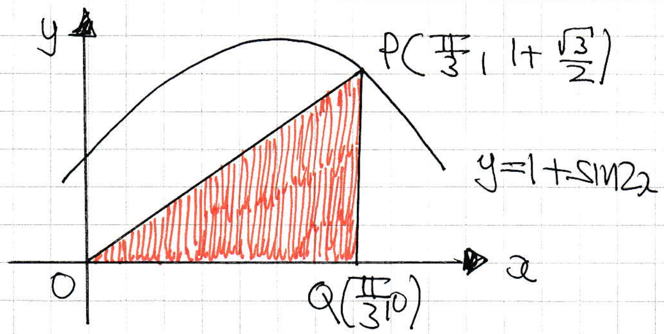
IYGB - MP2 PAPER N - QUESTION 1

- LOOKING AT THE DIAGRAM BELOW



- AREA UNDER THE CURVE BETWEEN $x=0$ & $x=\frac{\pi}{3}$ IS GIVEN BY

$$\int_0^{\frac{\pi}{3}} 1 + \sin 2x \, dx = \left[x - \tfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}}$$

$$= \left[\frac{\pi}{3} - \frac{1}{2}\left(-\frac{1}{2}\right)\right] - \left[0 - \frac{1}{2}\right]$$

$$= \frac{\pi}{3} + \frac{1}{4} + \frac{1}{2}$$

$$= \frac{\pi}{3} + \frac{3}{4}$$

- AREA OF THE TRIANGLE IS GIVEN BY

$$\frac{1}{2} \times \frac{\pi}{3} \times \left(1 + \frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + \frac{\pi\sqrt{3}}{12}$$

- REQUIRED AREA IS

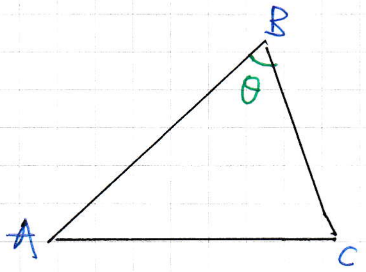
$$\left(\frac{\pi}{3} + \frac{3}{4}\right) - \left(\frac{\pi}{6} + \frac{\pi\sqrt{3}}{12}\right) = \frac{\pi}{6} + \frac{3}{4} - \frac{\pi\sqrt{3}}{12}$$

$$= \frac{1}{12}\left[2\pi + 9 - \pi\sqrt{3}\right]$$

AS REQUIRED

IYGB – MP2 PAPER N – QUESTION 2

$A(-3,0,1)$ $B(-1,4,1)$ $C(5,4,0)$



- $|\vec{AB}| = |\underline{b} - \underline{a}| = |(-1,4,1)-(-3,0,1)|$

$= |2,4,0| = \sqrt{4+16+0}$

$= \sqrt{20}$

- $|\vec{BC}| = |\underline{c} - \underline{b}| = |(5,4,0)-(-1,4,1)| = |6,0,-1| = \sqrt{36+0+1} = \sqrt{37}$
- $|\vec{CA}| = |\underline{a} - \underline{c}| = |(-3,0,1)-(5,4,0)| = |-8,-4,1| = \sqrt{64+16+1} = 9$

