IYGB - MP2 PAPER O - QUESTION 1

a) $b_{n+1} = 5b_n - 3$

- $b_1 = k$
- $b_2 = 5b_1 - 3 = 5k - 3$
- $b_3 = 5b_2 - 3 = 5(5k-3) - 3 = 25k - 18$
- $b_4 = 5b_3 - 3 = 5(25k-18) - 3 = 125k - 93$

b) $b_4 = 7$

$125k - 93 = 7$

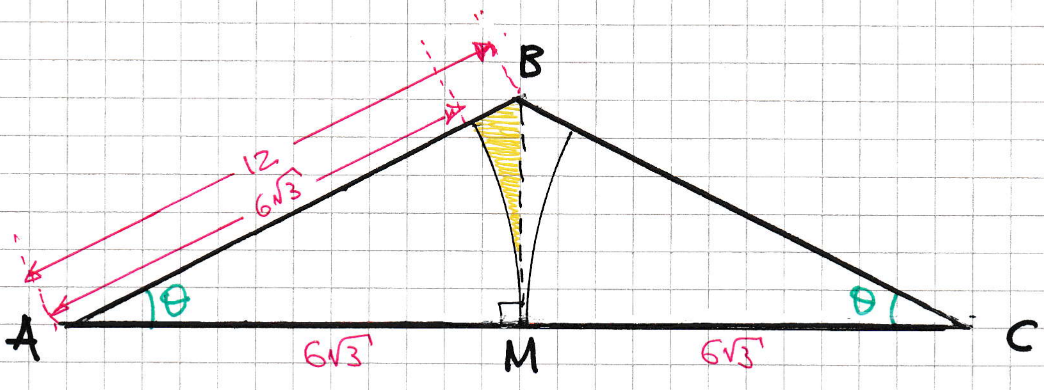
$125k = 100$

$k = \frac{100}{125}$

$k = \frac{4}{5}$

IYGB – MP2 PAPER O – QUESTION 2

a)

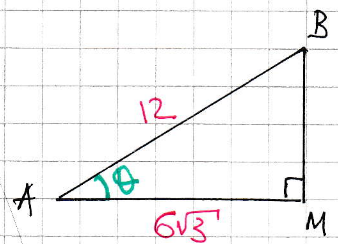


LOOKING AT THE RIGHT ANGLED TRIANGLE $\overset{\triangle}{ABM}$

$$\cos\theta = \frac{6\sqrt{3}}{12}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$



// AS REQUIRED

b) AREA OF $\overset{\triangle}{ABM}$

$$\frac{1}{2}|AB||AM|\sin\theta = \frac{1}{2}\times 12\times 6\sqrt{3}\times \sin\frac{\pi}{6} = 18\sqrt{3}$$

AREA OF SECTOR, CENTRE AT A & RADIUS $6\sqrt{3}$

$$\frac{1}{2}r^2\theta^c = \frac{1}{2}\times(6\sqrt{3})^2\times\frac{\pi}{6} = 9\pi$$

