IYGB – MP2 PAPER M – QUESTION 1

a) $f(x) = \dfrac{2-x}{\sqrt{1+x}} = (2-x)(1+x)^{-\frac{1}{2}}$

$$= (2-x)\left[1 + \frac{-\frac{1}{2}}{1}(x) + \frac{-\frac{1}{2}(-\frac{3}{2})}{1\times 2}(x)^2 + \frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})}{1\times 2\times 3}(x)^3 + O(x^4)\right]$$

$$= (2-x)\left(1 - \tfrac{1}{2}x + \tfrac{3}{8}x^2 - \tfrac{5}{16}x^3 + O(x^4)\right)$$

$$= 2 - x + \tfrac{3}{4}x^2 - \tfrac{5}{8}x^3 + O(x^4)$$
$$\quad - x + \tfrac{1}{2}x^2 - \tfrac{3}{8}x^3 + O(x^4)$$

$$= 2 - 2x + \tfrac{5}{4}x^2 - x^3 + O(x^4)$$

AS REQUIRED

b) $f(2x) = \dfrac{2-(2x)}{\sqrt{1+(2x)}} = 2 - 2(2x) + \tfrac{5}{4}(2x)^2 - (2x)^3 + O(x^4)$

$$= 2 - 4x + 5x^2 - 8x^3 + O(x^4)$$

IYGB - MP2 PAPER M - QUESTION 2

LOOKING AT THE TRANSFORMATIONS OF $(2,6)$

- MAX $(2,6)$ $\qquad y = f(x)$

$\downarrow$ TRANSLATION "LEFT" BY 2 UNITS $\downarrow$

- MAX $(0,6)$ $\qquad y = f(x+2)$

$\downarrow$ TRANSLATION "DOWN" BY 6 UNITS $\downarrow$

- MAX $(0,0)$ $\qquad y = f(x+2)-6$

$\downarrow$ REFLECTION ABOUT THE x AXIS $\downarrow$

- MIN $(0,0)$ $\qquad y = -\left[f(x+2)-6\right]$

$\therefore\ g(x) = 6 - f(x+2)$

IYGB - MP2 PAPER M - QUESTION 3

ASSERTION

FOR ALL REAL x, $(13x+1)^2+3>(5x-1)^2$

PROOF BY CONTRADICTION

SUPPOSE THAT FOR ALL REAL x, $(13x+1)^2+3 \leqslant (5x-1)^2$

THEN WE HAVE

$$\Rightarrow (13x+1)^2+3 \leqslant (5x-1)^2$$

$$\Rightarrow (169x^2+26x+1)+3 \leqslant 25x^2-10x+1$$

$$\Rightarrow 144x^2+36x+3 \leqslant 0$$

$$\Rightarrow \left(12x+\frac{3}{2}\right)^2-\frac{9}{4}+3 \leqslant 0$$

$$\Rightarrow \left(12x+\frac{3}{2}\right)^2+\frac{3}{4} \leqslant 0$$

WHICH IS A CONTRADICTION TO THE ASSERTION

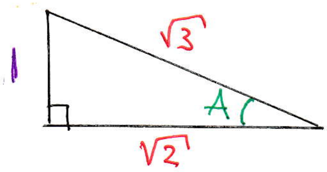
$\therefore$ BY CONTRADICTION, $(13x+1)^2+3>(5x-1)^2$

IYGB - MP2 PAPER M - QUESTION 4

- STARTING FROM $\cos 2A = \frac{1}{3}$ & NOTING THAT A IS OBTUSE

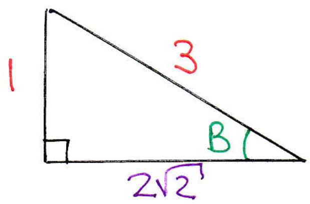
$$\Rightarrow \cos 2A = 2\cos^2 A - 1$$
$$\Rightarrow \frac{1}{3} = 2\cos^2 A - 1$$
$$\Rightarrow \frac{4}{3} = 2\cos^2 A$$
$$\Rightarrow \cos^2 A = \frac{2}{3}$$
$$\Rightarrow \cos A = -\sqrt{\frac{2}{3}} \qquad (\text{"A" IS OBTUSE})$$

