IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper N
Difficulty Rating: 3.5150/1.1268

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

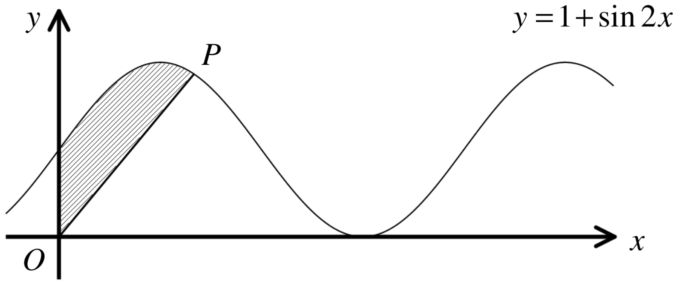
This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows the graph of the curve with equation

$$y = 1 + \sin 2x, \ x \in \mathbb{R}.$$

The point P lies on the curve where $x = \frac{\pi}{3}$.

Show that the area of the finite region bounded by the curve, the y axis and the straight line segment OP is exactly

$$\frac{1}{12}\left(2\pi + 9 - \pi\sqrt{3}\right). \qquad \textbf{(7)}$$

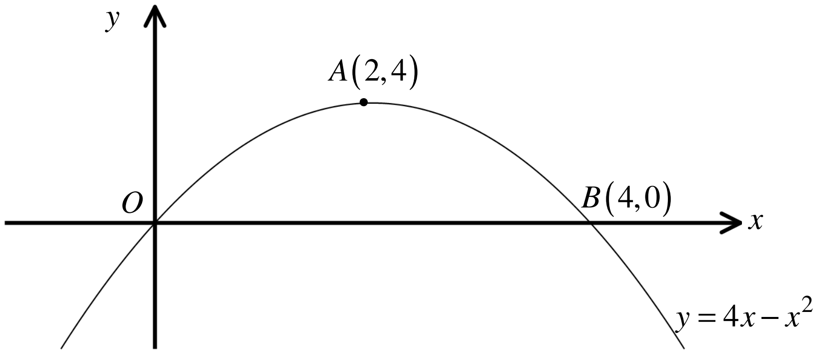
Question 2

Relative to a fixed origin O, the points A, B and C have respective position vectors

$$-3\mathbf{i} + \mathbf{k}, \qquad -\mathbf{i} + 4\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad 5\mathbf{i} + 4\mathbf{j}.$$

Calculate the size of the angle ABC and hence find the area of the triangle ABC. **(6)**

Question 3



The figure above shows part of the graph of the curve with equation

$$y = 4x - x^2, \; x \in \mathbb{R}.$$

The graph meets the coordinate axes at the origin O and $B(4,0)$, and has a stationary point at $A(2,4)$.

Sketch on separate diagrams, indicating the new coordinates of the points A and B, the graph of …

a) … $y = \left|4x - x^2\right|$. **(2)**

b) … $y = 4|x| - |x|^2$. **(2)**

c) … $y = \left|4|x| - |x|^2\right|$. **(1)**

Question 4

A cubic equation has the following equation.

$$x^3 + 1 = 4x, \quad x \in \mathbb{R}.$$

a) Show that the above equation has a root α, which lies between 0 and 1. **(2)**

