IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper P

Difficulty Rating: 3.7600/1.2500

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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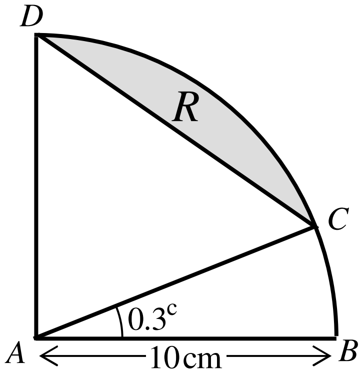
Question 1



The figure above shows a quarter circle ABD of radius 10 cm, whose centre is at A.

The point C lies on the arc BD so that the angle CAB is 0.3 radians.

The segment bounded by the semicircle and the chord CD is denoted by R.

a) Determine the area of R. **(4)**

b) Find the perimeter of R. **(3)**

Question 2

$OABC$ is a square, where O is the origin, and the vertices A and C have respective position vectors $2\mathbf{i}+4\mathbf{j}+4\mathbf{k}$ and $4\mathbf{i}+2\mathbf{j}-4\mathbf{k}$.

The point M is the midpoint of AB and the point N is the midpoint of MC.

The point D is such so that $\overrightarrow{AD}=\frac{3}{2}\overrightarrow{AB}$.

a) Find the position vectors of the points B, D and N. **(4)**

b) Deduce, showing your reasoning, that O, N and D are collinear. **(2)**

Question 3

Find the solution interval of the following modulus inequality.

$$|2x+1|+9<4x\,. \qquad (4)$$

Question 4

A species of tree is growing in height and the typical maximum height it can reach in its lifetime is 12 m.

The rate of growth of its height, $H\text{ m}$, is proportional to the difference between its height and the maximum height it can reach.

When a tree of this species was planted, it was 1 m in height and at that instant the tree was growing at the rate of 0.1 m per month.

a) Show clearly that

$$110\frac{dH}{dt}=12-H\,,$$

where t is the time, measured in months, since the tree was planted. **(2)**

b) Determine a simplified solution for the above differential equation, giving the answer in the form $H=f(t)$. **(5)**

c) Find, correct to 2 decimal places, the height of the tree after 5 **years**. **(1)**

d) Calculate, correct to the nearest year, the number of years it will take for the tree to reach a height of 11 m. **(2)**

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Question 5

The sum of the first 2 terms of an arithmetic progression is 40.

The sum of the first 4 terms of the same arithmetic progression is 130.

a) Determine the sum of the first 5 terms of the arithmetic progression. **(4)**

The sum of the first 2 terms of a geometric progression is 40.

The sum of the first 4 terms of the same geometric progression is 130.

b) Find the two possible values of the sum of the first 5 terms of the geometric progression. **(6)**

Question 6

A curve has equation

$$y = \frac{x+1}{x^3 + 2x + 1}.$$

It is given that the curve has a local maximum at the point M, whose approximate coordinates are $(-1.7, 0.1)$.

a) Show that the x coordinate of M is a solution of the equation

$$x = -\frac{3x^2 + 1}{2x^2}.$$ **(6)**

b) By using an iterative formula based on the equation of part **(a)**, determine the coordinates of M correct to three decimal places. **(3)**

c) State, with a reason, whether the convergence taking place using the formula of part **(b)** is of the "cobweb" type or the "staircase" type. **(1)**

Question 7

$$f(x) = \frac{e^{\sqrt[4]{x}}}{\sqrt{x}}, \quad x \in \mathbb{R}, \ x > 0 .$$

Find the value of

$$\int_0^1 f(x) \ dx ,$$

given further that the integral exists. **(8)**

Question 8

$$f(x) \equiv \left(\tfrac{1}{4} - x\right)^{-\frac{3}{2}}, \ |x| < \tfrac{1}{4}$$

a) Find the series expansion of $f(x)$, up and including the term in x^3. **(3)**

b) Use the result of part **(a)** to obtain the series expansion of

$$\sqrt{\tfrac{1}{4} - x}, \ |x| < \tfrac{1}{4},$$

up and including the term in x^3.

No credit will be given for obtaining a direct expansion in this part. **(5)**

Question 9

Fine sand is dropping on a horizontal floor at the constant rate of $5 \text{ cm}^3\text{s}^{-1}$ and forms a pile whose volume, V cm^3, and height, h cm, are connected by the formula

$$V = -2 + \sqrt{2h^3 + 3h + 8} .$$

Find the rate at which the height of the pile is increasing, when the height of the pile has reached 11 cm. **(7)**

Question 10

It is given that

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

If $\tan 3y = 3\tan x$ show that

$$\frac{dy}{dx} = \frac{1}{1+8\sin^2 x}. \qquad \textbf{(6)}$$

Question 11

It is given that

$$\tan\theta + \tan\varphi = 3,$$

$$\sin^2 x + 2\sin x + \sin(\theta+\varphi) = 3\cos\theta\cos\varphi - 1,$$

for $x \in \mathbb{R}$, $\theta \in \mathbb{R}$, $\varphi \in \mathbb{R}$.

Show that the above relationships imply that

$$\sin x = -1. \qquad \textbf{(5)}$$

Question 12

The functions f and g are defined by

$$f(x) = x^2,\ x \in \mathbb{R},\ x \ge 1$$

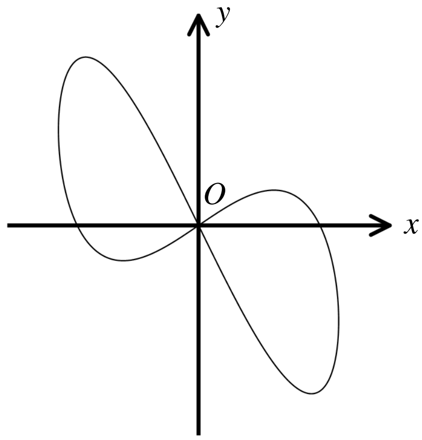
$$g(x) = x - 6,\ x \in \mathbb{R},\ x \le 10.$$

a) Find the domain and range of $fg(x)$. **(2)**

b) Show that the following equation has no solutions

$$fg(x) = g^{-1}(x) \qquad \textbf{(5)}$$

Question 13



The figure above shows the curve C with parametric equations

$$x = \cos\theta, \;\; y = \sin 2\theta - \cos\theta, \;\; 0 \le \theta < 2\pi .$$

a) Find an equation of the tangent to C at the point where $\theta = \frac{\pi}{4}$. **(5)**

b) Show that the tangent to C at the point where $\theta = \frac{5\pi}{4}$ is the same line as the tangent to C at the point where $\theta = \frac{\pi}{4}$. **(3)**

c) Show further that a Cartesian equation of the curve is

$$4x^2\left(1 - x^2\right) = \left(x + y\right)^2 .$$ **(4)**