- HENCE BY A STANDARD RIGHT ANGLED TRIANGLE



$$\therefore \boxed{\tan A = -\frac{1}{\sqrt{2}}}$$

- SIMILARLY $\sin B = \frac{1}{3}$ (B OBTUSE SO BOTH $\sin B$ & $\tan B$ ARE NEGATIVE)



$$\therefore \boxed{\tan B = -\frac{1}{2\sqrt{2}}}$$

- FINALLY BY THE COMPOUND ANGLE IDENTITIES

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}}{1 - \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)} = \frac{-\frac{3}{2\sqrt{2}}}{1 - \frac{1}{4}}$$

$$= -\frac{\frac{3}{2\sqrt{2}}}{\frac{3}{4}} = -\frac{12}{6\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

IYGB – MP2 PAPER M – QUESTION 5

- REWRITE THE EQUATION & DIFFERENTIATE THE PRODUCT.

$$\Rightarrow y = x(\ln x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 1 \times (\ln x)^{\frac{1}{2}} + x \times \frac{1}{2}(\ln x)^{-\frac{1}{2}} \times \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = (\ln x)^{\frac{1}{2}} + \frac{1}{2}(\ln x)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=e^4} = \sqrt{\ln e^4} + \frac{1}{2\sqrt{\ln e^4}}$$

$$= 2 + \frac{1}{4}$$

$$= \frac{9}{4}$$

- ALSO WHEN $x = e^4$, $y = e^4[\ln e^4]^{\frac{1}{2}}$ I.E $(e^4, 2e^4)$

- THUS WE HAVE THE EQUATION OF THE TANGENT

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 2e^4 = \frac{9}{4}(x - e^4)$$

$$\Rightarrow 4y - 8e^4 = 9x - 9e^4$$

$$\Rightarrow 4y = 9x - e^4$$

I.E $a = 4$
$b = 9$

IYGB - MP2 PAPER M - QUESTION 6

- START FORMING SOME EQUATIONS — LET a = 1ST TERM OF A.P.
  b = 1ST TERM OF G.P
  d = COMMON DIFFERENCE
  r = COMMON RATIO

- AP : $a \quad + \quad (a+d)$
  G.P : $b \quad + \quad br$
  NEW SERIES : $[a+b] \quad + \quad [a+d+br]$

- (I) $a+b = \frac{3}{8}$
  (II) $a+d+br = \frac{13}{16}$
  (III) $r = 2a$
  (IV) $d = 4b$

  SUBSTITUTE (III) & (IV) INTO (II) GIVES

  $a + 4b + 2ab = \frac{13}{16}$

- $\left\{\begin{array}{l} a+b = \frac{3}{8} \\ a+4b+2ab = \frac{13}{16} \end{array}\right\} \Rightarrow a = \frac{3}{8} - b$

$$\Rightarrow \frac{3}{8} - b + 4b + 2b\left(\frac{3}{8} - b\right) = \frac{13}{16}$$

$$\Rightarrow \frac{3}{8} + 3b + \frac{3}{4}b - 2b^2 = \frac{13}{16} \quad \Big) \times 16$$

$$\Rightarrow 6 + 48b + 12b - 32b^2 = 13$$

$$\Rightarrow 0 = 32b^2 - 60b + 7$$

$$\Rightarrow (8b-1)(4b-7) = 0$$

$$\Rightarrow b = \begin{cases} 1/8 \\ 7/4 \end{cases}$$

IYGB - MP2 PAPER M - QUESTION 7

- START BY THE SUBSTITUTION GIVEN

$$\Rightarrow t = \sqrt{x} + 3$$
$$\Rightarrow t = x^{\frac{1}{2}} + 3$$
$$\Rightarrow \frac{dt}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$
$$\Rightarrow \frac{dt}{dx} = \frac{1}{2x^{\frac{1}{2}}}$$
$$\Rightarrow 2x^{\frac{1}{2}}\,dt = dx$$
$$\Rightarrow dx = 2\sqrt{x}\,dt$$