BY THE COSINE RULE

$\Longrightarrow |AC|^2 = |AB|^2 + |BC|^2 - 2|AB||BC|\cos\theta$

$\Longrightarrow 9^2 = 20 + 37 - 2\sqrt{20}\sqrt{37}\cos\theta$

$\Longrightarrow 2\sqrt{20}\sqrt{37}\cos\theta = 20 + 37 - 81$

$\Longrightarrow \cos\theta = 0.441128\ldots$

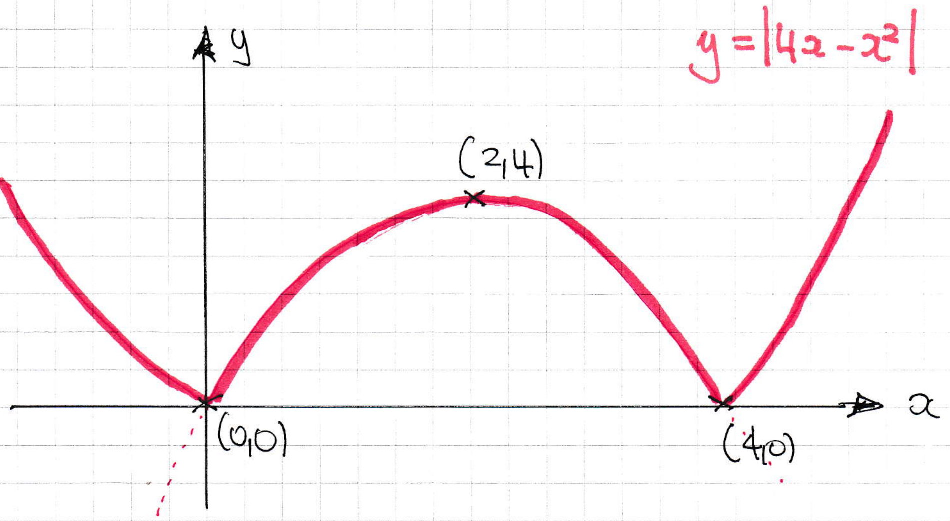
$\Longrightarrow \theta \simeq 116°$

FINALLY THE AREA IS GIVEN BY

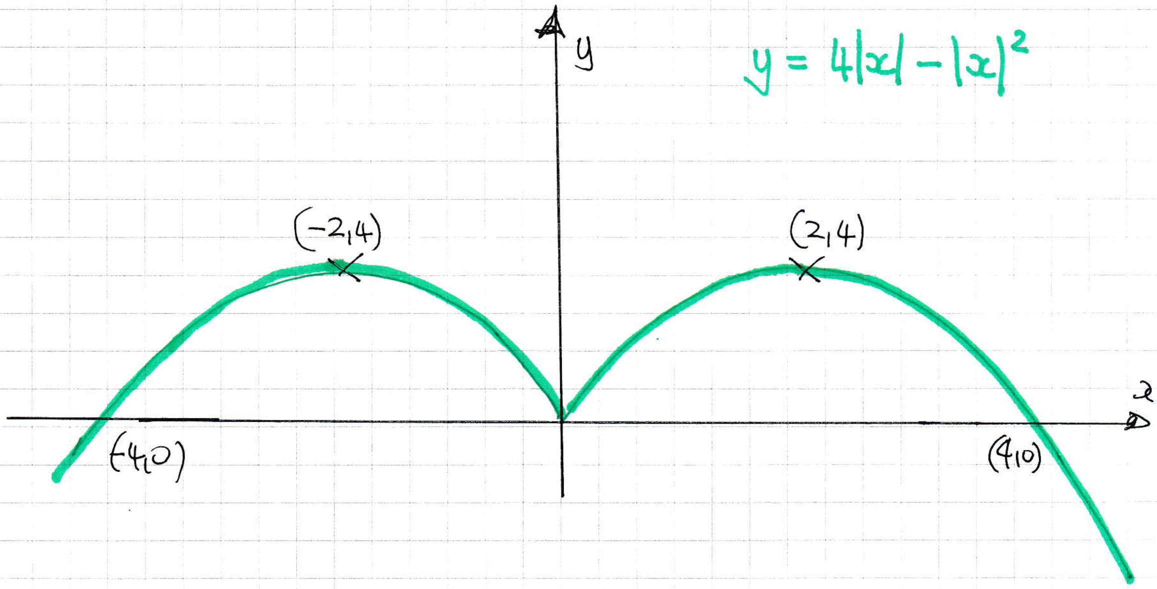
$\frac{1}{2}|AB||BC|\sin\theta = \frac{1}{2}\sqrt{20} \times \sqrt{37}\sin(116.^{\circ}..) \simeq 12.2$