THE AREA OF THE "YELLOW" REGION IS

$$18\sqrt{3} - 9\pi = 9(2\sqrt{3}-\pi)$$

THE REQUIRED AREA IS DOUBLE BY SYMMETRY

$$\text{AREA} = 2\times 9(2\sqrt{3}-\pi) = 18(2\sqrt{3}-\pi)$$

// AS REQUIRED

IYGB - MP2 - PAPER O - QUESTION 3

$f(x) = x+4 \ , \ x \in \mathbb{R}$

$g(x) = |2x+1| + 3 \ , \ x \in \mathbb{R}$

$\Rightarrow f(g(x)) > 12$

$\Rightarrow f(|2x+1|+3) > 12$

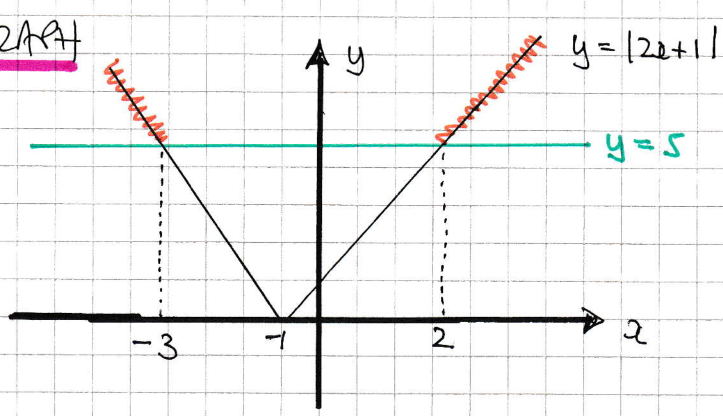
$\Rightarrow [|2x+1|+3]+4 > 12$

$\Rightarrow |2x+1| > 5$

SOLVING THE CORRESPONDING EQUATION

$\begin{cases} 2x+1 = 5 \\ 2x+1 = -5 \end{cases} \Rightarrow \begin{cases} 2x = 4 \\ 2x = -6 \end{cases} \Rightarrow \begin{matrix} x = 2 \\ x = -3 \end{matrix}$

SKETCHING A GRAPH



$\therefore \ x < -3 \quad \text{OR} \quad x > 2$

IYGB - MP2 PAPER O - QUESTION 4

a)

x	0	0.25	0.5	0.75	1
y	0	0.3429	0.6667	1.0286	1.5

b)

$$\int_0^1 \frac{3x}{2+x-x^2}\,dx \approx \frac{\text{"THICKNESS"}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$\approx \frac{0.25}{2}\left[0 + 1.5 + 2(0.3429 + 0.6667 + 1.0286)\right]$$

$$\approx 0.697 - 0.698$$

c)

$$\int_0^1 \frac{3x}{2+x-x^2}\,dx = \int_1^0 \frac{3x}{x^2-x-2}\,dx$$

$$= \int_1^0 \frac{3x}{(x-2)(x+1)}\,dx$$

PROCEED BY PARTIAL FRACTIONS

$$\frac{3x}{(x-2)(x+1)} \equiv \frac{A}{x-2} + \frac{B}{x+1}$$

$$\boxed{3x \equiv A(x+1) + B(x-2)}$$

- IF $x=-1 \implies -3 = -3B$
  $B = 1$
- IF $x = 2 \implies 6 = 3A$
  $A = 2$

IYGB - MP2 PAPER O - QUESTION 4

RETURNING TO THE INTEGRAL

$$\dots = \int_{1}^{0} \frac{2}{x-2} + \frac{1}{x+1} \, dx$$

$$= \Big[2\ln|x-2| + \ln|x+1| \Big]_{1}^{0}$$

$$= \Big[2\ln|-2| + \cancel{\ln 1} \Big] - \Big[\cancel{2\ln|-1|} + \ln 2 \Big]$$

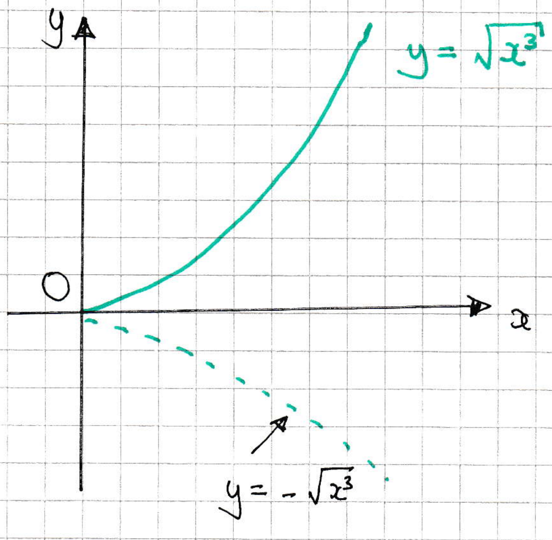
$$= 2\ln 2 - \ln 2$$

$$= \ln 2$$

$\simeq 0.693$

IYGB - MP2 PAPER O - QUESTION 5

a)



b)

$$y = \sqrt{x^3} \longmapsto \sqrt{(2x)^3}$$

$$\sqrt{8x^3} \longmapsto \sqrt{8(x-1)^3}$$

(HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$) THEN (TRANSLATION, "RIGHT" BY 1 UNIT)