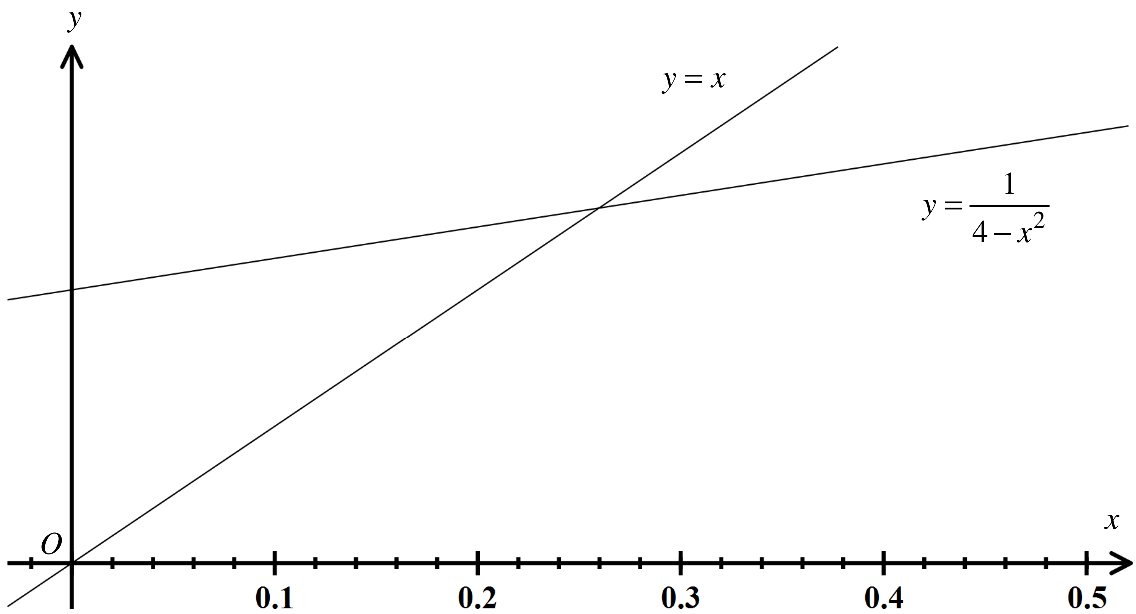
b) Show further that the above equation can be written as

$$x = \frac{1}{4 - x^2}. \qquad \textbf{(1)}$$

An iterative formula, based on the rearrangement of part **(b)**, is to be used to find α.

c) Starting with $x_1 = 0.1$, find to 4 decimal places, the value of x_2, x_3 and x_4. **(2)**

The diagram below is used to show the convergence of these iterations.



d) Draw, on a copy of this diagram, a "staircase" or "cobweb" pattern showing how the iterations of part **(c)** converge to α, marking the position of x_1, x_2, x_3 and x_4. **(2)**

Question 5

By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find

$$\int \frac{1}{4x^{\frac{1}{2}}\sqrt{x^{\frac{1}{2}}-1}}\,dx.$$ **(6)**

Question 6

The function f is given by

$$f : x \mapsto \frac{3}{x+2},\ x \in \mathbb{R},\ x \geq -1.$$

a) By sketching the graph of f, or otherwise, state its range. **(2)**

b) Determine an expression for $f^{-1}(x)$, the inverse of f. **(3)**

c) Find the domain and range of $f^{-1}(x)$. **(2)**

Question 7

Two variables x and y are related by

$$y = \tfrac{1}{4}\pi x^2 (4-x).$$

The variable y is changing with time t, at the constant rate of 0.2, in suitable units.

Find the rate at which x is changing with respect to t, when $x = 2$. **(6)**

Question 8

The curve C has equation

$$f(x)=\frac{x^2}{(x-a)^2},\ x\in\mathbb{R},\ x\neq a\ ,$$

where a is a non zero constant.

Given that $f'(2a)=-2$, determine the value of a. **(8)**

Question 9

Solve the differential equation

$$x(x+2)\frac{dy}{dx}=y,\quad x>0,\quad y>0$$

subject to the condition $y=2$ at $x=2$, giving the answer in the form $y^2=f(x)$. **(11)**

Question 10

Solve the following trigonometric equation

$$\tan 4x-\tan 2x=0,\ 0°\leq x<360°.$$ **(7)**

Question 11

The 1st, 3rd and 11th term of an arithmetic progression are the first three terms of a geometric progression.

It is further given that the sum of the first 13 terms of the arithmetic progression is 260.

Find, in any order, the common ratio of the geometric progression and the first term and common difference of the arithmetic progression. **(8)**

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Question 12

A curve C has implicit equation

$$xy(x-y)+16=0, \quad x \neq y, \quad x, y \neq 0.$$

Find the coordinates of the stationary point of C. **(9)**

Question 13

A curve is given parametrically by the equations

$$x=\frac{2t}{1+t^2}, \quad y=\frac{1-t^2}{1+t^2}, \quad t \in \mathbb{R}.$$

The point $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lies on this curve.

Show that an equation of the tangent at the point P is given by

$$x+y=\sqrt{2}.$$ **(13)**