IYGB – MP2 PAPER N – QUESTION 3

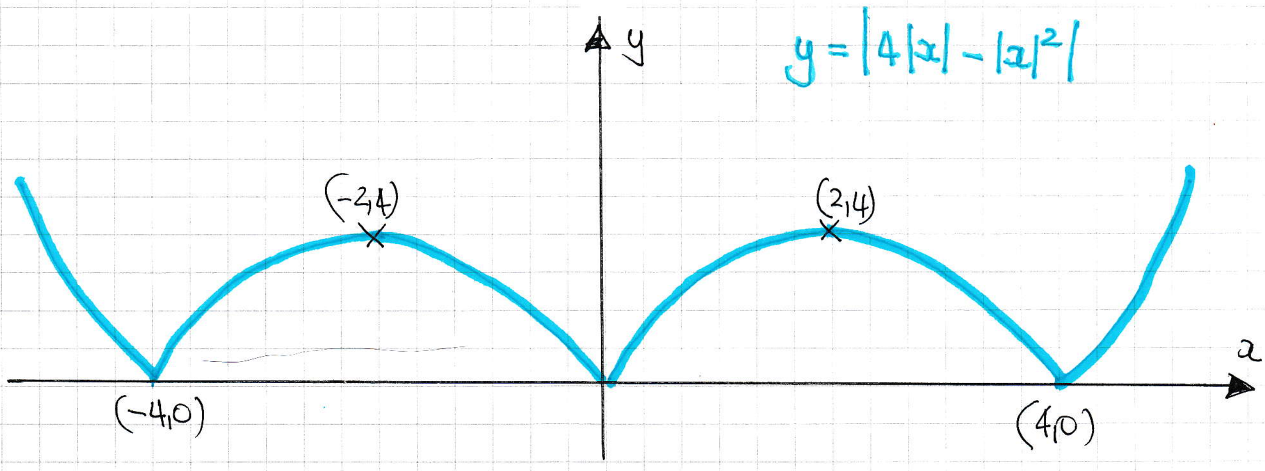
a)



b)



c)



IYGB - MP2 PAPER N - QUESTION 4

a) PROCEED AS FOLLOWS

$$\Rightarrow x^3+1=4x$$
$$\Rightarrow x^3-4x+1=0$$
$$\Rightarrow f(x)=x^3-4x+1$$

$$\left.\begin{array}{l} f(0)=1>0 \\ f(1)=-2<0 \end{array}\right\}$$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE INTERVAL $(0,1)$, THERE MUST BE AT LEAST ONE ROOT α IN THE INTERVAL

b) REARRANGE THE GIVEN EQUATION

$$\Rightarrow x^3+1=4x$$
$$\Rightarrow x^3-4x=-1$$
$$\Rightarrow x(x^2-4)=-1$$
$$\Rightarrow x=-\frac{1}{x^2-4}$$
$$\Rightarrow x=\frac{1}{4-x^2}$$

