IYGB - MP2 PAPER J - QUESTION 1

a) USING THE CO-ORDS GIVEN $\left(\frac{\pi}{2},-7\right)$ & $(\pi,1)$

$\left(\frac{\pi}{2},-7\right)$

$-7 = A\sec\pi + B$

$-7 = -A + B$

$(\pi,1)$

$1 = A\sec 2\pi + B$

$1 = A + B$

ADDING GIVES

$2B = -6$

$B = -3$ & $A = 4$

b) SETTING UP THE EQUATION

$\Rightarrow f\left(x+\frac{3\pi}{2}\right) = 5$

$\Rightarrow 4\sec\left[2\left(x+\frac{3\pi}{2}\right)\right] - 3 = 5$

$\Rightarrow 4\sec(2x+3\pi) - 3 = 5$

$\Rightarrow \sec(2x+3\pi) = 2$

$\Rightarrow \cos(2x+3\pi) = \frac{1}{2}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$$\left(\begin{array}{l} 2x+3\pi = \frac{\pi}{3} \pm 2\pi n \\ 2x+3\pi = \frac{5\pi}{3} \pm 2\pi n \end{array}\right. \qquad n = 0,1,2,3,\ldots$$

$$\left(\begin{array}{l} 2x = -\frac{8\pi}{3} \pm 2\pi n \\ 2x = -\frac{4\pi}{3} \pm 2\pi n \end{array}\right.$$

$$\left(\begin{array}{l} x = -\frac{4\pi}{3} \pm \pi n \\ x = -\frac{2\pi}{3} \pm \pi n \end{array}\right.$$

(RANGE $0 \leqslant x \leqslant 2\pi$)

$x = \frac{2\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{4\pi}{3}$

IYGB – MP2 PAPER L – QUESTION 2

DIFFERENTIATE WITH RESPECT TO x BY THE PRODUCT RULE

$$f(x) = e^{mx}(x^2+x)$$

$$f'(x) = me^{mx}(x^2+x) + e^{mx}(2x+1)$$

$$f'(x) = e^{mx}\left[m(x^2+x) + (2x+1)\right]$$

$$f'(x) = e^{mx}\left[mx^2+mx+2x+1\right]$$

SOLVING FOR ZERO FOR STATIONARY POINTS

$$e^{mx}\left[mx^2+mx+2x+1\right] = 0$$

$$mx^2+mx+2x+1 = 0 \qquad (e^{mx} \neq 0)$$

$$mx^2+(m+2)x+1 = 0$$

USING THE DISCRIMINANT b^2-4ac

$$(m+2)^2 - 4 \times m \times 1 = m^2+4m+4-4m$$

$$= m^2+4$$

$$\geqslant 4$$

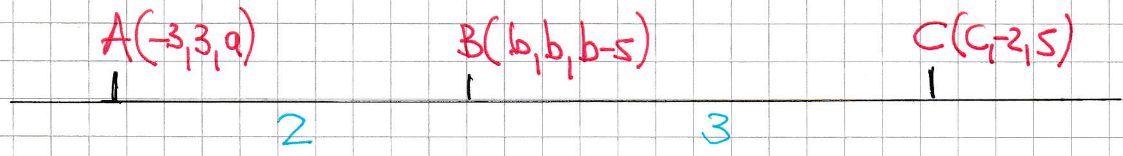
$$> 0$$

INDEED ALWAYS TWO REAL ROOTS

$\therefore$ ALWAYS 2 STATIONARY POINTS

IYGB - MP2 PAPER J - QUESTION 3

PUTTING THE INFORMATION IN A DIAGRAM



"CALCULATE" THE VECTORS $\vec{AB}$ & $\vec{BC}$

$$\vec{AB} = \underline{b} - \underline{a} = (b,b,b-5)-(-3,3,a) = (b+3,\ b-3,\ b-a-5)$$

$$\vec{BC} = \underline{c} - \underline{b} = (c,-2,5)-(b,b,b-5) = (c-b,\ -2-b,\ 10-b)$$

