$1 = \frac{3t-2}{t-1}$$

$$t-1 = 3t-2$$

$$1 = 2t$$

$$t = \frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{1}{2}} = 2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

FINALLY THE EQUATION OF THE TANGENT THROUGH $\left(1, -\frac{5}{2}\right)$

$$y - y_0 = m(x - x_0)$$

$$y + \frac{5}{2} = \frac{3}{4}(x-1)$$

$$4y + 10 = 3(x-1)$$

$$4y + 10 = 3x - 3$$

$$3x - 4y - 13 = 0$$

AS REQUIRED

IYGB - MP2 PAPER C - QUESTION 8

a) DETERMINE THE "GAP"

$$\frac{\ln 8 - \ln 2}{4} = \frac{3\ln 2 - \ln 2}{4} = \frac{2\ln 2}{4} = \frac{1}{2}\ln 2$$

$$\Rightarrow x_2 = \ln 2 + \tfrac{1}{2}\ln 2 = \tfrac{3}{2}\ln 2 \qquad y_2 = 2.090$$

$$\Rightarrow x_3 = \tfrac{3}{2}\ln 2 + \tfrac{1}{2}\ln 2 = 2\ln 2 \qquad y_3 = 3.2$$

$$\Rightarrow x_4 = 2\ln 2 + \tfrac{1}{2}\ln 2 = \tfrac{5}{2}\ln 2 \qquad y_4 = 4.807$$

b) USING THE STANDARD FORMULA

$$\int_{\ln 2}^{\ln 8} \frac{e^{2x}}{e^{x}+1}\,dx \approx \frac{\text{"THICKNESS"}}{2}\left[\text{FIRST} + \text{LAST} + 2\sum \text{REST}\right]$$

$$\approx \frac{\frac{1}{2}\ln 2}{2}\left[1.333 + 7.111 + 2(2.090 + 3.2 + 4.807)\right]$$

$$\approx \tfrac{1}{4}\ln 2 \times \left[28.638\right]$$

$$\approx 4.96$$

c) USING THE SUBSTITUTION GIVEN

- $u = e^{x}+1$
- $\frac{du}{dx} = e^{x}$
- $dx = \frac{du}{e^{x}}$

$x = \ln 2 \mapsto u = 3$

$x = \ln 8 \mapsto u = 9$

$e^{x} = u - 1$

$$\int_{\ln 2}^{\ln 2} \frac{e^{2x}}{e^{x}+1}\,dx = \int_{3}^{9} \frac{\cancel{e^{2x}}^{\,e^{x}}}{u}\,\frac{du}{\cancel{e^{x}}}$$

$$= \int_{3}^{9} \frac{e^{x}}{u}\,du = \int_{3}^{9} \frac{u-1}{u}\,du$$

$$= \int_{3}^{9} 1 - \frac{1}{u}\,du = \Big[u - \ln|u|\Big]_{3}^{9}$$

$$= (9 - \ln 9) - (3 - \ln 3) = 6 + \ln 3 - \ln 9$$

$$= 6 + \ln\left(\tfrac{1}{3}\right) = 6 - \ln 3$$

IYGB - MP2 PAPER C - QUESTION 9

a) FORMING A DIFFERENTIAL EQUATION

$$\Rightarrow \frac{dV}{dt} = -cA^2 \qquad (c = \text{PROPORTIONALITY CONSTANT})$$

$$\Rightarrow \frac{dV}{dr} \times \frac{dr}{dt} = -c\left(4\pi r^2\right)^2$$

$$\Rightarrow \cancel{4\pi r^2} \times \frac{dr}{dt} = -c(4\pi r^2)^{\cancel{2}}$$

$$\Rightarrow \frac{dr}{dt} = -c(4\pi r^2)$$

$$\Rightarrow \frac{dr}{dt} = -(4\pi c)r^2$$

$$\Rightarrow \frac{dr}{dt} = -kr^2$$

AS REQUIRED

t = TIME (hours)
A = SURFACE AREA (m^2)
r = RADIUS (m)
V = VOLUME (m^3)