ALTERNATIVE

$$y = \sqrt{x^3} \longmapsto y = \sqrt{(x-2)^3} \longmapsto \sqrt{(2x-2)^3}$$

$$\sqrt{8(x-1)^3}$$

(TRANSLATION "RIGHT" BY 2 UNITS) THEN (HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$)

IYGB - MP2 PAPER O - QUESTION 6

a) FORM A DIFFERENTIAL EQUATION

$$\Longrightarrow \frac{dV}{dt} = -kh^{\frac{1}{2}} \quad \longleftarrow \text{FROM THE CONTEXT}$$

$$\Longrightarrow \frac{dV}{dh} \times \frac{dh}{dt} = -kh^{\frac{1}{2}}$$

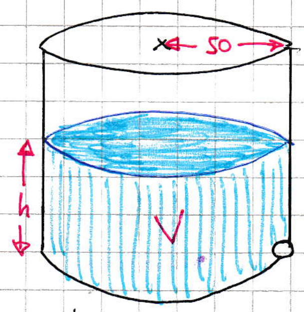
$$\Longrightarrow (2500\pi) \times \frac{dh}{dt} = -kh^{\frac{1}{2}}$$

OPPOSITE

$$\Longrightarrow \frac{dh}{dt} = -\frac{k}{2500\pi}h^{\frac{1}{2}}$$

$$\Longrightarrow \frac{dh}{dt} = -Ah^{\frac{1}{2}}$$

// AS REQUIRED



"$V = \pi r^2 h$"

$V = \pi \times 50^2 \times h$

$V = 2500\pi h$

$\frac{dV}{dh} = 2500\pi$

b) SOLVING THE DIFFERENTIAL EQUATION BY SEPARATING VARIABLES

$$\Longrightarrow dh = -Ah^{\frac{1}{2}}\,dt$$

$$\Longrightarrow \frac{1}{h^{\frac{1}{2}}}\,dh = -A\,dt$$

$$\Longrightarrow h^{-\frac{1}{2}}\,dh = -A\,dt$$

$$\Longrightarrow \int h^{-\frac{1}{2}}\,dh = \int -A\,dt$$

$$\Longrightarrow \boxed{2h^{\frac{1}{2}} = -At + C}$$

IYGB – MP2 PAPER O – QUESTION 6

APPLY CONDITION $t=0,\ h=100$

$$\Rightarrow 2\times 100^{\frac{1}{2}} = 0 + C$$

$$\Rightarrow C = 20$$

$$\Rightarrow \boxed{2h^{\frac{1}{2}} = 20 - At}$$

APPLY CONDITION $t=2,\ h=64$

$$\Rightarrow 2\times 64^{\frac{1}{2}} = 20 - A\times 2$$

$$\Rightarrow 16 = 20 - 2A$$

$$\Rightarrow 2A = 4$$

$$\Rightarrow A = 2$$

$$\Rightarrow 2h^{\frac{1}{2}} = 20 - 2t$$

$$\Rightarrow \boxed{h^{\frac{1}{2}} = 10 - t}$$

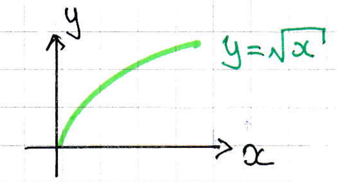
FINALLY WITH $h=1$

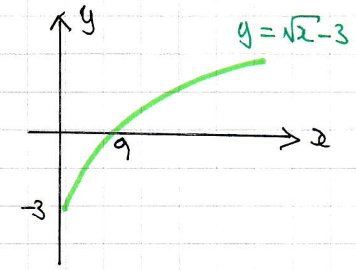
$$\Rightarrow 1^{\frac{1}{2}} = 10 - t$$

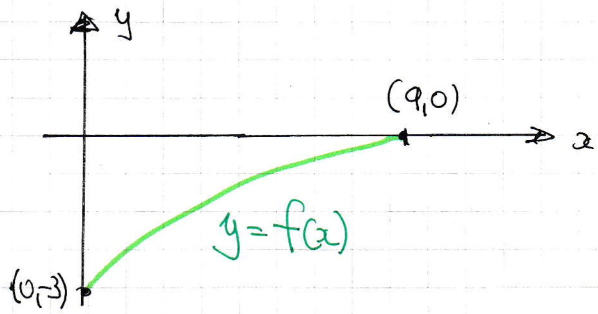
$$\Rightarrow t = 9$$

IYGB - MP2 - PAPER O - QUESTION 7

a)







b) LOOKING AT THE GRAPH ABOVE

$$-3 \leq f(x) \leq 0$$

c)