LOOKING AT $\underline{j}$

$$\frac{b-3}{-2-b} = \frac{2}{3} \Longrightarrow 3b-9 = -4-2b$$

$$\Longrightarrow 5b = 5$$

$$\Longrightarrow b = 1$$

LOOKING AT $\underline{i}$

$$\frac{b+3}{c-b} = \frac{2}{3} \Longrightarrow 3b+9 = 2c-2b$$

$$\Longrightarrow 3+9 = 2c-2$$

$$\Longrightarrow 14 = 2c$$

$$\Longrightarrow c = 7$$

LOOKING AT $\underline{k}$

$$\frac{b-a-5}{10-b} = \frac{2}{3} \Longrightarrow 3b-3a-15 = 20-2b$$

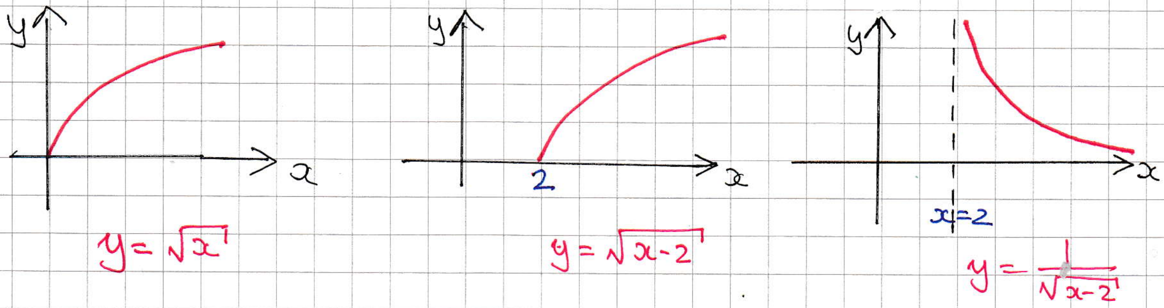
$$\Longrightarrow 3-3a-15 = 20-2$$

$$\Longrightarrow -30 = 3a$$

$$\Longrightarrow a = -10$$

IYGB - MP2 PAPER I - QUESTION 4

a) ATTEMPTING TO SKETCH THE GRAPH OF $f(x)$



THE RANGE OF $f(x)$ IS

$$f(x) \in \mathbb{R},\ f(x) > 0$$

b) LET $y = f(x)$ FOR SIMPLICITY

$$\Longrightarrow y = \frac{1}{\sqrt{x-2}}$$

$$\Longrightarrow y^2 = \frac{1}{x-2}$$

$$\Longrightarrow x-2 = \frac{1}{y^2}$$

$$\Longrightarrow x = \frac{1}{y^2} + 2$$

$$\Longrightarrow f^{-1}(x) = \frac{1}{x^2} + 2$$

	$f(x)$	$f^{-1}(x)$
D	$x > 2$	$x > 0$
R	$f(x) > 0$	$f^{-1}(x) > 2$

DOMAIN OF $f^{-1}(x)$: $x > 0$

RANGE OF $f^{-1}(x)$: $f^{-1}(x) > 2$

c) SOLVING THE GIVEN EQUATION, IN ORDER TO DETERMINE "WHAT IS THE PROBLEM" WITH THE ROOTS

$$\Longrightarrow \frac{1}{x^2} + 2 = -\frac{3}{x}$$

$$\Longrightarrow 1 + 2x^2 = -3x$$

IYGB – MP2 PAPER T – QUESTION 4

$$\Rightarrow \quad 2x^2 + 3x + 1 = 0$$

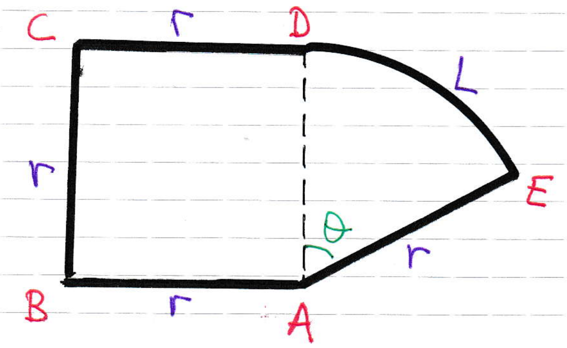
$$\Rightarrow \quad (2x+1)(x+1) = 0$$

$$\Rightarrow \quad x = \begin{cases} -\frac{1}{2} \\ -1 \end{cases}$$

NEITHER SOLUTION IS POSSIBLE AS THE DOMAIN OF $f^{-1}(x)$ IS $x > 0$

IYGB - MP2 PAPER J - QUESTION 5

LOOKING AT THE DIAGRAM



- AREA $= r^2 + \frac{1}{2}r^2\theta$

$48 = r^2 + \frac{1}{2}r^2\theta$

- PERIMETER $= 4r + L$

$28 = 4r + r\theta$

FOR SECTORS

AREA $= \frac{1}{2}r^2\theta^c$

ARCLENGTH $= r\theta^c$