AS REQUIRED

c) WRITING THE ABOVE EQUATION AS A RECURRENCE RELATION

$$x_{n+1}=\frac{1}{4-x_n^2}$$

$x_1=0.1$

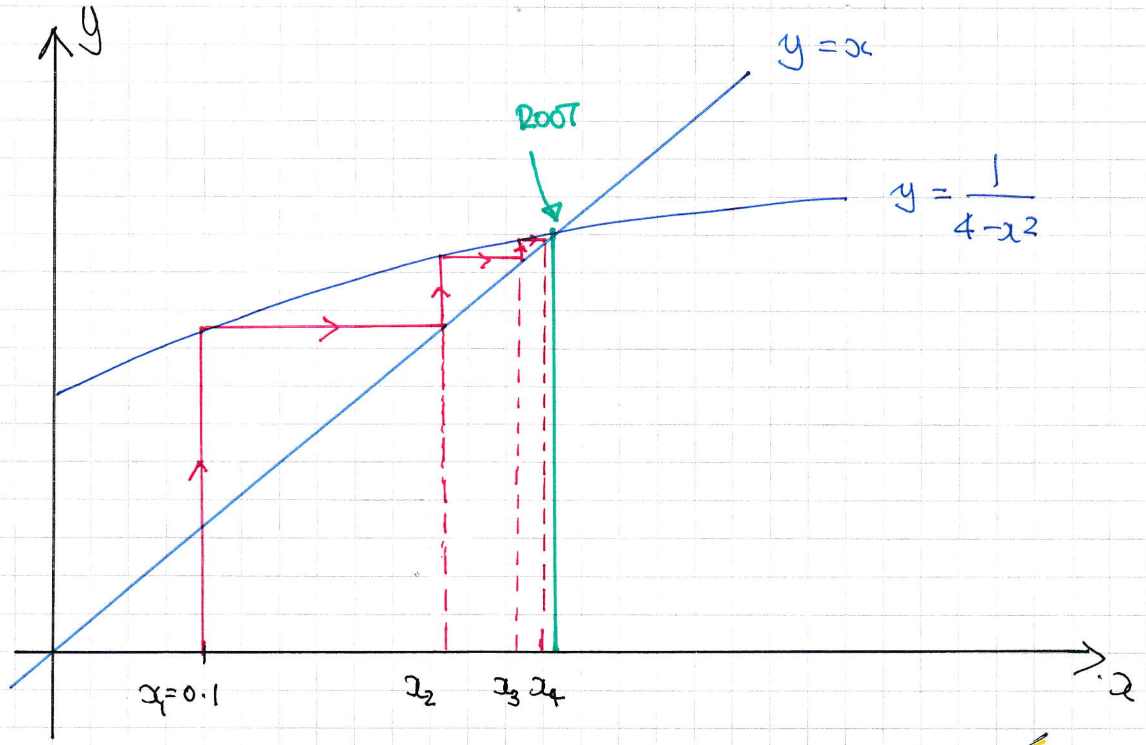
$x_2=0.25062656\ldots \simeq 0.2506$

$x_3=0.25398848\ldots \simeq 0.2540$

$x_4=0.25409797\ldots \simeq 0.2541$

IYGB - MP2 PAPER N - QUESTION 4

d)



IYGB – MP2 PAPER N – QUESTION 5

USING THE SUBSTITUTION GIVEN

$$\Rightarrow u = x^{\frac{1}{2}}$$

$$\Rightarrow u^2 = x$$

$$\Rightarrow x = u^2$$

$$\Rightarrow \frac{dx}{du} = 2u$$

$$\Rightarrow dx = 2u\,du$$

TRANSFORMING THE INTEGRAL WE OBTAIN

$$\int \frac{1}{4x^{\frac{1}{2}}\sqrt{x^{\frac{1}{2}}-1}}\,dx = \int \frac{1}{\cancel{4u}\sqrt{u-1}}(\cancel{2u}\,du)$$

$$= \int \frac{1}{2}(u-1)^{-\frac{1}{2}}\,du$$

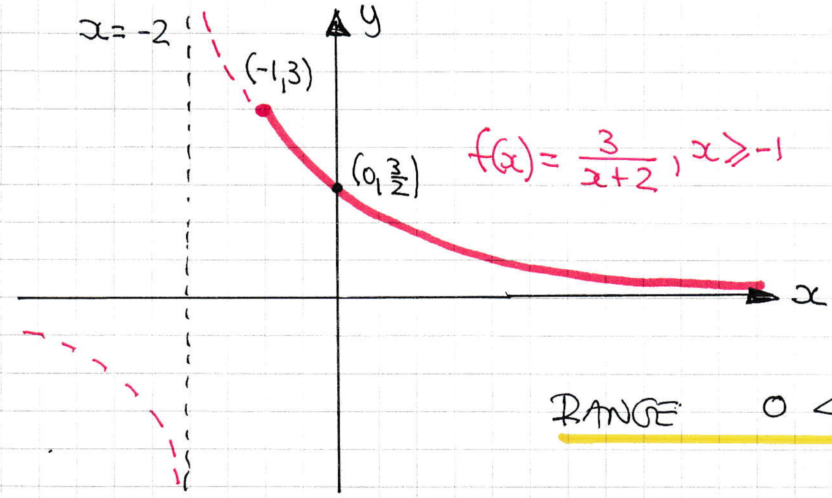
$$= \frac{\frac{1}{2}}{\frac{1}{2}}(u-1)^{\frac{1}{2}} + C$$

$$= (u-1)^{\frac{1}{2}} + C$$

$$= \sqrt{\sqrt{x}-1} + C$$

IYGB - MP2 PAPER N - QUESTION 6

a)



RANGE $0 < f(x) \leq 3$

b)