LIMITS

$x = 1 \longmapsto t = 4$

$x = 36 \longmapsto t = 9$

- HENCE THE INTEGRAL TRANSFORMS AS FOLLOWS

$$\int_1^{36} \frac{1}{\sqrt{x^{3/2} + 3x}}\,dx = \int_4^9 \frac{1}{\sqrt{x^{\frac{3}{2}} + 3x}}\,(2\sqrt{x})\,dt$$

$$= \int_4^9 \frac{2\sqrt{x}}{\sqrt{x(x^{\frac{1}{2}} + 3)}}\,dt$$

$$= \int_4^9 \frac{2\sqrt{x}}{\sqrt{xt}}\,dt$$

$$= \int_4^9 \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}\sqrt{t}}\,dt$$

$$= \int_4^9 2t^{-\frac{1}{2}}\,dt$$

$$= \left[4t^{\frac{1}{2}}\right]_4^9 = \left(4 \times 9^{\frac{1}{2}}\right) - \left(4 \times 4^{\frac{1}{2}}\right)$$

$$= 12 - 8 = 4$$

IYGB – MP2 PAPER M – QUESTION 8

a) $y = e^{-x}\ln x \quad , \quad x > 0$

$$\Rightarrow \frac{dy}{dx} = -e^{-x} \times \ln x + e^{-x} \times \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = e^{-x}\left[\frac{1}{x} - \ln x\right]$$

SOLVING FOR ZERO TO FIND STATIONARY POINTS

$$\Rightarrow \frac{1}{x} - \ln x = 0 \qquad e^{-x} \neq 0$$

$$\Rightarrow 1 - x\ln x = 0$$

$$\Rightarrow f(x) = 1 - x\ln x$$

- $f(1) = 1 \quad > 0$
- $f(2) = -0.386.. \quad < 0$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE INTERVAL $(1,2)$ THERE IS AT LEAST ONE ROOT IN THE INTERVAL, I.E HERE A STATIONARY POINT

IYGB - MP2 PAPER M - QUESTION 8

b) • $f(x) = 1 - x\ln x$

• $f'(x) = -\left[1 \times \ln x + x \times \frac{1}{x}\right] = -\left[\ln x + 1\right]$

$= -1 - \ln x$

BY THE NEWTON RAPHSON METHOD

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{1 - x_n \ln x_n}{-1 - \ln x_n}$$

$$\Rightarrow x_{n+1} = x_n + \frac{1 - x_n \ln x_n}{1 + \ln x_n}$$

$$\Rightarrow x_{n+1} = \frac{x_n + \cancel{x_n \ln x_n} + 1 - \cancel{x_n \ln x_n}}{1 + \ln x_n}$$

$$\Rightarrow \boxed{x_{n+1} = \frac{x_n + 1}{1 + \ln x_n}}$$

NOW USING AS A STARTING VALUE $x_1 = 1.5$, IE A VALUE HALFWAY IN THE INTERVAL WE OBTAIN

$x_1 = 1.5$

$x_2 = 1.778770590\ldots$

$x_3 = 1.763266078\ldots$

$x_4 = 1.763222835\ldots$

$x_5 = 1.763222834\ldots$

$x_6 = 1.763222834$

$\therefore x \approx 1.76322283$

8 d.p

IYGB - MP2 PAPER M - QUESTION 9

a) $x = 2\cos t \qquad y = 4\sin t \qquad 0 \le t \le \frac{\pi}{2}$

- $\frac{dx}{dt} = -2\sin t$ • $\frac{dy}{dt} = 4\cos t$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-2\sin t} = -2\cot t$

