IYGB - MP2 PAPER G - QUESTION 1

a) CONVERTING THE LIMITS FROM x INTO t

- $x=2 \Rightarrow t^2-2=2$, $t^2=4$, $t=+2$, $(t \geqslant 0)$
- $x=14 \Rightarrow t^2-2=14 \Rightarrow t^2=16 \Rightarrow t=+4$, $t \geqslant 0$

SETTING UP THE INTEGRAL

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt = \int_2^4 \overset{y(t)}{(6t)}\overset{\frac{dx}{dt}}{(2t)}\,dt$$

$$= \int_2^4 12t^2\,dt$$

T=4

b) EVALUATING THE INTEGRAL

$$\text{AREA} = \Big[4t^3\Big]_2^4 = 256-32 = 224$$

IYGB - MP2 PAPER G - QUESTION 2

SUPPOSE THAT THERE EXIST INTEGERS m & n SO THAT

$$3n + 21m = 137$$

THEN WE HAVE

$$3(n+7m) = 137$$

$$n+7m = \frac{137}{3}$$

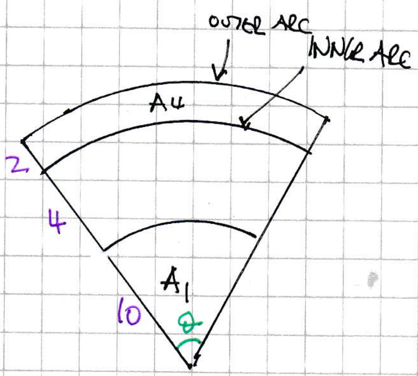
$$n+7m = 45\tfrac{2}{3}$$

BUT n IS AN INTEGER AND $7m$ MUST ALSO BE AN INTEGER, SO $n+7m$ HAS TO BE AN INTEGER & NOT $45\frac{2}{3}$

THIS IS A CONTRADICTION, SO THE ASSERTION $3n+21m=137$ CAN BE SATISFIED BY INTEGERS IS FALSE

IYGB - MP2 PAPER G - QUESTION 3

LOOKING AT THE DIAGRAM



$P_1 = 10 + 10 + 10\theta$ ← "rθ"

$P_1 = 20 + 10\theta$

$P_4 = 2 + 2 + \underset{\text{INNER ARC}}{14\theta} + \underset{\text{OUTER ARC}}{16\theta}$

$P_4 = 4 + 30\theta$

SETTING UP AN EQUATION

$$P_4 = 1.4 \times P_1$$

$$4 + 30\theta = 1.4(20 + 10\theta)$$

$$4 + 30\theta = 28 + 14\theta$$

$$16\theta = 24$$

$$\theta = 1.5^c$$

IYGB - MP2 PAPER G - QUESTION 4

a) $f(x) = \frac{1}{\sqrt{1+4x}} = (1+4x)^{-\frac{1}{2}} = 1 + \frac{-\frac{1}{2}}{1}(4x) + \frac{-\frac{1}{2}(-\frac{3}{2})}{1\times2}(4x)^2 + \frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})}{1\times2\times3}(4x)^3 + \dots$

$f(x) = 1 - 2x + 6x^2 - 20x^3 + O(x^4)$ //

b) Now $f(x+x^2)$

$$\therefore f(x+x^2) = 1 - 2(x+x^2) + 6(x+x^2)^2 - 20(x+x^2)^3 + O(x^4)$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$= \text{-------} + 6[\dots + x^3 + \dots] - 20[x^3 + O(x^4)] + O(x^4)$$

$$= \text{-----} + 12x^3 - 20x^3 + O(x^4)$$

$\therefore [x^3] = -8$ //

IYGB - FP2 PAPER G - QUESTION 5

a) BY THE PRODUCT RULE

I $y = (x+1)^2 e^{2x}$

$$\frac{dy}{dx} = 2(x+1)e^{2x} + (x+1)^2 e^{2x} \times 2$$

$$\frac{dy}{dx} = 2e^{2x}(x+1) + 2e^{2x}(x+1)^2$$

FACTORIZE $2e^{2x}(x+1)$

$$\frac{dy}{dx} = 2e^{2x}(x+1)\left[1 + (x+1)\right]$$

$$\frac{dy}{dx} = 2e^{2x}(x+1)(x+2)$$

AS REQUIRED

II) DIFFERENTIATE AGAIN "AFTER REGROUPING"

$$\frac{dy}{dx} = 2e^{2x}(x^2+3x+2)$$

$$\frac{d^2y}{dx^2} = 4e^{2x}(x^2+3x+2) + 2e^{2x}(2x+3)$$

FACTORIZE $2e^{2x}$

$$\frac{d^2y}{dx^2} = 2e^{2x}\left[2(x^2+3x+2) + (2x+3)\right]$$

$$\frac{d^2y}{dx^2} = 2e^{2x}\left[2x^2+6x+4 + 2x+3\right]$$

$$\frac{d^2y}{dx^2} = 2e^{2x}\left[2x^2+8x+7\right]$$

AS REQUIRED

ALTERNATIVE FOR a(II)

