IYGB - MP2 PAPER E - QUESTION 1

a) $y = (2x + \ln x)^3$ (CHAIN RULE)

$$\frac{dy}{dx} = 3(2x+\ln x)^2 \times \left(2+\frac{1}{x}\right) = 3\left(2+\frac{1}{x}\right)(2x+\ln x)^2$$

b) $y = \frac{x^2}{3x-1}$ (QUOTIENT RULE)

$$\frac{dy}{dx} = \frac{(3x-1)\times(2x) - x^2 \times 3}{(3x-1)^2} = \frac{6x^2-2x-3x^2}{(3x-1)^2} = \frac{3x^2-2x}{(3x-1)^2}$$

c) $y = \sin^4 3x = (\sin 3x)^4$ (CHAIN RULE)

$$\frac{dy}{dx} = 4(\sin 3x)^3 \times \cos 3x \times 3 = 12\sin^3 3x \cos 3x$$

IYGB - MP2 PAPER E - QUESTION 2

a) FILLING A STANDARD TABLE

x	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
$e^{\tan^2 x}$	1	1.074	1.396	2.718	20.086
	FIRST	REST	REST	REST	LAST

BY THE TRAPEZIUM RULE

$$\int_0^{\frac{\pi}{3}} e^{\tan^2 x}\, dx \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \sum \text{REST}\right]$$

$$\simeq \frac{\pi/12}{2}\left[1 + 20.086 + 2(1.074 + 1.396 + 2.718)\right]$$

$$\simeq 4.12$$

b) USING $1 + \tan^2 x \equiv \sec^2 x$

$$\int_0^{\pi/3} e^{\sec^2 x}\, dx = \int_0^{\pi/3} e^{1+\tan^2 x}\, dx = \int_0^{\pi/3} e^1 \times e^{\tan^2 x}\, dx = e\int e^{\tan^2 x}\, dx$$

$$\simeq e \times 4.12 \simeq 11.2$$

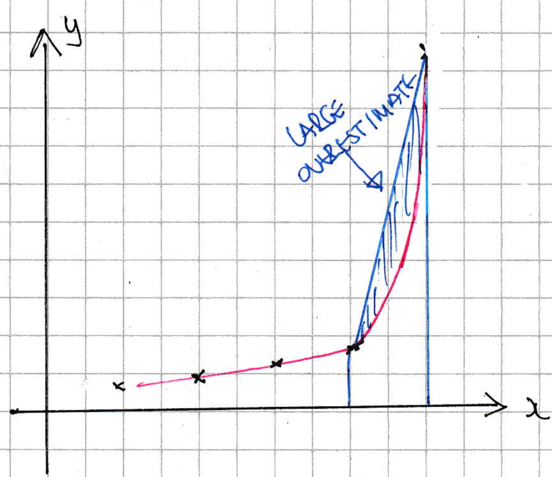
c) BOTH GRAPHS OF $y = e^{\tan^2 x}$ & $y = e^{\sec^2 x}$ ARE STRICTLY INCREASING

BUT GROW VERY VERY RAPIDLY AT THE END

THIS WILL CREATE LARGE OVERESTIMATES FOR BOTH, SEE DIAGRAM

$\therefore$ NOT LIKELY TO BE ACCURATE

AND BOTH OVERESTIMATE



IYGB - MP2 PAPER E - QUESTION 3

SUPPOSE THAT IF p & q WERE POSITIVE INTEGERS

$$\frac{p}{q} + \frac{q}{p} < 2$$

THEN PROCEED AS FOLLOWS

$$\Rightarrow \frac{p^2+q^2}{pq} < 2$$

$$\Rightarrow p^2 + q^2 < 2pq \qquad (pq > 0)$$

$$\Rightarrow p^2 - 2pq + q^2 < 0$$

$$\Rightarrow (p-q)^2 < 0$$

THIS IS A CONTRADICTION AS A SQUARED QUANTITY IS NEGATIVE

$$\therefore \quad \frac{p}{q} + \frac{q}{p} \geqslant 2$$

IYGB - MP2 PAPER E - QUESTION 4

$A(t,3,2)$ $\quad B(5,2,2t)$ $\quad |\vec{AB}| = \sqrt{21}$

$\Rightarrow |\vec{AB}| = \sqrt{21}$ (GIVEN)