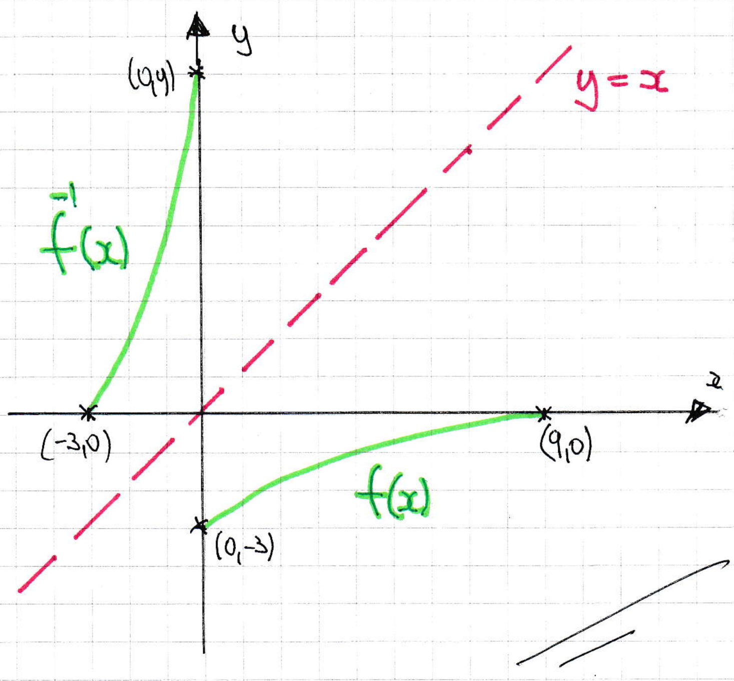
$$y = \sqrt{x} - 3$$

$$y + 3 = \sqrt{x}$$

$$x = (y+3)^2$$

$$f^{-1}(x) = (x+3)^2$$

d)



IYGB - MP2 PAPER O - QUESTION 8

a)
$$\cos 3x = \cos(\overset{A}{2x} + \overset{B}{x})$$
$$= \cos 2x \cos x - \sin 2x \sin x$$
$$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$$
$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$$
$$= 2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x$$
$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$
$$= 4\cos^3 x - 3\cos x$$

AS REQUIRED

b) PROCEED AS FOLLOWS

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$
$$\cos 6\theta = \cos(3 \times 2\theta) = 4\cos^3 2\theta - 3\cos 2\theta$$

TRANSFORMING THE EQUATION

$$\Longrightarrow 2 + \cos 6\theta \sec 2\theta = 0$$
$$\Longrightarrow 2 + \left[4\cos^3 2\theta - 3\cos 2\theta\right]\sec 2\theta = 0$$
$$\Longrightarrow 2 + 4\cos^3 2\theta \sec 2\theta - 3\cos 2\theta \sec 2\theta = 0$$
$$\Longrightarrow 2 + 4\cos^2 2\theta - 3 = 0$$

IYGB - MP2 PAPER O - QUESTION 8

$\Rightarrow \quad 4\cos^2 2\theta = 1$

$\Rightarrow \quad 4\left(\frac{1}{2} + \frac{1}{2}\cos 4\theta\right) = 1$

$\Rightarrow \quad 2 + 2\cos 4\theta = 1$

$\Rightarrow \quad 2\cos 4\theta = -1$

$\Rightarrow \quad \cos 4\theta = -\frac{1}{2}$

$\arccos\left(-\frac{1}{2}\right) = 120°$

$\Rightarrow \begin{cases} 4\theta = 120° \pm 360n \\ 4\theta = 240° \pm 360n \end{cases} \qquad n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} \theta = 30° \pm 90n \\ \theta = 60° \pm 90n \end{cases}$

$\Rightarrow \quad \theta = 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$