TIDY THE EQUATIONS

$$\begin{cases} 96 = 2r^2 + r^2\theta \\ 28 = 4r + r\theta \quad \times r \end{cases}$$

$$\begin{cases} 96 = 2r^2 + r^2\theta \\ 28r = 4r^2 + r^2\theta \end{cases}$$

SUBTRACTING THE EQUATIONS

$\Longrightarrow 96 - 28r = -2r^2$

$\Longrightarrow 2r^2 - 28r + 96 = 0$

$\Longrightarrow r^2 - 14r + 48 = 0$

$\Longrightarrow (r-6)(r-8) = 0$

$r = \begin{cases} 6 \\ 8 \end{cases}$

USING $28 = 4r + r\theta$

$\Longrightarrow r\theta = 28 - 4r$

$\Longrightarrow \theta = \frac{28}{r} - 4$

$\Longrightarrow \theta = \begin{cases} \frac{28}{6} - 4 = \frac{2}{3} \\ \frac{28}{8} - 4 = \cancel{-\frac{1}{2}} \end{cases}$

ONLY SOLUTION IS $r = 6$ AND $\theta = \frac{2}{3}$ //

IYGB - MP2 PAPER J - QUESTION 6

THE RADIUS "R" & THE AREA "A" ARE RELATED BY

$$A = \pi R^2$$
$$\frac{dA}{dR} = 2\pi R$$

NOW CONTINUE AS FOLLOWS

$$\Rightarrow \frac{dA}{dt} = \frac{dA}{dR} \times \frac{dR}{dt}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi R \times \frac{d}{dt}\left(10(1-e^{-kt})\right)$$

$$\Rightarrow \frac{dA}{dt} = 2\pi R \times 10 \frac{d}{dt}\left(1-e^{-kt}\right)$$

$$\Rightarrow \frac{dA}{dt} = 20\pi R \times \frac{d}{dt}\left(1-e^{-kt}\right)$$

$$\Rightarrow \frac{dA}{dt} = 20\pi R \times \left(+ke^{-kt}\right)$$

$$\Rightarrow \frac{dA}{dt} = 20\pi \times 10(1-e^{-kt}) \times ke^{-kt}$$

$$\Rightarrow \frac{dA}{dt} = 200\pi k e^{-kt}(1-e^{-kt})$$

$$\Rightarrow \frac{dA}{dt} = 200\pi k\left(e^{-kt} - e^{-2kt}\right)$$

// AS REQUIRED

IYGB - MP2 PAPER J - QUESTION 7

a) PROCEED AS FOLLOWS

$$\Rightarrow y = \arctan x$$

$$\Rightarrow \tan y = x$$

$$\Rightarrow x = \tan y$$

$$\Rightarrow \frac{dx}{dy} = \sec^2 y$$

$$\Rightarrow \frac{dx}{dy} = 1 + \tan^2 y$$

$$\Rightarrow \frac{dx}{dy} = 1 + x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$$