$\Rightarrow |\underline{b} - \underline{a}| = \sqrt{21}$

$\Rightarrow |(5,2,2t)-(t,3,2)| = \sqrt{21}$

$\Rightarrow |5-t, -1, 2t-2| = \sqrt{21}$

$\Rightarrow \sqrt{(5-t)^2+(-1)^2+(2t-2)^2} = \sqrt{21}$ (DEFINITION OF THE MODULUS OF A VECTOR)

$\Rightarrow \sqrt{25-10t+t^2+1+4t^2-8t+4} = \sqrt{21}$

$\Rightarrow \sqrt{5t^2-18t+30} = \sqrt{21}$

$\Rightarrow 5t^2-18t+30 = 21$

$\Rightarrow 5t^2-18t+9 = 0$

$\Rightarrow (5t-3)(t-3) = 0$

$\Rightarrow t = \begin{cases} \frac{3}{5} \\ 3 \end{cases}$

IYGB - MP2 PAPER E - QUESTION 5

a) FORM A MODEL

20 PARTS OF 10 m

DEPTH	COST
10	£5000
20	£6200
30	£7400
⋮	
200	?

$$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{20} = \frac{20}{2}\left[2\times5000 + 19\times1200\right]$$

$$\Rightarrow S_{20} = 10\left[10000 + 22800\right]$$

$$\Rightarrow S_{20} = 328000$$

i.e. £328,000

b) "WORKING BACKWARDS"

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 15000000 = \frac{n}{2}\left[2\times5000 + (n-1)\times1200\right]$$

$$\Rightarrow 15000000 = \frac{n}{2}\left[10000 + 1200n - 1200\right]$$

$$\Rightarrow 15000000 = \frac{n}{2}\left[8800 + 1200n\right]$$

$$\Rightarrow 15000000 = 4400n + 600n^2$$

$$\Rightarrow 600n^2 + 4400n + 15000000 = 0$$

$\div 100$

$$\Rightarrow 6n^2 + 44n - 150000 = 0$$

$\div 2$

$$\Rightarrow 3n^2 + 22n - 75000 = 0$$

BY THE QUADRATIC FORMULA

$$n = \frac{-22 \pm \sqrt{484 + 900000}}{6} = \begin{cases} \cancel{-161.823\ldots} \\ 154.489\ldots \simeq 154 \end{cases}$$

$\therefore$ DEPTH OF 1540 m

(ACCEPT 1544 OR 1545)

IYGB - MP2 PAPER E - QUESTION 6

RELATING V, x AND t AS FOLLOWS

$$V = x^3 e^{-x^2}$$

$$\frac{dV}{dx} = (3x^2) \times e^{-x^2} + x^3 \times e^{-x^2}(-2x)$$

$$\frac{dV}{dx} = 3x^2 e^{-x^2} - 2x^4 e^{-x^2}$$

NOW WE HAVE

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$\frac{dV}{dt} = \left(3x^2 e^{-x^2} - 2x^4 e^{-x^2}\right)\frac{dx}{dt}$$

WHEN $x = \frac{1}{2}$ & $\frac{dx}{dt} = 0.01$

$$\frac{dV}{dt} = \left(3 \times \frac{1}{4} e^{-\frac{1}{4}} - 2 \times \frac{1}{16} e^{-\frac{1}{4}}\right) \times 0.01$$

$$\frac{dV}{dt} = \left(\frac{3}{4} e^{-\frac{1}{4}} - \frac{1}{8} e^{-\frac{1}{4}}\right) \times \frac{1}{100}$$

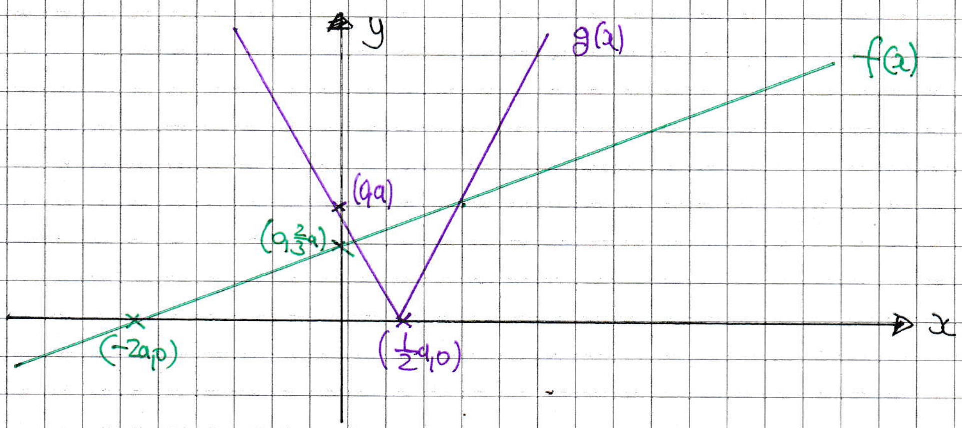
$$\frac{dV}{dt} = \frac{5}{800} e^{-\frac{1}{4}}$$