LET $f(x) = \dfrac{3}{x+2}$

$$y = \frac{3}{x+2}$$

$$yx + 2y = 3$$

$$yx = 3 - 2y$$

$$x = \frac{3-2y}{y}$$

$$\therefore f^{-1}(x) = \frac{3-2x}{x}$$

c)

USING A TWO WAY TABLE

	$f(x)$	$f^{-1}(x)$
D	$x \geq -1$	$0 < x \leq 3$
R	$0 < f(x) \leq 3$	$f^{-1}(x) \geq -1$

$\therefore$ DOMAIN: $0 < x \leq 3$

RANGE $f^{-1}(x) \geq -1$

IYGB – MP2 PAPER N – QUESTION 7

$$y = \frac{1}{4}\pi x^2(4-x) \quad \& \quad \frac{dy}{dt} = 0.2$$

$$\Rightarrow y = \frac{1}{4}\pi(4x^2 - x^3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4}\pi(8x - 3x^2)$$

NOW WE REQUIRE $\frac{dx}{dt}$, FOLLOWED BY $\left.\frac{dx}{dt}\right|_{x=2}$

$$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{1}{\frac{1}{4}\pi(8x-3x^2)} \times 0.2$$

$$\frac{dx}{dt} = \frac{0.8}{\pi x(8-3x)}$$

$$\left.\frac{dx}{dt}\right|_{x=2} = \frac{0.8}{\pi \times 2 \times (8 - 3\times 2)} = \frac{1}{5\pi} \simeq 0.0637$$

IYGB – MP2 PAPER N – QUESTION 8

$$f(x) = \frac{x^2}{(x-a)^2}, \quad x \in \mathbb{R}, \quad x \neq a$$

DIFFERENTIATING BY THE QUOTIENT RULE

$$\Rightarrow f'(x) = \frac{(x-a)^2 \times 2x - x^2 \times 2(x-a)^1}{(x-a)^4}$$

$$\Rightarrow f'(x) = \frac{2x(x-a)^{\cancel{2}\,1} - 2x^2\cancel{(x-a)}}{(x-a)^{\cancel{4}\,3}}$$

$$\Rightarrow f'(x) = \frac{2x(x-a) - 2x^2}{(x-a)^3}$$

$$\Rightarrow f'(x) = \frac{\cancel{2x^2} - 2ax - \cancel{2x^2}}{(x-a)^3}$$

$$\Rightarrow f'(x) = -\frac{2ax}{(x-a)^3}$$

NOW USING $f'(2a) = -2$

$$\Rightarrow -2 = -\frac{2a(2a)}{(2a-a)^3}$$

$$\Rightarrow -2 = -\frac{4a^2}{a^3}$$

$$\Rightarrow -2 = -\frac{4}{a}$$

$$\Rightarrow -2a = -4$$

$$\Rightarrow a = 2$$

IYGB - MP2 PAPER N - QUESTION 9

SEPARATING VARIABLES

$$\Longrightarrow x(x+2)\frac{dy}{dx} = y$$

$$\Longrightarrow x(x+2)\,dy = y\,dx$$

$$\Longrightarrow \frac{1}{y}\,dy = \frac{1}{x(x+2)}\,dx$$

BY PARTIAL FRACTIONS, FOR THE R.H.S

$$\frac{1}{x(x+2)} \equiv \frac{A}{x} + \frac{B}{x+2}$$

$$\boxed{1 \equiv A(x+2) + Bx}$$

- $x=0 \Longrightarrow 1 = 2A$
  $\Longrightarrow A = \frac{1}{2}$
- $x=-2 \Longrightarrow 1 = -2B$
  $\Longrightarrow B = -\frac{1}{2}$

RETURNING TO THE O.D.E, & INTEGRATING SUBJECT TO (2,2)

$$\Longrightarrow \int_{y=2}^{y} \frac{1}{y}\,dy = \int_{x=2}^{x} \frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2}\,dx$$

$$\Longrightarrow \int_{y=2}^{y} \frac{2}{y}\,dy = \int_{x=2}^{x} \frac{1}{x} - \frac{1}{x+2}\,dx$$

$$\Longrightarrow \Big[2\ln|y|\Big]_{y=2}^{y} = \Big[\ln|x| - \ln|x+2|\Big]_{x=2}^{x}$$