// AS REQUIRED

b) $y = 2\arctan x - 3\ln(1+x^2) - 7x^2$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2} - 3\left(\frac{2x}{1+x^2}\right) - 14x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2} - \frac{6x}{1+x^2} - 14x$$

SOLVING FOR ZERO

$$\Rightarrow \frac{2}{1+x^2} - \frac{6x}{1+x^2} - 14x = 0$$

$$\Rightarrow 2 - 6x - 14x(x^2+1) = 0$$

IYGB - MP2 PAPER J - QUESTION 7

$$\Rightarrow \quad 2 - 6x - 14x^3 - 14x = 0$$

$$\Rightarrow \quad 0 = 14x^3 + 20x - 2$$

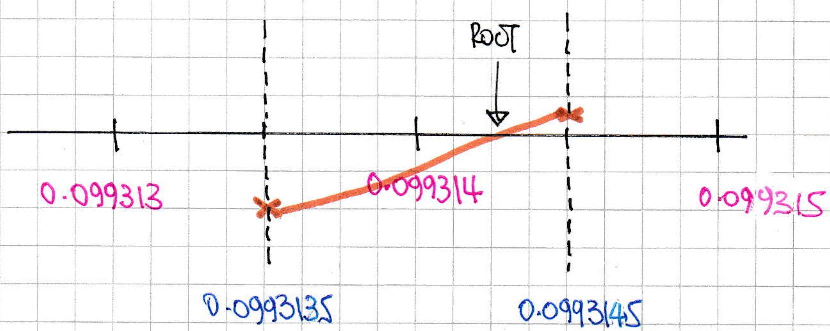
$$\Rightarrow \quad 7x^3 + 10x - 1 = 0$$

// AS REQUIRED

c) WRITING THE ABOVE EQUATION IN FUNCTION FORM

- $f(x) = 7x^3 + 10x - 1$

$f(0.0993135) = -0.000008\ldots < 0$
$f(0.0993145) = +0.000002\ldots > 0$



- AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE ABOVE INTERVAL,

$$0.0993135 < \text{ROOT} < 0.0993145$$

THUS THE ROOT IS 0.099314, TO 6 DECIMAL PLACES

IYGB - MP2 PAPER J - QUESTION 8

a) USING THE RESULT GIVEN

$$S_8 = \sum_{r=1}^{8} u_r = 128 - 2^{7-8} = 128 - 2^{-1} = 127.5$$

b) USING THE SUMMATION

$$\Rightarrow u_8 = S_8 - S_7$$

$$\Rightarrow u_8 = 127.5 - \left[128 - 2^{7-7}\right]$$

$$\Rightarrow u_8 = 127.5 - \left[128 - 1\right]$$

$$\Rightarrow u_8 = 0.5$$

c) FIND THE FIRST TERM

$$\Rightarrow a = u_1 = S_1$$

$$\Rightarrow a = 128 - 2^{7-1}$$

$$\Rightarrow a = 64$$

EITHER

$$\Rightarrow u_8 = ar^7$$

$$\Rightarrow \frac{1}{2} = 64 \times r^7$$

$$\Rightarrow r^7 = \frac{1}{128}$$

$$\Rightarrow r = \sqrt[7]{\frac{1}{128}}$$

$$\Rightarrow r = \frac{1}{2}$$

OR

$$\Rightarrow S_2 = u_1 + u_2 = 128 - 2^{7-2}$$

$$\Rightarrow u_1 + u_2 = 128 - 32$$

$$\Rightarrow 64 + u_2 = 96$$

$$\Rightarrow u_2 = 32$$

$$\therefore r = \frac{u_2}{u_1} = \frac{32}{64} = \frac{1}{2}$$

IYGB – MP2 PAPER J – QUESTION 9

a) FILL IN THE TABLE

x	$\frac{\pi}{6}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
y	$\frac{16}{3}$	$32-16\sqrt{3}$	4	$32-16\sqrt{3}$	$\frac{16}{3}$