$$\approx 0.00487 \text{ m}^3\text{s}^{-1}$$

IYGB - MP2 PAPER E - QUESTION 7

$\boxed{f(x) = \frac{1}{3}(x+2a),\ x \in \mathbb{R}}$ $\boxed{g(x) = |2x-a|,\ x \in \mathbb{R}}$

a)



b) TO FIND POINTS OF INTERSECTION SOLVE SIMULTANEOUS EQUATIONS

$\Rightarrow |2x-a| = \frac{1}{3}(x+2a)$ (TWO SOLUTIONS ARE EXPECTED)

$$\Rightarrow \begin{cases} 2x-a = \frac{1}{3}(x+2a) \\ 2x-a = -\frac{1}{3}(x+2a) \end{cases}$$

$$\Rightarrow \begin{cases} 6x-3a = x+2a \\ 6x-3a = -x-2a \end{cases}$$

$$\Rightarrow \begin{cases} 5x = 5a \\ 7x = a \end{cases}$$

$$\Rightarrow x = \begin{cases} a \\ \frac{1}{7}a \end{cases}$$

USING $y = \frac{1}{3}(x+2a)$

$$\Rightarrow y = \begin{cases} \frac{1}{3}(a+2a) = \frac{1}{3} \times 3a = a \\ \frac{1}{3}\left(\frac{1}{7}a+2a\right) = \frac{1}{3} \times \frac{15}{7}a = \frac{5}{7}a \end{cases}$$

$\therefore$ REQUIRED CO-ORDINATES ARE (a,a) & $\left(\frac{1}{7}a, \frac{5}{7}a\right)$ //

IYGB - MP2 PAPER E - QUESTION 8

SEPARATE VARIABLES AND INTEGRATE

$$\Rightarrow \frac{dy}{dx} \sec x = y^2 - y$$

$$\Rightarrow dy \sec x = (y^2 - y)\,dx$$

$$\Rightarrow \frac{1}{y^2-y}\,dy = \frac{1}{\sec x}\,dx$$

$$\Rightarrow \int \frac{1}{y^2-y}\,dy = \int \cos x\,dx$$

THE L.H.S REQUIRES PARTIAL FRACTIONS

$$\frac{1}{y^2-y} = \frac{1}{y(y-1)} = \text{BY INSPECTION (OR FULL METHOD)} \dots = \frac{1}{y-1} - \frac{1}{y}$$

RETURNING TO THE DIFFERENTIAL EQUATION

$$\Rightarrow \int \frac{1}{y-1} - \frac{1}{y}\,dy = \int \cos x\,dx$$

$$\Rightarrow \ln|y-1| - \ln|y| = \sin x + C$$

$$\Rightarrow \ln\left|\frac{y-1}{y}\right| = \sin x + C$$

$$\Rightarrow \frac{y-1}{y} = e^{\sin x + C}$$

$$\Rightarrow \frac{y-1}{y} = e^{\sin x} \times e^{C}$$

$$\Rightarrow \frac{y-1}{y} = Ae^{\sin x} \qquad (A = e^{C})$$

IYGB - MP2 PAPER E - QUESTION 8

APPLY CONDITION $x=0$, $y=\frac{1}{2}$

$$\frac{\frac{1}{2}-1}{\frac{1}{2}} = Ae^{\sin 0}$$

$$-1 = A$$

THUS WE FINALLY HAVE

$$\frac{y-1}{y} = -e^{\sin x}$$

$$1-\frac{1}{y} = -e^{\sin x}$$

$$1+e^{\sin x} = \frac{1}{y}$$

$$y = \frac{1}{1+e^{\sin x}}$$

// AS REQUIRED

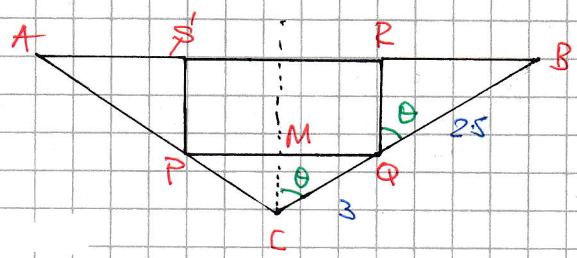
IYGB – MP2 PAPER E – QUESTION 9

a) LOOKING AT THE DIAGRAM, TRIANGLES $\overset{\triangle}{MCQ}$ & $\overset{\triangle}{RBQ}$