ALTERNATIVE FROM $4\cos^2 2\theta = 1$

$\cos^2 2\theta = \frac{1}{4}$

$\cos 2\theta = \pm\frac{1}{2}$

AND SOLVE FROM THERE

IYGB - MP2 PAPER O - QUESTION 9

REARRANGE THE EQUATION, AND WRITE IT AS A FUNCTION

$$\Rightarrow x^3 + x = 3$$

$$\Rightarrow x^3 + x - 3 = 0$$

$$\Rightarrow f(x) = x^3 + x - 3$$

SET UP A RECURRENCE RELATION BASED ON NEWTON RAPHSON

- $f'(x) = 3x^2 + 1$

- $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 + x_n - 3}{3x_n^2 + 1}$$

$$\Rightarrow x_{n+1} = \frac{3x_n^3 + \cancel{x_n} - (x_n^3 + \cancel{x_n} - 3)}{3x_n^2 + 1}$$

$$\Rightarrow \boxed{x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}}$$

PRODUCING ITERATIONS, STARTING WITH $x_1 = 1.25$, USING THIS FORMULA

$$\Rightarrow x_1 = 1.25$$

$$\Rightarrow x_2 = 1.214285714$$

$$\Rightarrow x_3 = 1.213412176$$

$$\Rightarrow x_4 = 1.213411663$$

$$\Rightarrow x_5 = 1.213411663$$

$\therefore \ \alpha = 1.213411$ // 6 d.p

IYGB - MP2 PAPER O - QUESTION 10

METHOD A – WITHOUT IMPLICIT DIFFERENTIATION

- START BY REARRANGING THE EQUATION OF THE CURVE FOR x

$$\Rightarrow y = \frac{\ln y}{x-y}$$

$$\Rightarrow xy - y^2 = \ln y$$

$$\Rightarrow xy = y^2 + \ln y$$

$$\Rightarrow x = y + \frac{\ln y}{y}$$

- WHEN $y=e$

$$\Rightarrow x = e + \frac{\ln e}{e} = e + \frac{1}{e} \qquad \therefore P\left(e+\frac{1}{e}, e\right)$$

- DIFFERENTIATE WITH RESPECT TO y

$$\Rightarrow \frac{dx}{dy} = 1 + \frac{y \times \frac{1}{y} - \ln y \times 1}{y^2}$$

$$\Rightarrow \frac{dx}{dy} = 1 + \frac{1-\ln y}{y^2}$$

$$\Rightarrow \left.\frac{dx}{dy}\right|_{y=e} = 1 + \frac{1-\ln e}{e^2} = 1 + \cancel{\frac{1-1}{e^2}} = 1$$

$$\Rightarrow \frac{dy}{dx} = 1$$

- EQUATION OF TANGENT AT $P\left(e+\frac{1}{e}, e\right)$

$$\Rightarrow y - e = 1\left(x - e - \frac{1}{e}\right)$$

$$\Rightarrow y \cancel{- e} = x \cancel{- e} - \frac{1}{e}$$

$$\Rightarrow \frac{1}{e} = x - y \qquad \therefore e(x-y) = 1$$

IYGB – MP2 PAPER O – QUESTION 10

METHOD B – BY IMPLICIT DIFFERENTIATION

- FIRSTLY WHEN $y = e$

$$\Rightarrow y = \frac{\ln y}{x-y}$$

$$\Rightarrow e = \frac{\ln e}{x-e}$$

$$\Rightarrow e = \frac{1}{x-e}$$

$$\Rightarrow x - e = \frac{1}{e}$$

$$\Rightarrow x = e + \frac{1}{e} \qquad \therefore P\left(e+\frac{1}{e}, e\right)$$

- MULTIPLY THE DENOMINATOR ACROSS AND DIFFERENTIATE W.R.T x

$$\Rightarrow yx - y^2 = \ln y$$

$$\Rightarrow \frac{d}{dx}(yx - y^2) = \frac{d}{dx}(\ln y)$$

$$\Rightarrow x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$

- EVALUATE THE ABOVE EXPRESSION AT $P\left(e+\frac{1}{e}, e\right)$

$$\Rightarrow \left(e+\frac{1}{e}\right)\left.\frac{dy}{dx}\right|_P + e - 2e\left.\frac{dy}{dx}\right|_P = \frac{1}{e}\left.\frac{dy}{dx}\right|_P$$

$$\Rightarrow e = \left(\frac{1}{e} + 2e - e - \frac{1}{e}\right)\left.\frac{dy}{dx}\right|_P$$

$$\Rightarrow e = e\left.\frac{dy}{dx}\right|_P$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_P = 1$$

