IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper J

Difficulty Rating: 3.965/1.3759

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

$$f(x) = A\sec 2x + B,\ 0 \le x < 2\pi.$$

The graph of $f(x)$, where A and B are non zero constants, passes through the points $\left(\frac{\pi}{2}, -7\right)$ and $(\pi, 1)$.

a) Determine the value of A and the value of B. **(4)**

b) Solve the equation

$$f\left(x + \frac{3\pi}{2}\right) = 5.$$ **(6)**

Question 2

The function f is given by

$$f(x) \equiv e^{mx}\left(x^2 + x\right),\ x \in \mathbb{R},$$

where m is a non zero constant.

Show that f has two stationary points, for all non zero values of m. **(7)**

Question 3

The points $A(-3,3,a)$, $B(b,b,b-5)$ and $C(c,-2,5)$, where a, b and c are scalar constants, are referred relative to a fixed origin O.

Ii is further given that A, B and C are collinear and the ratio $\left|\overrightarrow{AB}\right| : \left|\overrightarrow{BC}\right| = 2:3$.

Use vector algebra to find the value of a, the value of b and the value of c. **(6)**

Question 4

The function $f(x)$ is defined by

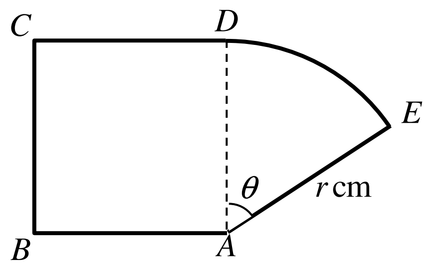
$$f(x) = \frac{1}{\sqrt{x-2}}, \quad x \in \mathbb{R}, \; x > 2.$$

a) Find the range of $f(x)$. **(1)**

b) Determine a simplified expression for $f^{-1}(x)$, further stating the domain and range of $f^{-1}(x)$. **(5)**

c) Show that the equation $f^{-1}(x) = -\frac{3}{x}$ has no real solutions. **(4)**

Question 5



A minor sector ADE with radius r cm, subtends an angle of θ radians at A.

The sector is attached to a square $ABCD$, forming a composite shape S, as shown in figure above.

The area and the perimeter of S are $48\,\text{cm}^2$ and $28\,\text{cm}$, respectively.

By forming and solving two equations, find the value of r and the value of θ. **(7)**

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Question 6

The radius R of a circle, in cm, at time t seconds is given by

$$R = 10\left(1 - e^{-kt}\right),$$

where k is a positive constant and $t > 0$.

Show that if A is the area of the circle, in cm^2, then

$$\frac{dA}{dt} = 200\pi k\left(e^{-kt} - e^{-2kt}\right). \qquad (6)$$

Question 7

$$y = \arctan x, \quad x \in \mathbb{R}.$$

a) By writing $y = \arctan x$ as $x = \tan y$ show that

$$\frac{dy}{dx} = \frac{1}{1+x^2}. \qquad (4)$$

The curve C has equation

$$y = 2\arctan x - 3\ln\left(1+x^2\right) - 7x^2, \quad x \in \mathbb{R}.$$

b) Show that the x coordinate of the stationary point of C is a solution of the cubic equation

$$7x^3 + 10x - 1 = 0. \qquad (5)$$

c) Hence show further that the x coordinate of the stationary point of C is 0.099314, correct to 6 decimal places. **(2)**

Question 8

It is given that

$$\sum_{r=1}^{n} u_r = 128 - 2^{7-n},$$

where u_r is the r^{th} term of a geometric progression.

a) Find the sum of the first 8 terms of the progression. **(2)**

b) Determine the value of u_8. **(3)**

c) Find the common ratio of the progression. **(3)**

Question 9

$$y = \left(1+\cot^2 x\right)\sec^2 x, \quad 0 < x < \tfrac{1}{2}\pi.$$

a) Calculate the three missing values of x in the following table.

x	$\frac{1}{6}\pi$				$\frac{1}{3}\pi$
y	$\frac{16}{3}$	$32-16\sqrt{3}$	4	$32-16\sqrt{3}$	$\frac{16}{3}$

(1)

b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \left(1+\cot^2 x\right)\sec^2 x \; dx.$$ **(3)**

c) Use an appropriate integration method to find an exact simplified value for

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \left(1+\cot^2 x\right)\sec^2 x \; dx.$$ **(5)**

Question 10

A curve C is defined implicitly by

$$\sin 2x \cot y = 1, \quad x \in \mathbb{R}, \quad y \in \mathbb{R}, \ 0 < x < \frac{\pi}{2}, \ 0 < y < \frac{\pi}{2}.$$

a) Show clearly that

$$\frac{dy}{dx} = \cot 2x \sin 2y. \qquad (4)$$

The point $A\left(\frac{\pi}{4}, \frac{\pi}{12}\right)$ is a turning point of C.

b) Use $\frac{d^2y}{dx^2}$ to show that A is a local maximum. (4)

Question 11

An object is moving in such a way so that its coordinates relative to a fixed origin O are given by

$$x = 4\cos t - 3\sin t + 1, \qquad y = 3\cos t + 4\sin t - 1,$$

where t is the time in seconds.

Initially the object was at the point with coordinates $(5,2)$.

a) Show that the motion of the particle is governed by the differential equation

$$\frac{dy}{dx} = \frac{1-x}{1+y}. \qquad (5)$$

b) Find, in exact form, the possible values of the y coordinate of the object when its x coordinate is 2. (7)

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Question 12

$$f(x) \equiv \sqrt[3]{1+12x}, \ x \in \mathbb{R}.$$

It is given that the equation

$$f(x) + (6x-5)^2 = 24 - 15x$$

has a solution α, which is numerically small.

Use a quadratic approximation for $f(x)$ to find an approximate value for α. **(6)**

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