$|MQ| = 3\sin\theta$

&

$|RQ| = 2.5\cos\theta$



$\Rightarrow |PQ| = 2|MQ| = 2 \times 3\sin\theta = 6\sin\theta$

$\Rightarrow \text{PERIMETER} = 2 \times 6\sin\theta + 2 \times \underbrace{2.5\cos\theta}_{|RQ|}$

$\Rightarrow \text{PERIMETER} = 12\sin\theta + 5\cos\theta$

AS REQUIRED

b) USING THE COMPOUND ANGLE IDENTITY FOR $\sin(A+B)$

$$12\sin\theta + 5\cos\theta \equiv R\sin(\theta+\alpha)$$
$$12\sin\theta + 5\cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$$
$$12\sin\theta + 5\cos\theta \equiv (R\cos\alpha)\sin\theta + (R\sin\alpha)\cos\theta$$

$\left.\begin{array}{l} R\cos\alpha = 12 \\ R\sin\alpha = 5 \end{array}\right\}$ SQUARE & ADD $\quad R = \sqrt{12^2+5^2}$

$R = 13$

DIVIDE THE EQUATIONS SIDE BY SIDE

$\tan\alpha = \frac{5}{12}$

$\alpha = 22.62°$

$\therefore\ 12\sin\theta + 5\cos\theta \approx 13\sin(\theta + 22.62°)$

IYGB - MP2 PAPER E - QUESTION 9

c) SETTING $P=10$

$$\Rightarrow 5\cos\theta + 12\sin\theta = 10$$

$$\Rightarrow 13\sin(\theta + 22.62°) = 10$$

$$\Rightarrow \sin(\theta + 22.62°) = \frac{10}{13}$$

$$\arcsin\left(\frac{10}{13}\right) = 50.28°$$

$$\Rightarrow \begin{cases} \theta + 22.62 = 50.28 \pm 360n \\ \theta + 22.62 = 129.72 \pm 360n \end{cases} \qquad n = 0,1,2,3,\ldots$$

$$\Rightarrow \begin{cases} \theta = 27.7 \pm 360n \\ \theta = 107.10 \pm 360n \end{cases}$$

$$\therefore \theta = 27.7°$$

IYGB – MP2 PAPER E – QUESTION 10

a) EXPAND & COMPARE

$$\Rightarrow 6\sin x \equiv A(\cos x + \sin x) + B(\cos x - \sin x)$$
$$\Rightarrow 6\sin x \equiv (A+B)\cos x + (A-B)\sin x$$

$$\Rightarrow \left.\begin{array}{l} A+B=0 \\ A-B=6 \end{array}\right\}$$ ADDING & SUBSTACTING GIVES

$A = 3$
$B = -3$

b) USING PART (a)

$$\int \frac{6\sin x}{\cos x + \sin x}\,dx = \int \frac{3(\cos x + \sin x) - 3(\cos x - \sin x)}{\cos x + \sin x}\,dx$$

$$= \int \frac{3(\cos x + \sin x)}{\cos x + \sin x} - \frac{3(\cos x - \sin x)}{\cos x + \sin x}\,dx$$

$$= \int 3 - 3\left(\frac{-\sin x + \cos x}{\cos x + \sin x}\right)\,dx$$

THIS IS OF THE FORM $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$

$$= 3x - 3\ln|\cos x + \sin x| + C$$

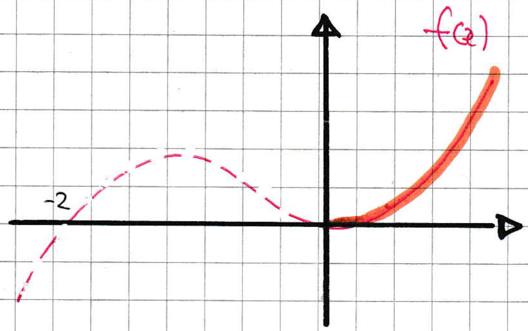
IYGB - MP2 PAPER E - QUESTION 11

a) EASIEST IS TO SHOW THAT f IS AN INCREASING FUNCTION IN ITS DOMAIN

$\Rightarrow f(x) = x^2(x+2) \qquad x>0$

$\Rightarrow f(x) = x^3 + 2x^2$

$\Rightarrow f'(x) = 3x^2 + 2x$



IF $x>0$, $f'(x)>0$

$\therefore$ f IS AN INCREASING FUNCTION, & HENCE INVERTIBLE

b) THE SOLUTION SET OF $f(x) = f^{-1}(x)$ IS IDENTICAL TO THAT OF $f(x) = x$ OR INDEED $f^{-1}(x) = x$