$\Rightarrow \left.\frac{dy}{dx}\right|_{t=\theta} = -2\cot\theta$

EQUATION OF TANGENT THROUGH $(2\cos\theta, 4\sin\theta)$

$\Rightarrow y - 4\sin\theta = -2\cot\theta\,(x - 2\cos\theta)$

$\Rightarrow y - 4\sin\theta = -\frac{2\cos\theta}{\sin\theta}(x - 2\cos\theta)$

$\Rightarrow y\sin\theta - 4\sin^2\theta = (-2\cos\theta)x + 4\cos^2\theta$

$\Rightarrow y\sin\theta + 2x\cos\theta = 4\cos^2\theta + 4\sin^2\theta$

$\Rightarrow y\sin\theta + 2x\cos\theta = 4(\cos^2\theta + \sin^2\theta)$

$\Rightarrow y\sin\theta + 2x\cos\theta = 4$

AS REQUIRED

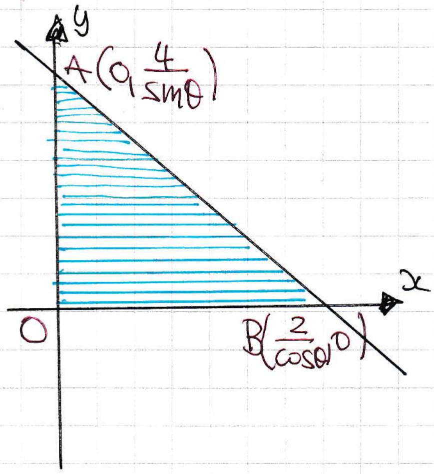
b) WHEN $x=0$, $y\sin\theta = 4$

$y = \frac{4}{\sin\theta}$ $\qquad A\left(0, \frac{4}{\sin\theta}\right)$

WHEN $y=0$, $2x\cos\theta = 4$

$x = \frac{2}{\cos\theta}$ $\qquad B\left(\frac{2}{\cos\theta}, 0\right)$

IYGB - MP2 PAPER M - QUESTION 9



AREA OF $\overset{\triangle}{OAB}$ $= \frac{1}{2}|OA||OB|$

$= \frac{1}{2} \times \frac{4}{\sin\theta} \times \frac{2}{\cos\theta}$

$= \frac{8}{2\sin\theta\cos\theta}$

$= \frac{8}{\sin 2\theta}$

AS $0 \leqslant \theta \leqslant \frac{\pi}{2}$ $\Longrightarrow$ $\sin 2\theta$ LIES BETWEEN 0 & 1

$\Longrightarrow$ $\frac{1}{\sin 2\theta}$ IS AT LEAST 1

$\Longrightarrow$ $\frac{8}{\sin 2\theta}$ IS AT LEAST 8

$\Longrightarrow$ MINIMUM AREA OF 8 UNITS

$\Longrightarrow$ WHICH OCCURS WHEN $\theta = \frac{\pi}{4}$

$\Longrightarrow$ $P(\sqrt{2}, 2\sqrt{2})$

IYGB - MP2 PAPER M - QUESTION 10

a) SEPARATING THE VARIABLES

$$\Rightarrow \frac{dy}{dt} = k(1-2y)(1-3y)$$

$$\Rightarrow 1\,dy = k(1-2y)(1-3y)\,dt$$

$$\Rightarrow \frac{1}{(1-2y)(1-3y)}\,dy = k\,dt$$

BY PARTIAL FRACTIONS

$$\frac{1}{(1-2y)(1-3y)} \equiv \frac{A}{1-2y} + \frac{B}{1-3y}$$

$$\boxed{1 \equiv A(1-3y) + B(1-2y)}$$

- If $y=\frac{1}{2}$, $1 = -\frac{1}{2}A$, $A=-2$
- If $y=\frac{1}{3}$, $1 = \frac{1}{3}B$, $B=3$

RETURNING TO THE O.D.E

$$\Rightarrow \int \frac{-2}{1-2y} + \frac{3}{1-3y}\,dy = \int k\,dt$$

$$\Rightarrow \ln|1-2y| - \ln|1-3y| = kt + C$$

$$\Rightarrow \ln\left|\frac{1-2y}{1-3y}\right| = kt + C$$

AS REQUIRED

IYGB - MP2 PAPER M - QUESTION 10

b) WHEN $t=0$, $y=0$

$$\Rightarrow \cancel{\ln 1} = 0 + C$$

$$\Rightarrow C = 0$$

$$\Rightarrow \ln\left|\frac{1-2y}{1-3y}\right| = kt \longrightarrow$$

WHEN $t=\ln 4$, $y=\frac{1}{4}$

$$\ln\left|\frac{1-\frac{1}{2}}{1-\frac{3}{4}}\right| = k\ln 4$$

$$\ln\left|\frac{1/2}{1/4}\right| = 2k\ln 2$$

$$\ln 2 = 2k\ln 2$$

$$1 = 2k$$

$$k = \frac{1}{2}$$

$$\Rightarrow \frac{1-2y}{1-3y} = e^{kt}$$

$$\Rightarrow 1-2y = (1-3y)e^{kt}$$

$$\Rightarrow 1-2y = e^{kt} - 3ye^{kt}$$

$$\Rightarrow 3ye^{kt} - 2y = e^{kt} - 1$$

$$\Rightarrow y(3e^{kt}-2) = e^{kt} - 1$$

$$\Rightarrow \boxed{y = \frac{e^{kt}-1}{3e^{kt}-2}}$$

$$\Rightarrow y = \frac{e^{\frac{1}{2}t}-1}{3e^{\frac{1}{2}t}-2}$$

$$\Rightarrow y = \frac{e^{\frac{1}{2}t}e^{-\frac{1}{2}t} - 1\times e^{-\frac{1}{2}t}}{3e^{\frac{1}{2}t}e^{-\frac{1}{2}t} - 2\times e^{-\frac{1}{2}t}}$$