$$\left[\frac{\frac{\pi}{3}-\frac{\pi}{6}}{4} = \frac{\frac{\pi}{6}}{4} = \frac{\pi}{24} \leftarrow \text{GAP}\right]$$

b) BY THE TRAPEZIUM RULE

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x(1+\cot^2 x)\,dx \simeq \text{“}\frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]\text{”}$$

$$\simeq \frac{\pi/24}{2}\left[\frac{16}{3}+\frac{16}{3}+2\left(32-16\sqrt{3}+4+32-16\sqrt{3}\right)\right]$$

$$\simeq \frac{\pi}{48}\times 35.81541\ldots$$

$$\simeq 2.34411\ldots$$

$$\simeq 2.34$$

b)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x(1+\cot^2 x)\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + \sec^2 x\cot^2 x\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + \frac{1}{\cancel{\cos^2 x}}\times\frac{\cancel{\cos^2 x}}{\sin^2 x}\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + \operatorname{cosec}^2 x\,dx$$

NOTING THE DIFFERENTIALS

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = \operatorname{cosec}^2 x$$

$$= \left[\tan x - \cot x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

IYGB – MP2 PAPER J – QUESTION 9

$$= \left(\sqrt{3} - \tfrac{1}{3}\sqrt{3}\right) - \left(\tfrac{1}{3}\sqrt{3} - \sqrt{3}\right)$$

$$= \tfrac{4}{3}\sqrt{3}$$

ALTERNATIVE INTEGRATION

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x(1+\cot^2 x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \operatorname{cosec}^2 x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x \sin^2 x}\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(\cos x \sin x)^2}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\left(\frac{1}{2}\sin 2x\right)^2}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\frac{1}{4}\sin^2 2x}\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4\operatorname{cosec}^2 2x\,dx = \Big[-2\cot 2x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$= \Big[2\cot 2x\Big]_{\frac{\pi}{3}}^{\frac{\pi}{6}} = \left[\frac{2}{\tan 2x}\right]_{\frac{\pi}{3}}^{\frac{\pi}{6}}$$

$$= \tfrac{2}{3}\sqrt{3} - \left(-\tfrac{2}{3}\sqrt{3}\right) = \tfrac{4}{3}\sqrt{3}$$

IYGB - MP2 PAPER J - QUESTION 10

a) DIFFERENTIATE WITH RESPECT TO x – BY PRODUCT RULE ON L.H.S

$$\Rightarrow \frac{d}{dx}\left(\sin 2x \cot y\right) = \frac{d}{dx}(1)$$

$$\Rightarrow 2\cos 2x \cot y + \sin 2x\left(-\operatorname{cosec}^2 y\right)\frac{dy}{dx} = 0$$

$$\Rightarrow 2\cos 2x \cot y = \sin 2x \operatorname{cosec}^2 y \frac{dy}{dx}$$

$$\Rightarrow \frac{2\cos 2x}{\sin 2x} = \frac{\operatorname{cosec}^2 y}{\cot y}\frac{dy}{dx}$$

$$\Rightarrow 2\cot 2x = \operatorname{cosec}^2 y \tan y \frac{dy}{dx}$$

$$\Rightarrow 2\cot 2x = \frac{1}{\sin^2 y}\frac{\sin y}{\cos y}\frac{dy}{dx}$$

$$\Rightarrow 2\cot 2x = \frac{1}{\sin y \cos y}\frac{dy}{dx}$$

$$\Rightarrow (2\sin y\cos y)\cot 2x = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \cot 2x \sin 2y$$

AS REQUIRED

b) DIFFERENTIATE AGAIN W.R.T x BY THE PRODUCT RULE

$$\Rightarrow \frac{d^2y}{dx^2} = -2\operatorname{cosec}^2 2x \sin 2y + \cot 2x\left(2\cos 2y \frac{dy}{dx}\right)$$