IYGB - MP2 PAPER N - QUESTION 9

$$\Rightarrow \quad 2\ln|y| - 2\ln 2 = \left(\ln|x| - \ln|x+2|\right) - \left(\ln 2 - \ln 4\right)$$

$$\Rightarrow \quad \ln y^2 - \ln 4 = \ln\left|\frac{x}{x+2}\right| - \ln\frac{1}{2}$$

$$\Rightarrow \quad \ln\left(\frac{y^2}{4}\right) = \ln\left|\frac{x}{x+2}\right| + \ln 2$$

$$\Rightarrow \quad \ln\left(\frac{y^2}{4}\right) = \ln\left|\frac{2x}{x+2}\right|$$

$$\Rightarrow \quad \frac{y^2}{4} = \frac{2x}{x+2}$$

$$\Rightarrow \quad y^2 = \frac{8x}{x+2}$$

IYGB - MP2 PAPER N - QUESTION 10

METHOD A

$$\Rightarrow \tan 4x - \tan 2x = 0$$
$$\Rightarrow \tan 4x = \tan 2x$$
$$\Rightarrow 4x = 2x \pm 180n \qquad n = 0,1,2,3,\ldots$$
$$\Rightarrow 2x = 0^\circ \pm 180n$$
$$\Rightarrow x = 0^\circ \pm 90n$$

$$x = 0^\circ, 90^\circ, 180^\circ, 270^\circ$$

METHOD B

using $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\Rightarrow \tan(2\times 2x) - \tan 2x = 0$$
$$\Rightarrow \frac{2\tan 2x}{1-\tan^2 2x} - \tan 2x = 0$$
$$\Rightarrow 2\tan 2x - \tan 2x\left(1-\tan^2 2x\right) = 0$$
$$\Rightarrow \tan 2x\left[2-\left(1-\tan^2 2x\right)\right] = 0$$
$$\Rightarrow \tan 2x\,\cancel{\left(1+\tan^2 2x\right)} = 0$$

NO SOLUTIONS

$$\Rightarrow \tan 2x = 0$$

IYGB - MP2 PAPER N - QUESTION 10

$$\Longrightarrow \quad 2x = 0 \pm 180n \qquad n = 0,1,2,3,\ldots$$

$$\Longrightarrow \quad x = 0 \pm 90n$$

$$\therefore \; x = 0^\circ, 90^\circ, 180^\circ, 270^\circ$$

AS BEFORE

IYGB - MP2 PAPER N - QUESTION 11

STARTING THE MODELLING WITH A DIAGRAM

$$\begin{tikzcd}[column sep=large] \text{A.P.} & u_1 & u_3 & u_{11} \\ & a \arrow[r, bend right, "\times r"'] & a+2d \arrow[r, bend right, "\times r"'] & a+10d \end{tikzcd}$$

FROM THE ABOVE WE HAVE

$$\left.\begin{aligned} ar &= a+2d \\ (a+2d)r &= a+10d \end{aligned}\right\}$$

DIVIDING THE EQUATIONS "UPWARDS"

$$\Longrightarrow \frac{a+2d}{a} = \frac{a+10d}{a+2d}$$

$$\Longrightarrow (a+2d)^2 = a(a+10d)$$

$$\Longrightarrow \cancel{a^2} + 4ad + 4d^2 = \cancel{a^2} + 10ad$$

$$\Longrightarrow 4d^2 = 6ad$$

$$\Longrightarrow 2d^2 = 3ad$$

$d \neq 0$

$$\Longrightarrow 2d = 3a$$

NOW WE MAKE USE OF $S_{13} = 260$ FOR THE ARITHMETIC SERIES

$$\Longrightarrow \frac{13}{2}\left[2a + 12d\right] = 260$$

$$\Longrightarrow 13\left[a + 6d\right] = 260$$

$$\Longrightarrow a + 6d = 20$$

IYGB - MP2 PAPER N - QUESTION 11

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{r} 2d = 3a \\ a+6d = 20 \end{array}\right\} \Rightarrow \left.\begin{array}{r} 6d = 9a \\ a+6d = 20 \end{array}\right\}$$

