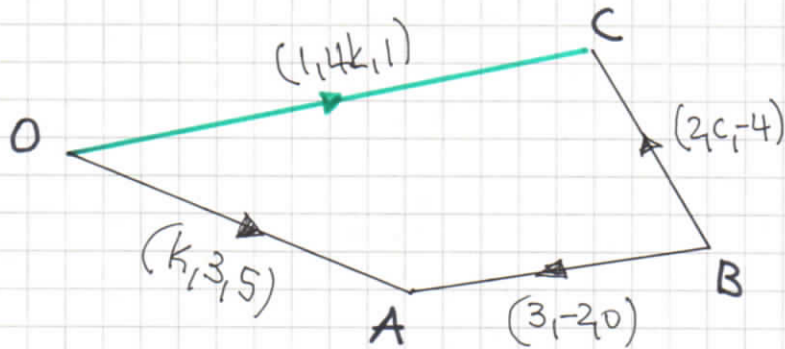


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1YGB-MP2 PAPER 2 - QUESTION 1

STARTING WITH A VECTOR DIAGRAM



$$\begin{aligned} A & (k, 3, 5) \\ \vec{BA} & = (3, -2, 0) \\ \vec{BC} & = (2, c, -4) \\ C & (1, 4k, 1) \end{aligned}$$

FORMING A VECTOR EQUATION

$$\Rightarrow \vec{OA} + \vec{AB} + \vec{BC} = \vec{OC}$$

$$\Rightarrow (k, 3, 5) - (3, -2, 0) + (2, c, -4) = (1, 4k, 1)$$

$$\Rightarrow (k-1, c+5, 1) = (1, 4k, 1)$$

$$[i]: k-1=1 \Rightarrow \underline{k=2}$$

$$[j]: c+5=4k$$

$$c+5=8$$

$$\underline{c=3}$$

FINALLY WE CAN FIND THE DISTANCE BC

$$\Rightarrow |\vec{BC}| = |2, 3, -4|$$

$$\Rightarrow |\vec{BC}| = \sqrt{2^2 + 3^2 + (-4)^2}$$

$$\Rightarrow |\vec{BC}| = \sqrt{4+9+16}$$

$$\Rightarrow |\vec{BC}| = \underline{\underline{\sqrt{29} \approx 5.39}}$$

-|-

IYGB - MP2 PAPER 2 - QUESTION 2

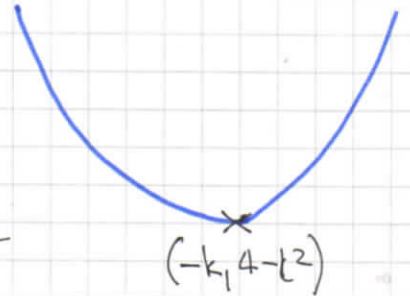
a) COMPLETING THE SQUARE

$$f(x) = x^2 + 2kx + 4, \quad x \in \mathbb{R}$$

$$f(x) = (x+k)^2 - k^2 + 4$$

$f(x)$ HAS A MINIMUM VALUE OF $4 - k^2$

$$\underline{f(x) \geq 4 - k^2} //$$



b) $f(g(2)) = 4$

$$\Rightarrow f(3 - k \times 2) = 4$$

$$\Rightarrow f(3 - 2k) = 4$$

$$\Rightarrow (3 - 2k)^2 + 2k(3 - 2k) + 4 = 4$$

$$\Rightarrow 9 - 12k + 4k^2 + 6k - 4k^2 = 0$$

$$\Rightarrow 9 = 6k$$

$$\Rightarrow \underline{k = \frac{3}{2}} //$$

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YOB - MP2 PAPER R - QUESTIONS 3