- AND THE EQUATION OF THE TANGENT CAN BE FOUND AS BEFORE

IYGB - MP2 PAPER O - QUESTION 11

STARTING WITH THE y EQUATION & CREATE COSINES

$$\Rightarrow y = \sin\theta - \tan\theta \qquad [x = \cos\theta]$$

$$\Rightarrow y = \sin\theta - \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow y = \sin\theta\left(1 - \frac{1}{\cos\theta}\right)$$

$$\Rightarrow y = \sin\theta\left(\frac{\cos\theta - 1}{\cos\theta}\right)$$

$$\Rightarrow y^2 = \sin^2\theta\left(\frac{\cos\theta - 1}{\cos\theta}\right)^2$$

$$\Rightarrow y^2 = (1 - \cos^2\theta)\frac{(\cos\theta - 1)^2}{\cos^2\theta}$$

$$\Rightarrow y^2 = \frac{(1 - x^2)(x - 1)^2}{x^2}$$

AS REQUIRED

ALTERNATIVE APPROACH

$$\Rightarrow y = \sin\theta - \tan\theta$$

$$\Rightarrow y^2 = (\sin\theta - \tan\theta)^2$$

$$\Rightarrow y^2 = \sin^2\theta - 2\sin\theta\tan\theta + \tan^2\theta$$

$$\Rightarrow y^2 = (\cancel{1} - \cos^2\theta) - \frac{2\sin^2\theta}{\cos\theta} + (\sec^2\theta \cancel{- 1})$$

$$\Rightarrow y^2 = \sec^2\theta - \cos^2\theta - \frac{2(1 - \cos^2\theta)}{\cos\theta}$$

$$\Rightarrow y^2 = \frac{1}{\cos^2\theta} - \cos^2\theta - \frac{2}{\cos\theta} + 2\cos\theta$$

IYGB - MP2 PAPER O - QUESTION 11

$$\Rightarrow y^2 = \frac{1}{x^2} - x^2 - \frac{2}{x} + 2x$$

$$\Rightarrow y^2 = \frac{1 - x^4 - 2x + 2x^3}{x^2}$$

$$\Rightarrow y^2 = \frac{(1-x^2)(1+x^2) - 2x(1-x^2)}{x^2}$$

$$\Rightarrow y^2 = \frac{(1-x^2)(1+x^2-2x)}{x^2}$$

$$\Rightarrow y^2 = \frac{(1-x^2)(x-1)^2}{x^2}$$

AS BEFORE

IYGB – MP2 PAPER O – QUESTION 12

REWRITING THE FIRST EQUATION

$$\frac{1}{n}\sum_{r=1}^{n} x_r = 2 \implies \boxed{\sum_{r=1}^{n} x_r = 2n}$$

NOW PROCEED AS FOLLOWS

$$\implies \sqrt{\frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}\left[\sum_{r=1}^{n} x_r\right]^2} = 3$$

$$\implies \frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}\left[\sum_{r=1}^{n} x_r\right]^2 = 9$$

$$\implies \frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}(2n)^2 = 9$$

$$\implies \frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - 4 = 9$$

$$\implies \frac{1}{n}\sum_{r=1}^{n}(x_r)^2 = 13$$

$$\implies \boxed{\sum_{r=1}^{n}(x_r)^2 = 13n}$$

FINALLY WE HAVE

$$\implies \sum_{r=1}^{n}(x_r+1)^2 = \sum_{r=1}^{n}\left[(x_r)^2 + 2x_r + 1\right]$$

IYGB - MP2 PAPER O - QUESTION 12

$$= \sum_{r=1}^{n} (x_r)^2 + 2\sum_{r=1}^{n} (x_r) + \sum_{r=1}^{n} 1$$

$$= 13n + 2(2n) + n$$

$$= 18n$$