$$\Rightarrow a = 20 - 9a$$

$$\Rightarrow 10a = 20$$

$$\Rightarrow \underline{a = 2}$$

$$\Rightarrow \underline{d = 3}$$

$$\Rightarrow ar = a + 2d$$

$$\Rightarrow 2r = 2 + 6$$

$$\Rightarrow \underline{r = 4}$$

IYGB - MP2 PAPER N - QUESTION 12

EXPANDING & DIFFERENTIATING IMPLICITLY

$$\Rightarrow xy(x-y)+16=0$$

$$\Rightarrow x^2y - xy^2 + 16 = 0$$

$$\Rightarrow \frac{d}{dx}(x^2y) - \frac{d}{dx}(xy^2) = \frac{d}{dx}(-16)$$

$$\Rightarrow 2xy + x^2\frac{dy}{dx} - y^2 - x\left(2y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow 2xy + x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$$

FOR STATIONARY POINTS $\frac{dy}{dx}=0$.

$$\Rightarrow 2xy - y^2 = 0$$

$$\Rightarrow 2x - y = 0 \qquad (y \neq 0)$$

$$\Rightarrow y = 2x$$

SUBSTITUTE INTO THE EQUATION

$$\Rightarrow x(2x)[x-2x)+16=0$$

$$\Rightarrow -2x^3+16=0$$

$$\Rightarrow 16 = 2x^3$$

$$\Rightarrow 8 = x^3$$

$$\Rightarrow x = 2$$

$$\therefore (2,4)$$

IYGB - MP2 PAPER N - QUESTION 13

START BY OBTAINING THE GRADIENT FUNCTION

$$x = \frac{2t}{1+t^2} \qquad\qquad y = \frac{1-t^2}{1+t^2}$$

$$\frac{dx}{dt} = \frac{(1+t^2)\times 2 - 2t(2t)}{(1+t^2)^2} \qquad\qquad \frac{dy}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{2+2t^2-4t^2}{(1+t^2)^2} \qquad\qquad \frac{dy}{dt} = \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{2-2t^2}{(1+t^2)^2} \qquad\qquad \frac{dy}{dt} = -\frac{4t}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4t}{2-2t^2} = \frac{-2t}{1-t^2} = \frac{2t}{t^2-1}$$

USING THE x EQUATION

$$\Rightarrow \frac{2t}{1+t^2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2}(1+t^2) = 4t$$

$$\Rightarrow 1+t^2 = 2\sqrt{2}t$$

$$\Rightarrow t^2 - 2\sqrt{2}t + 1 = 0$$

$$\Rightarrow (t-\sqrt{2})^2 - 2 + 1 = 0$$

$$\Rightarrow (t-\sqrt{2})^2 = 1$$

$$\Rightarrow t - \sqrt{2} = \pm 1$$

$$\therefore t = \begin{cases} 1+\sqrt{2} \\ -1+\sqrt{2} \end{cases}$$

IYGB - MP2 PAPER N - QUESTION 13

VERIFY WITH THE y EQUATION (OR SOLVING)