$\Rightarrow f(x) = x$

$\Rightarrow x^2(x+2) = x$

$\Rightarrow x(x+2) = 1 \qquad (x \neq 0)$

$\Rightarrow x^2 + 2x = 1$

$\Rightarrow (x+1)^2 - 1 = 1$

$\Rightarrow (x+1)^2 = 2$

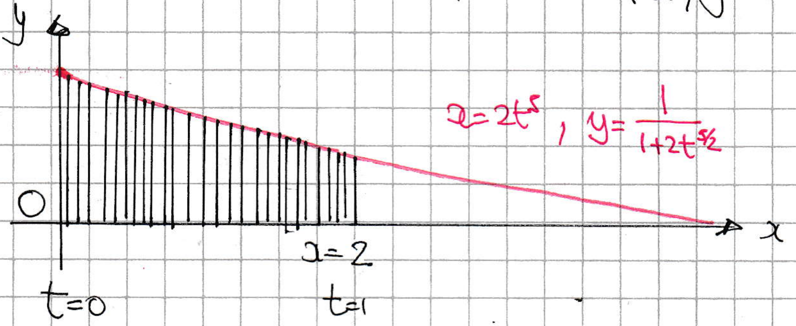
$\Rightarrow x+1 = \pm\sqrt{2}$

$\Rightarrow x = \begin{cases} -1+\sqrt{2} \\ \cancel{-1-\sqrt{2}} \end{cases} \qquad x>0$

IYGB - MP2 PAPER E - QUESTION 12

START WITH A SKETCH

- BY INSPECTION $t=0 \Longrightarrow (0,1)$
- AS t INCREASES, x INCREASES
- AS t INCREASES, y DECREASES, BUT STAYS POSITIVE



$t=0 \quad x=0$

$t=1 \quad x=2$

SETTING UP A PARAMETRIC INTEGRAL

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt = \int_0^1 \frac{1}{1+2t^{5/2}} \times 10t^4\,dt$$

$$= \int_0^1 \frac{10t^4}{1+2t^{5/2}}\,dt.$$

NOW BY RECOGNITION (OR SUBSTITUTION SHOWN LATER)

$$\text{AREA} = \int_0^1 \frac{10t^4}{1+2t^{5/2}}\,dt = \int_0^1 \frac{5t^{\frac{3}{2}}(2t^{\frac{5}{2}}+1) - 5t^{3/2}}{1+2t^{5/2}}\,dt$$

$$= \int_0^1 5t^{\frac{3}{2}} - \frac{5t^{3/2}}{1+2t^{5/2}}\,dt$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$= \left[2t^{\frac{5}{2}} - \ln|1+2t^{\frac{5}{2}}|\right]_0^1 = (2-\ln 3) - (0 - \cancel{\ln 1})$$

$$= \underline{2 - \ln 3}$$

IYGB - MP2 PAPER E - QUESTION 12

ALTERNATIVE BY SUBSTITUTION

$$\text{AREA} = \int_0^1 \frac{10t^4}{1+2t^{5/2}}\,dt$$

$$= \int_1^3 \frac{10t^4}{u} \frac{du}{5t^{3/2}}$$

$$= \int_1^3 \frac{2t^{5/2}}{u}\,du$$

$$= \int_1^3 \frac{u-1}{u}\,du$$

$$= \int_1^3 1 - \frac{1}{u}\,du$$

$$= \Big[u - \ln|u|\Big]_1^3$$

$$= (3 - \ln 3) - (1 - \cancel{\ln 1})$$

$$= 2 - \ln 3$$

AS BEFORE

LET $u = 1 + 2t^{5/2}$

$\frac{du}{dt} = 5t^{3/2}$

$dt = \frac{du}{5t^{3/2}}$

$2t^{5/2} = u - 1$

$t=0 \quad u=1$

$t=1 \quad u=3$