$$\frac{dy}{dx} = 2e^{2x}(x+1)(x+2)$$

← TRIPLE PRODUCT

$$\frac{d}{dx}(uvw) = \frac{du}{dx}vw + \frac{dv}{dx}uw + \frac{dw}{dx}uv$$

$$\frac{dy}{dx} = 4e^{2x}(x+1)(x+2) + 2e^{2x}\times 1\times(x+2) + 2e^{2x}(x+1)\times 1$$

$$= 2e^{2x}\left[(x+1)(x+2) + (x+2) + 2(x+1)\right]$$ ETC ETC

IYGB – MP2 PAPER G – QUESTION 5

b) SETTING $\frac{dy}{dx} = 0$

$$\Rightarrow 2(x+1)(x+2)e^{2x} = 0$$

$$\Rightarrow x = \begin{cases} -1 \\ -2 \end{cases} \qquad (e^{2x} \neq 0).$$

FIND THE y CO-ORDINATES

$$y_{(-1)} = 0 \qquad \& \qquad y_{(-2)} = (-1)^2 e^{-4} = e^{-4}$$

$$(-1, 0) \qquad\qquad (-2, \tfrac{1}{e^4})$$

CHECK THE NATURE

$$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = 2(2-8+7)e^{-2} = \frac{2}{e^2} > 0$$

(−1, 0) IS A LOCAL MINIMUM

$$\left.\frac{d^2y}{dx^2}\right|_{x=-2} = 2(8-16+7)e^{-4} = -\frac{2}{e^4} < 0$$

$(-2, \frac{1}{e^4})$ IS A LOCAL MAXIMUM

IYGB - MP2 PAPER G - QUESTION 6

a) DIFFERENTIATE EACH PARAMETRIC WITH RESPECT TO t

$x = t + \ln t$ $\qquad$ $y = t - \ln t$

$\frac{dx}{dt} = 1 + \frac{1}{t}$ $\qquad$ $\frac{dy}{dt} = 1 - \frac{1}{t}$

(NOT ACTUALLY NEEDED)

NOW OBTAIN THE GRADIENT FUNCTION & SETTING IT TO ZERO

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0 \implies \frac{dy}{dt} = 0$$

$$\implies 1 - \frac{1}{t} = 0$$

$$\implies \frac{1}{t} = 1$$

$$\implies t = 1$$

$\therefore$ STATIONARY POINT $(1, 1)$

$(\ln 1 = 0)$

b) BY ELIMINATION WE CAN WORK AS FOLLOWS

$x = t + \ln t$

$y = t - \ln t$

ADDING & SUBTRACTING WE OBTAIN

$$\left.\begin{array}{l} x + y = 2t \\ x - y = 2\ln t \end{array}\right\} \implies \frac{1}{2}(x+y) = t$$

$\Downarrow$

SUBSTITUTE INTO THE OTHER

$$\implies x - y = 2\ln\left[\frac{1}{2}(x+y)\right]$$

$$\implies e^{x-y} = e^{2\ln\left[\frac{1}{2}(x+y)\right]}$$

IYGB - MP2 PAPER G - QUESTION 6

$$\Rightarrow \quad e^{x-y} = e^{\ln\left[\frac{1}{2}(x+y)\right]^2}$$

$$\Rightarrow \quad e^{x-y} = \left[\frac{1}{2}(x+y)\right]^2$$

$$\Rightarrow \quad e^{x-y} = \frac{1}{4}(x+y)^2$$

$$\Rightarrow \quad 4e^{x-y} = (x+y)^2 \quad //$$

AS REQUIRED

ALTERNATIVE BY VERIFICATION

- $LHS = 4e^{x-y} = 4e^{(t+\ln t)-(t-\ln t)} = 4e^{2\ln t} = 4e^{\ln t^2}$

$= 4t^2$

- $RHS = (x+y)^2 = \left[(t+\ln t)+(t-\ln t)\right]^2 = (2t)^2 = 4t^2$

INDEED THE CORRECT CARTESIAN EQUATION

IYGB – MP2 PAPER G – QUESTION 7

$A(1,8,t-1)$ & $B(2t-1,4,3t-1)$

- START BY DETERMINING AN EXPRESSION, IN TERMS OF t, FOR $|\overrightarrow{AB}|$

$$\Rightarrow |\overrightarrow{AB}| = |\underline{b}-\underline{a}| = |(2t-1,4,3t-1)-(1,8,t-1)|$$

$$\Rightarrow |\overrightarrow{AB}| = |2t-2, -4, 3t| = \sqrt{(2t-2)^2+(-4)^2+(2t)^2}$$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{4t^2-8t+4+16+4t^2} = \sqrt{8t^2-8t+20}$$