IF $t = 1+\sqrt{2}$

$t^2 = 1+2\sqrt{2}+2$

$t^2 = 3+2\sqrt{2}$

$$y = \frac{1-(3+2\sqrt{2})}{1+(3+2\sqrt{2})}$$

$$y = \frac{-2-2\sqrt{2}}{4+2\sqrt{2}}$$

$$y = -\frac{2+2\sqrt{2}}{4+2\sqrt{2}}$$

$$y = -\frac{1+\sqrt{2}}{2+\sqrt{2}}$$

$$y = -\frac{(1+\sqrt{2})(2-\sqrt{2})}{4-2}$$

$$y = -\frac{\cancel{2}-\sqrt{2}+2\sqrt{2}-\cancel{2}}{2}$$

$$y = -\frac{\sqrt{2}}{2}$$

IF $t = -1+\sqrt{2}$

$t^2 = 1-2\sqrt{2}+2$

$t^2 = 3-2\sqrt{2}$

$$y = \frac{1-(3-2\sqrt{2})}{1+(3-2\sqrt{2})}$$

$$y = \frac{-2+2\sqrt{2}}{4-2\sqrt{2}}$$

$$y = -\frac{2-2\sqrt{2}}{4-2\sqrt{2}}$$

$$y = -\frac{1-\sqrt{2}}{2-\sqrt{2}}$$

$$y = -\frac{(1-\sqrt{2})(2+\sqrt{2})}{4-2}$$

$$y = -\frac{\cancel{2}-\sqrt{2}-2\sqrt{2}-\cancel{2}}{2}$$

$$y = +\frac{\sqrt{2}}{2}$$

$\therefore\ t = -1+\sqrt{2}$

$$\left.\frac{dy}{dx}\right|_{t=-1+\sqrt{2}} = \frac{2\overset{t}{(-1+\sqrt{2})}}{\underset{t^2}{(3-2\sqrt{2})}-1} = \frac{-2+2\sqrt{2}}{2-2\sqrt{2}} = \frac{-1+\sqrt{2}}{1-\sqrt{2}}$$

$$= \frac{-(1-\sqrt{2})}{1-\sqrt{2}} = -1$$

IYGB – MP2 – PAPER N – QUESTION 13

FINALLY WE HAVE THE EQUATION OF THE TANGENT

$$y - \frac{\sqrt{2}}{2} = -1\left(x - \frac{\sqrt{2}}{2}\right)$$

$$y - \frac{\sqrt{2}}{2} = -x + \frac{\sqrt{2}}{2}$$

$$x + y = \sqrt{2}$$

ALTERNATIVE BY VERIFICATION

SOLVING SIMULTANEOUSLY

$$x + y = \sqrt{2} \qquad x = \frac{2t}{1+t^2} \qquad y = \frac{1-t^2}{1+t^2}$$

$$\Longrightarrow \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \sqrt{2}$$

$$\Longrightarrow 2t + 1 - t^2 = \sqrt{2}(1+t^2)$$

$$\Longrightarrow 2t + 1 - t^2 = \sqrt{2} + \sqrt{2}t^2$$

$$\Longrightarrow 0 = (1+\sqrt{2})t^2 - 2t + (\sqrt{2}-1) = 0$$

$$\Longrightarrow 0 = (\sqrt{2}-1)(1+\sqrt{2})t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)(\sqrt{2}-1) = 0(\sqrt{2}-1)$$

$$\Longrightarrow 0 = t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)^2 = 0$$

$$\Longrightarrow \underbrace{t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)^2}_{\text{PERFECT SQUARE}} = 0$$

IYGB - MP2 PAPER N - QUESTION 13

$$\Rightarrow \left[t - (\sqrt{2}-1)\right]^2 = 0$$

$\Rightarrow$ REPEATED ROOT AT $t = \sqrt{2}-1$

INDEED A TANGENT

AND FOR THE POINT OF TANGENCY

- $t = \sqrt{2}-1$
- $t^2 = (\sqrt{2}-1)^2 = 2 - 2\sqrt{2} + 1 = 3 - 2\sqrt{2}$

$$x = \frac{2t}{1+t^2}$$

$$x = \frac{2(\sqrt{2}-1)}{1+3-2\sqrt{2}} = \frac{2(\sqrt{2}-1)}{4-2\sqrt{2}} \quad \frac{\sqrt{2}-1}{2-\sqrt{2}}$$

$$= \frac{(\sqrt{2}-1)(2+\sqrt{2})}{4-2} = \frac{2\sqrt{2}+\cancel{2}-\cancel{2}-\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$y = \frac{1-t^2}{1+t^2}$$

$$y = \frac{1-(3-2\sqrt{2})}{1+(3-2\sqrt{2})} = \frac{-2+2\sqrt{2}}{4-2\sqrt{2}} = \frac{-1+\sqrt{2}}{2-\sqrt{2}}$$

$$= \frac{(-1+\sqrt{2})(2+\sqrt{2})}{4-2} = \frac{\cancel{-2}-\sqrt{2}+2\sqrt{2}\cancel{+2}}{2} = \frac{\sqrt{2}}{2}$$

$\therefore$ TANGENT AT $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$