$$\Rightarrow y = \frac{1-e^{-\frac{1}{2}t}}{3-2e^{-\frac{1}{2}t}}$$

AS REQUIRED

c) AS $t \to +\infty$, $e^{-\frac{1}{2}t} \to 0$, SO $y \to \frac{1}{3}$

LIMITING VALUE OF $\frac{1}{3}$

IYGB - MP2 PAPER M - QUESTION 11

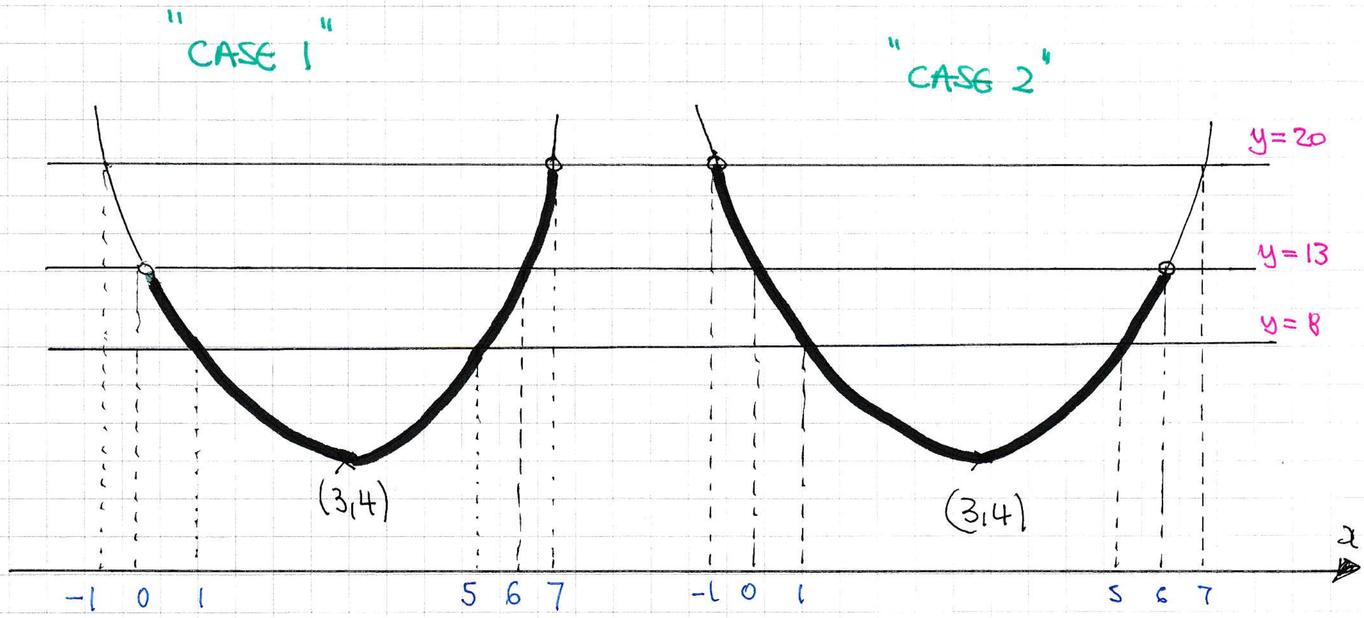
$f(x) = x^2 - 6x + 13$, DOMAIN TO BE DETERMINED