- TO MINIMIZE THIS DISTANCE PROCEED BY ONE OF TWO METHODS

BY COMPLETING THE SQUARE

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{8(t^2-t+\tfrac{5}{2})}$$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{8\left[(t-\tfrac{1}{2})^2-\tfrac{1}{4}+\tfrac{5}{2}\right]}$$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{8(t-\tfrac{1}{2})^2-2+20}$$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{8(t-\tfrac{1}{2})^2+18}$$

$$\therefore |\overrightarrow{AB}|_{MIN} = \sqrt{18} = 3\sqrt{2}$$

(IT OCCURS WHEN $t=\frac{1}{2}$)

BY CALCULUS

- LET $f(t) = |AB|^2 = 8t^2-8t+20$

$$f'(t) = 16t-8$$

- SOLVE FOR ZERO

$$16t-8=0$$
$$16t=8$$
$$t=\tfrac{1}{2}$$

- $f(\frac{1}{2}) = 8(\frac{1}{2})^2-8(\frac{1}{2})+20$

$$= 2-4+20$$
$$= 18$$

$$\therefore f(t)_{MIN} = |AB|^2_{MIN} = 18$$

$$\therefore |AB|_{MIN} = \sqrt{18}$$

IYGB - MP2 PAPER G - QUESTION 8

a) SETTING A DIFFERENTIAL EQUATION

IN FLOW : $\frac{dV}{dt} = 0.05$

OUT FLOW : $\frac{dV}{dt} = -\frac{4V}{5}$

NET FLOW : $\frac{dV}{dt} = 0.05 - \frac{4V}{5}$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{20} - \frac{4V}{5} \quad \times(-20)$$

$$\Rightarrow -20\frac{dV}{dt} = -1 + 16V$$

$$\Rightarrow -20\frac{dV}{dt} = 16V - 1$$

// AS REQUIRED

b) SOLVING BY SEPARATING VARIABLES

$$\Rightarrow -20\,dV = (16V-1)\,dt$$

$$\Rightarrow -\frac{20}{16V-1}\,dV = 1\,dt$$

$$\Rightarrow \int \frac{-20}{16V-1}\,dV = \int 1\,dt$$

$$\Rightarrow -\frac{5}{4}\ln|16V-1| = t + C$$

$$\Rightarrow \ln|16V-1| = -\frac{4}{5}t + C$$

$$\Rightarrow 16V - 1 = e^{-\frac{4}{5}t + C}$$

$$\Rightarrow 16V - 1 = e^{-\frac{4}{5}t} \times e^{C}$$

IYGB − MP2 PAPER G − QUESTION 8

$$\Longrightarrow \quad 16V = 1 + Ae^{-\frac{4}{5}t} \qquad (A = e^{c})$$

$$\Longrightarrow \quad V = \frac{1}{16} + Ae^{-\frac{4}{5}t}$$

APPLY THE INITIAL CONDITION $t=0 \quad V=4$

$$\Longrightarrow \quad 4 = \frac{1}{16} + A$$

$$\Longrightarrow \quad A = \frac{63}{16}$$

$$\Longrightarrow \quad V = \frac{1}{16} + \frac{63}{16}e^{-\frac{4}{5}t}$$

$$\Longrightarrow \quad V = \frac{1}{16}\left[1 + 63e^{-\frac{4}{5}t}\right]$$

// AS REQUIRED

c) As $t \to \infty$, $e^{-\frac{4}{5}} \to 0$

$$\therefore V \to \frac{1}{16} = 0.0625$$

∴ THE VOLUME WILL TEND TO 0.0625 m^3

(62.5 litres)

IYGB - MP2 PAPER G - QUESTION 8

$$\Rightarrow 16V = 1 + Ae^{-\frac{4}{5}t} \qquad (A = e^c)$$

$$\Rightarrow V = \frac{1}{16} + Ae^{-\frac{4}{5}t}$$

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$$\Rightarrow 4 = \frac{1}{16} + A$$

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$$\Rightarrow V = \frac{1}{16} + \frac{63}{16}e^{-\frac{4}{5}t}$$

$$\Rightarrow V = \frac{1}{16}\left[1 + 63e^{-\frac{4}{5}t}\right]$$

// AS REQUIRED

c) AS $t \to \infty$, $e^{-\frac{4}{5}} \to 0$

$$\therefore V \to \frac{1}{16} = 0.0625$$

$\therefore$ THE VOLUME WILL TEND TO 0.0625 m^3

(62.5 litres)

IYGB - MP2 PAPER G - QUESTION 9

a) $\frac{dA}{dt} = +360$ (GIVEN)

