# IYGB GCE

## Mathematics MP2

## Advanced Level

### Practice Paper O

Difficulty Rating: 3.5700/1.1523

### Time: 2 hours

**Candidates may use any calculator allowed by the regulations of this examination.**

**Information for Candidates**

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 100.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

**Question 1**

A sequence $b_1$, $b_2$, $b_3$, $b_4$, … is given by

$$b_{n+1} = 5b_n - 3, \quad b_1 = k,$$

where $k$ is a non zero constant.

**a)** Find the value of $b_4$ in terms of $k$. **(3)**

**b)** Given that $b_4 = 7$, determine the value of $k$. **(2)**

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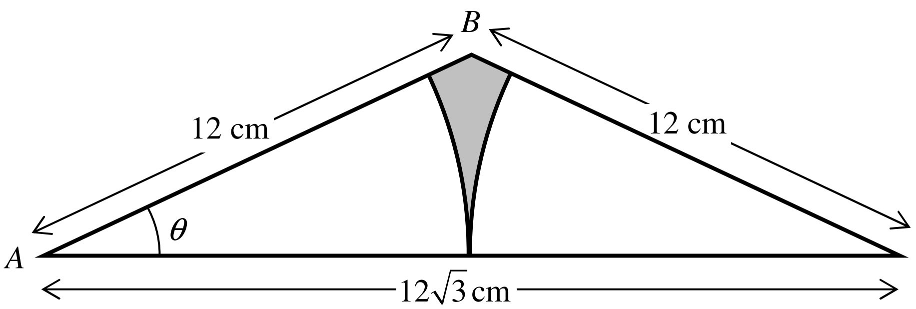
**Question 2**




An isosceles triangle $ABC$ has $AC = 12\sqrt{3}$ cm and $AB = BC = 12$ cm.

The angle $BAC$ is $\theta$ radians.

Two identical arcs centred at $A$ and $C$ are drawn inside the triangle. These arcs meet at a point on $AC$, as shown in the figure above.

**a)** Show that $\theta = \frac{1}{6}\pi$. **(2)**

**b)** Show that the area of the shaded region in the above figure is

$$18\left(2\sqrt{3} - \pi\right)\text{cm}^2.$$ **(5)**

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**Question 3**

The functions $f$ and $g$ are defined by

$$f(x) = x + 4, \quad x \in \mathbb{R}$$

$$g(x) = |2x+1| + 3, \quad x \in \mathbb{R}.$$

Solve the inequality

$$fg(x) > 12. \qquad (6)$$

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**Question 4**

$$y = \frac{3x}{2 + x - x^2}$$

**a)** Calculate the three missing values of $y$ in the following table.

| $x$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| $y$ | 0 | | | | 1.5 |

(1)

**b)** Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_0^1 \frac{3x}{2 + x - x^2} \, dx. \qquad (3)$$

**c)** Use a suitable method to find the exact value of

$$\int_0^1 \frac{3x}{2 + x - x^2} \, dx. \qquad (7)$$

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**Question 5**

A semi cubical parabola $C$ has equation

$$y = \sqrt{x^3}\,,\ x \in \mathbb{R}\,,\ x \geq 0\,.$$

**a)** Sketch the graph of $C$. **(1)**

**b)** Describe fully a sequence of two transformations which map the graph of $C$ onto the graph with equation

$$y = \sqrt{8(x-1)^3}\,,\ x \in \mathbb{R}\,,\ x \geq 1\,.$$ **(4)**

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**Question 6**

A container is in the shape of a hollow right circular cylinder of base radius 50 cm and height 100 cm.

The container is filled with water and is standing upright on horizontal ground. Water is leaking out of a hole on the side of the container which is 1 cm above the ground.

Let $h$ cm be the height of the water in the container, where $h$ is measured from the ground, and $t$ minutes be the time from the instant since $h = 100$.

The rate at which the volume of the water is decreasing is directly proportional to the square root of the height of the water in the container.

**a)** By relating the volume and the height of the water in the container, show that

$$\frac{dh}{dt} = -Ah^{\frac{1}{2}},$$

where $A$ is a positive constant. **(4)**

$\left[\text{volume of a cylinder of radius } r \text{ and height } h \text{ is given by } \pi r^2 h\right]$

When $t = 2$, $h = 64$.

**b)** Determine the value of $t$, by which no more water leaks out of the container. **(8)**

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**Question 7**

The function $f$ is satisfies

$$f(x)=\sqrt{x}-3,\ x\in\mathbb{R},\ 0\le x\le 9.$$

**a)** Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes. **(3)**

**(2)**

**b)** State the range of $f(x)$.

**c)** Find an expression for $f^{-1}(x)$. **(2)**

**d)** Sketch in the same set of axes as that of part **(a)** the graph of $f^{-1}(x)$.

The sketch must include the coordinates of the points where the graph of $f^{-1}(x)$ meets the coordinate axes, and how the graph of $f^{-1}(x)$ is related to the graph of $f(x)$. **(3)**

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**Question 8**

It is given that

$$\cos 3x \equiv 4\cos^3 x-3\cos x.$$

**a)** Prove the validity of the above trigonometric identity. **(5)**

**b)** Hence, or otherwise solve the trigonometric equation

$$2+\cos 6\theta \sec 2\theta = 0,\ 0°\le\theta<360°.$$ **(7)**

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**Question 9**

It is known that the cubic equation below has a root $\alpha$, which is close to $1.25$.

$$x^3 + x = 3.$$

Use an iterative formula based on the Newton Raphson method to find the value of $\alpha$, correct to $6$ decimal places. **(7)**

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**Question 10**

The curve $C$ has equation

$$y = \frac{\ln y}{x - y}, \quad y > 0.$$

Show that the equation of the tangent to $C$ at the point where $y = \mathrm{e}$ can be written as

$$\mathrm{e}(x - y) = 1. \qquad \textbf{(10)}$$

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**Question 11**

A curve is defined by the parametric equations

$$x = \cos\theta, \quad y = \sin\theta - \tan\theta, \qquad 0 \le \theta < 2\pi.$$

Show that a Cartesian equation of the curve is given by

$$y^2 = \frac{(x-1)^2\left(1-x^2\right)}{x^2}. \qquad \textbf{(7)}$$

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**Question 12**

It is given that

$$\frac{1}{n}\sum_{r=1}^{n} x_r = 2 \qquad \text{and} \qquad \sqrt{\frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}\left(\sum_{r=1}^{n} x_r\right)^2} = 3 .$$

Determine, in terms of $n$, the value of

$$\sum_{r=1}^{n}(x_r + 1)^2 .$$

**(8)**





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