BUT $\left.\frac{dy}{dx}\right|_{\left(\frac{\pi}{4}, \frac{\pi}{12}\right)} = 0$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\left(\frac{\pi}{4}, \frac{\pi}{12}\right)} = -2\operatorname{cosec}^2\left(\frac{\pi}{2}\right)\sin\frac{\pi}{6} = -2\times 1\times\frac{1}{2} = -1 < 0$$

INDEED A LOCAL MAX

IYGB - MP2 PAPER J - QUESTION 11

a) DIFFERENTIATING THE GIVEN PARAMETRIC EQUATIONS

- $x = 4\cos t - 3\sin t + 1$

$$\frac{dx}{dt} = -4\sin t - 3\cos t$$

- $y = 3\cos t + 4\sin t - 1$

$$\frac{dy}{dt} = -3\sin t + 4\cos t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\sin t + 4\cos t}{-4\sin t - 3\cos t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4\cos t - 3\sin t}{-(3\cos t + 4\sin t)}$$

BUT $x - 1 = 4\cos t - 3\sin t$
$y + 1 = 3\cos t + 4\sin t$

$$\Rightarrow \frac{dy}{dx} = \frac{x-1}{-(y+1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{1+y}$$ // AS REQUIRED

b) SOLVING BY SEPARATION OF VARIABLES

$$\Rightarrow dy = \frac{1-x}{1+y}\,dx$$

$$\Rightarrow (1+y)\,dy = (1-x)\,dx$$

$$\Rightarrow \int 1+y \; dy = \int 1-x \; dx$$

$$\Rightarrow y + \frac{1}{2}y^2 = x - \frac{1}{2}x^2 + C$$

IYGB - MP2 PAPER J - QUESTION 11

$$\Longrightarrow \quad 2y + y^2 = 2x - x^2 + C$$

APPLY THE CONDITION $(5,2)$

$$\Longrightarrow \quad (2\times2) + 2^2 = 2\times5 - 5^2 + C$$

$$\Longrightarrow \quad 8 = -15 + C$$

$$\Longrightarrow \quad C = 23$$

$$\Longrightarrow \quad y^2 + 2y + x^2 - 2x = 23 \qquad \text{(I.E A CIRCLE)}$$

FINALLY $x=2$

$$\Longrightarrow \quad y^2 + 2y + 4 - 4 = 23$$

$$\Longrightarrow \quad y^2 + 2y = 23$$

$$\Longrightarrow \quad y^2 + 2y + 1 = 24$$

$$\Longrightarrow \quad (y+1)^2 = 24$$

$$\Longrightarrow \quad y+1 = \pm\sqrt{24}$$

$$\Longrightarrow \quad y = \begin{cases} -1 + 2\sqrt{6} \\ -1 - 2\sqrt{6} \end{cases}$$

IYGB - MP2 PAPER J - QUESTION 12

EXPAND $f(x)$ UP TO x^2

$$f(x) = (1+12x)^{\frac{1}{3}} = 1 + \frac{\frac{1}{3}}{1}(12x) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{1\times 2}(12x)^2 + O(x^3)$$

$$= 1 + 4x - 16x^2 + O(x^3)$$

NOW LOOKING AT THE EQUATION

$$f(x) + (6x-5)^2 = 24 - 15x$$

$$\left[1 + 4x - 16x^2 + O(x^3)\right] + \left[36x^2 - 60x + 25\right] = 24 - 15x$$

FOR $|x| << 1$ WE HAVE

$$1 + 4x - 16x^2 + 36x^2 - 60x + 25 \approx 24 - 15x$$

$$20x^2 - 41x + 2 = 0$$

BY QUADRATIC FORMULA OF FACTORIZING

$$(x-2)(20x-1) = 0$$

$$x = \begin{cases} \frac{1}{20} \\ \cancel{2} \end{cases}$$

$|x| < \frac{1}{12}$