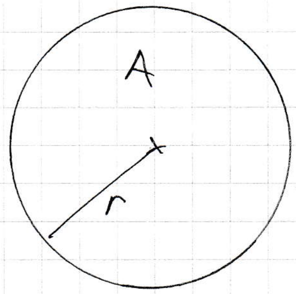
$\Longrightarrow \frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$\Longrightarrow \frac{dr}{dt} = \frac{1}{2\pi r} \times 360$

$\Longrightarrow \frac{dr}{dt} = \frac{180}{\pi r}$

$\Longrightarrow \left.\frac{dr}{dt}\right|_{r=100} = \frac{180}{100\pi} = \frac{9}{5\pi}$

$\simeq 0.573 \text{ ms}^{-1}$



$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dA} = \frac{1}{2\pi r}$

b) "CONSTANT RATE" OF 360 m^2 PER SECOND

$$\Longrightarrow \begin{array}{ll} t=0 & A=0 \\ t=1 & A=360 \\ t=2 & A=360\times 2 \\ \vdots & \\ t=60 & A=360\times 60 = 21600 \end{array}$$

USING $A = \pi r^2$

$\Longrightarrow \pi r^2 = 21600$

$\Longrightarrow r = 82.918...$

FINALLY

$\left.\frac{dr}{dt}\right|_{t=1\,\text{min}} = \left.\frac{dr}{dt}\right|_{t=60\,\text{s}} = \left.\frac{dr}{dt}\right|_{r=82.918..} = \frac{180}{\pi \times 82.918..} \simeq 0.691 \text{ ms}^{-1}$

IYGB - MP2 PAPER G - QUESTION 10

BREAK DOWN THE DOUBLE ARGUMENTS & TIDY

$$\Rightarrow 64\cos2\theta\cos\theta + 32\sin2\theta\sin\theta = 27$$

$$\Rightarrow 64(2\cos^2\theta - 1)\cos\theta + 32(2\sin\theta\cos\theta)\sin\theta = 27$$

$$\Rightarrow 128\cos^3\theta - 64\cos\theta + 64\sin^2\theta\cos\theta = 27$$

$$\Rightarrow 128\cos^3\theta - 64\cos\theta + 64(1-\cos^2\theta)\cos\theta = 27$$

$$\Rightarrow 128\cos^3\theta \cancel{- 64\cos\theta + 64\cos\theta} - 64\cos^3\theta = 27$$

$$\Rightarrow 64\cos^3\theta = 27$$

$$\Rightarrow \cos^3\theta = \frac{27}{64}$$

$$\Rightarrow \cos\theta = \frac{3}{4}$$

IYGB - MP2 PAPER G - QUESTION 11

GENERATE TERMS TO SEE THE PATTERN

$$\sum_{r=1}^{20}(2r+x) = (2+x)+(4+x)+(6+x)+ \ldots (40+x)$$

... AN ARITHMETIC PROGRESSION WITH ...

- $a = 2+x$
- $d = 2$
- $L = 40+x$
- $n = 20$

USING $S_n = \frac{n}{2}\left[a+L\right]$

$$S_{20} = \frac{20}{2}\left[(2+x)+(40+x)\right]$$

$$S_{20} = 10\left(2x+42\right)$$

FINALLY SOLVE THE EQUATION

$$10(2x+42) = 280$$

$$2x+42 = 28$$

$$2x = -14$$

$$x = -7$$

IYGB – MP2 PAPER G – QUESTION 12

a)

$$f(x) \equiv 2 - \sqrt{x-1}\,, \quad x \geqslant 1$$

USING THE HINT GIVEN

$$\Rightarrow f(x)\,g(x) = 1$$

$$\Rightarrow g(x) = \frac{1}{f(x)} = \frac{1}{2-\sqrt{x-1}}$$

$$\Rightarrow \frac{1}{f(x)} = \frac{(2+\sqrt{x-1})}{(2-\sqrt{x-1})(2+\sqrt{x-1})}$$

$$\Rightarrow \frac{1}{f(x)} = \frac{2+\sqrt{x-1}}{4-(x-1)}$$

$$\Rightarrow \frac{1}{f(x)} = \frac{2+\sqrt{x-1}}{5-x} \quad //$$

b) USING PART (a)

$$\int \frac{5-x}{2-\sqrt{x-1}}\,dx = \int \frac{5-x}{f(x)}\,dx = \int (5-x)\times\frac{1}{f(x)}\,dx$$

$$= \int \cancel{(5-x)}\,\frac{2+\sqrt{x-1}}{\cancel{5-x}}\,dx = \int 2+\sqrt{x-1}\,dx$$

$$= \int 2 + (x-1)^{\frac{1}{2}}\,dx$$

$$= 2x + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$$