$t=0, \; r=5$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

b) SOLVING THE O.D.E BY SEPARATING VARIABLES

$$\Rightarrow dr = -kr^2 dt$$

$$\Rightarrow -\frac{1}{r^2}\,dr = k\,dt$$

$$\Rightarrow \int -\frac{1}{r^2}\,dr = \int k\,dt$$

$$\Rightarrow \boxed{\frac{1}{r} = kt + B}$$

APPLY THE INITIAL CONDITION, $t=0 \quad r=5$

$$\Rightarrow \frac{1}{5} = 0 + B$$

$$\Rightarrow B = \frac{1}{5}$$

$$\Rightarrow \boxed{\frac{1}{r} = kt + \frac{1}{5}}$$

IYGB - MP2 PAPER C - QUESTION 9

APPLY BOUNDARY CONDITION $t=10$, $r=4.8$

$$\Rightarrow \frac{1}{4.8} = k \times 10 + \frac{1}{5}$$

$$\Rightarrow \frac{5}{24} = 10k + \frac{1}{5}$$

$$\Rightarrow 10k = \frac{1}{120}$$

$$\Rightarrow k = \frac{1}{1200}$$

$$\therefore \boxed{\frac{1}{r} = \frac{t}{1200} + \frac{1}{5}}$$

FINALLY WE OBTAIN

when $r=4$

$$\frac{1}{4} = \frac{t}{1200} + \frac{1}{5} \quad \times 1200$$

$$300 = t + 240$$

$$t = 60$$

IYGB - MP2 PAPER C - QUESTION 10

a) SWITCH INTO SINES AND COSINES & USE THE QUOTIENT RULE

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

b) PROCEED AS FOLLOWS

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{dy}{dx} = -(1 + \cot^2 x)$$

$$\Rightarrow \frac{dy}{dx} = -(1 + y^2)$$

$$\Rightarrow \frac{dy}{dx} = -1 - y^2$$

DIFFERENTIATE W.R.T x

$$\Rightarrow \frac{d^2y}{dx^2} = 0 - 2y\frac{dy}{dx}.$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2y\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2y\left[-1 - y^2\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2y(1 + y^2)$$

AS REQUIRED

OR BY DIRECT DIFFERENTIATION

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2\operatorname{cosec}x(-\operatorname{cosec}x\cot x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\cot x\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\cot x(1 + \cot^2 x)$$

BUT $y = \cot x$

$$\Rightarrow \frac{d^2y}{dx^2} = 2y(1 + y^2)$$

AS BEFORE

IYGB - MP2 PAPER C - QUESTION 11

a) USING THE COMPOUND ANGLE IDENTITY FOR $\cos(A-B)$

$$\sqrt{3}\sin x + \cos x \equiv R\cos(x-\alpha)$$
$$\equiv R\cos x\cos\alpha + R\sin x\sin\alpha$$
$$\equiv (R\sin\alpha)\sin x + (R\cos\alpha)\cos x$$

- $R\sin\alpha = \sqrt{3}$
- $R\cos\alpha = 1$

SQUARE & ADD YIELDS: $R = \sqrt{(\sqrt{3})^2+1^2}$

$R = 2$

DIVIDING SIDE BY SIDE: $\tan\alpha = \sqrt{3}$

$\alpha = 60°$

$$\therefore f(x) = 2\cos(x-60°)$$

ALTERNATIVE BY MANIPULATION

$$\sqrt{3}\sin x + \cos x = 2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right] = 2\left[\underbrace{\tfrac{1}{2}}_{\cos 60}\cos x + \underbrace{\tfrac{\sqrt{3}}{2}}_{\sin 60}\sin x\right]$$

$$= 2\left[\cos 60\cos x + \sin 60 \sin x\right] = 2\cos(60-x)$$

$$= 2\cos(x-60) \quad \text{(COSINE IS EVEN)}$$

b) MAX VALUE OF $f(x) = 2\cos(x-60°)$ IS 2

(AS $-1 \leq \cos(x-60°) \leq 1$)

TO GET THIS MAX VALUE OF 2, $\cos(x-60) = +1$

$$\Longrightarrow \cos(x-60) = 1$$
$$x - 60 = 0 \quad \text{(PRIMARY VALUE)}$$
$$x = 60$$

IYGB - MP2 PAPER C - QUESTION 11

c) NOTING THE SIMILARITY/ANALOGY TO PART (a) & (b)

$$T(t) = 32 + 2\cos\left(\underset{x}{15t} - 60\right)$$

$$\therefore T_{MAX} = 32 + 2$$

$$T_{MAX} = 34^\circ C$$

& USING PART (b)

$$15t = 60$$
$$t = 4$$

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d) SOLVING THE EQUATION $T = 30.5$

$$\Rightarrow 30.5 = 32 + \sqrt{3}\sin(15t) + \cos(15t)$$
$$\Rightarrow -1.5 = 2\cos(15t - 60)$$
$$\Rightarrow \cos(15t - 60) = -0.75$$

$\arccos(-0.75) = 138.59$

$$\Rightarrow \begin{cases} 15t - 60 = 138.59 \pm 360n \\ 15t - 60 = 221.41 \pm 360n \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \begin{cases} 15t = 198.59 \pm 360n \\ 15t = 281.41 \pm 360n \end{cases}$$

$$\Rightarrow \begin{cases} t = 13.24 \pm 24n \\ t = 18.76 \pm 24n \end{cases}$$

$$\therefore t_1 = 13.24 \quad \therefore \ 13:14 \quad \longleftarrow \quad 0.24 \times 60 = 14.4$$

$$t_2 = 18.76 \quad \therefore \ 18:46 \quad \longleftarrow \quad 0.76 \times 60 = 45.6$$