- $f(x) = 8$, 2 SOLUTIONS
- $f(x) = 13$, 1 SOLUTION
- $f(x) = 20$, NO SOLUTION

COMPLETE THE SQUARE TO LOCATE THE MINIMUM

$$f(x) = (x-3)^2 - 3^2 + 13$$
$$f(x) = (x-3)^2 + 4$$

TWO CASES TO CONSIDER - DRAW A GRAPH IN EACH CASE



LARGEST POSSIBLE REAL DOMAIN OF $f(x)$ IS

EITHER $0 < x < 7$ OR $-1 < x < 6$

IYGB - MP2 PAPER U - QUESTION 12

$$\int \frac{1}{1+\cos 2x}\,dx = \int \frac{1}{1+(2\cos^2 x-1)}\,dx$$

$$= \int \frac{1}{2\cos^2 x}\,dx$$

$$= \int \frac{1}{2}\sec^2 x\,dx$$

$$= \frac{1}{2}\tan x + C$$

IYGB - MP2 PAPER M - QUESTION 13

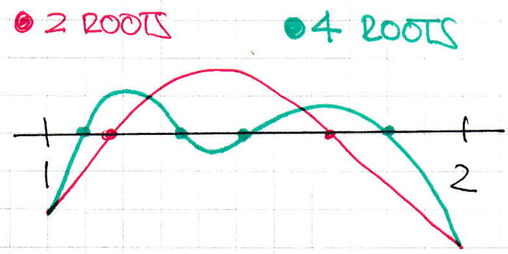
$$f(x) = \frac{50x^2 - 142x + 95}{2x - 5}$$

a) $x \in \mathbb{R}$, $x \neq \frac{5}{2}$ (DENOMINATOR ZERO) //

b) I) $f(1) = -1 < 0$

$f(2) = -11 < 0$

NO CHANGE OF SIGN

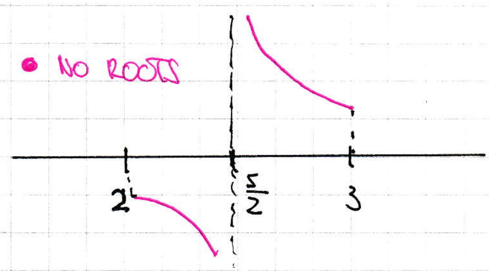


AS THE FUNCTION HAS NO DISCONTINUITIES BETWEEN 1 & 2 EITHER THERE ARE NO SOLUTIONS IN THIS INTERVAL OR THERE IS AN EVEN NUMBER OF SOLUTIONS (SEE DIAGRAM)

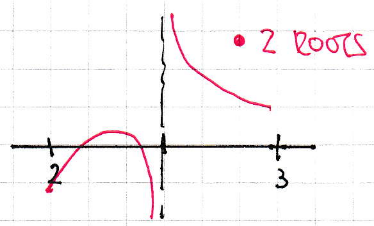
II) $f(2) = -11 < 0$

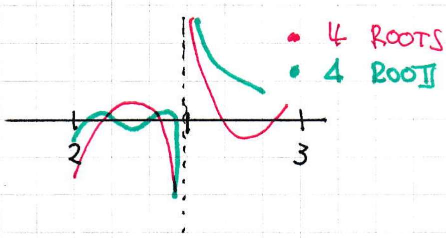
$f(3) = 119 > 0$

CHANGE OF SIGN



THE FUNCTION HAS A DISCONTINUITY AT $x = 2.5$ (ASYMPTOTE) EITHER THERE ARE NO SOLUTIONS IN THE INTERVAL OR THERE IS AN EVEN NUMBER OF ROOTS (SEE DIAGRAMS)





IYGB - MP2 PAPER M - QUESTION 13

c) BY LONG DIVISION

$$
\begin{array}{r|l|l}
2x-5 & 50x^2 - 142x + 95 & 25x - \frac{17}{2} \\
 & -50x^2 + 125x & \\
\hline
 & -17x + 95 & \\
 & +17x - \frac{85}{2} & \\
\hline
 & \frac{105}{2} &
\end{array}
$$

$$\therefore \; f(x) = 25x - \frac{17}{2} + \frac{105/2}{2x-5}$$

$\therefore$ $A = 25$

$B = -\frac{17}{2}$

$C = \frac{105}{2}$

d)

$$f(x) = 25x - \frac{17}{2} + \frac{105}{2}(2x-5)^{-1}$$

$$\Rightarrow f'(x) = 25 - \frac{105}{2}(2x-5)^{-2} \times 2$$

$$\Rightarrow f'(x) = 25 - \frac{105}{(2x-5)^2}$$

SOLVING FOR ZERO

$$\Rightarrow \frac{105}{(2x-5)^2} = 25$$

$$\Rightarrow (2x-5)^2 = \frac{21}{5}$$

$$\Rightarrow 2x-5 = \pm\sqrt{\frac{21}{5}}$$

$$\Rightarrow x = \frac{1}{2}\left[5 \pm \sqrt{\frac{21}{5}}\right] = \begin{cases} 3.525 \\ 1.475 \end{cases}$$

3 d.p.