START FORMING EQUATIONS AS FOLLOWS

$$\begin{array}{ccc} u_2 & & u_3 & & u_9 \\ a+d & & a+2d & & a+8d \end{array}$$

$\xrightarrow{\quad \times r \quad}$ $\xrightarrow{\quad \times r \quad}$

← $u_n = a + (n-1)d$

$$\Rightarrow \begin{cases} (a+d)r = a+2d \\ (a+2d)r = a+8d \end{cases}$$

ELIMINATE THE COMMON RATIO r , BY DIVISION

$$\Rightarrow \frac{a+d}{a+2d} = \frac{a+2d}{a+8d}$$

$$\Rightarrow (a+d)(a+8d) = (a+2d)^2$$

$$\Rightarrow \cancel{a^2} + 8ad + ad + 8d^2 = \cancel{a^2} + 4ad + 4d^2$$

$$\Rightarrow 4d^2 + 9ad = 0$$

$$\Rightarrow d(4d + 9a) = 0$$

$$\Rightarrow 9a + 4d = 0 \quad (d \neq 0)$$

$$\Rightarrow d = -\frac{9}{4}a$$

NOW RETURNING & PICKING ONE OF THE ORIGINAL EQUATIONS WHICH CONTAIN

a, d & r

$$\Rightarrow (a+d)r = a+2d$$

$$\Rightarrow \left(a - \frac{9}{4}a\right)r = a + 2\left(-\frac{9}{4}a\right)$$

$$\Rightarrow -\frac{1}{4}ar = -\frac{3}{2}a$$

$$\Rightarrow \frac{1}{4}r = \frac{3}{2} \quad a \neq 0$$

$$\Rightarrow r = 6$$

→

1YGB - MP2 PAPER 2 - QUESTION 4

FINALLY THE AREA OF THE TRIANGLE $\triangle OAC$

$$\begin{aligned}\text{AREA} &= \frac{1}{2} |OA| |OC| \sin \frac{\pi}{4} = \frac{1}{2} xy \times \frac{\sqrt{2}}{2} = \frac{1}{4} \sqrt{2} xy \\ &= \frac{1}{4} \sqrt{2} (6\sqrt{6} + 6\sqrt{2}) (6 + 6\sqrt{3}) \\ &= \frac{1}{4} \sqrt{2} \times 6 (\sqrt{6} + \sqrt{2}) \times 6 (1 + \sqrt{3}) = 9\sqrt{2} (\sqrt{6} + \sqrt{2}) (1 + \sqrt{3}) \\ &= 9\sqrt{2} (2\sqrt{6} + 4\sqrt{2}) = 9\sqrt{2} \times 2 (\sqrt{6} + 2\sqrt{2}) \\ &= 18(\sqrt{12} + 4) = 18(2\sqrt{3} + 4) = 36(\sqrt{3} + 2)\end{aligned}$$

THE SHADDED AREA IS GIVEN BY

$$\begin{aligned}\text{AREA OF SECTOR} - \text{AREA OF TRIANGLE} \\ &= 18\pi(2 + \sqrt{3}) - 36(2 + \sqrt{3}) \\ &= 18(2 + \sqrt{3}) [\pi - 2] \\ &= \underline{18(2 + \sqrt{3})(\pi - 2)}\end{aligned}$$

LYGB - MP2 PAPER 2 - QUESTION 5

LOCATE THE CO.ORDINATES OF THE MINIMUM BY DIFFERENTIATION

$$f(x) = e^{nx} + ke^{-nx}$$

$$f'(x) = ne^{nx} - nke^{-nx}$$

so we $f'(x) = 0$

$$\Rightarrow ne^{nx} - nke^{-nx} = 0$$

$$\Rightarrow e^{nx} - ke^{-nx} = 0 \quad n \neq 0$$

$$\Rightarrow e^{nx} = ke^{-nx}$$

$$\Rightarrow e^{nx} = \frac{k}{e^{nx}}$$

$$\Rightarrow (e^{nx})^2 = k$$

$$\Rightarrow e^{nx} = +\sqrt{k} \quad e^{nx} > 0$$

NEXT WE CAN FIND THE y CO.ORDINATE - WE DON'T REQUIRE x

$$\Rightarrow y = e^{nx} + ke^{-nx}$$

$$\Rightarrow y = e^{nx} + \frac{k}{e^{nx}}$$

$$\Rightarrow y = \sqrt{k} + \frac{k}{\sqrt{k}}$$

$$\Rightarrow y = \sqrt{k} + \sqrt{k}$$

$$\Rightarrow y = 2\sqrt{k}$$

\therefore THE RANGE IS $f(x) \geq 2\sqrt{k}$

YGB - MP2 PAPER 2 - QUESTION 6

a) COLLECTING ALL THE INFORMATION

$$\frac{dV}{dt} = -kV^2$$

↑
RATE

↑↑↑
DEPRECIATING

↑↑↑
VALUE SQUARED
PROPORTIONAL

V = value, in thousands
 t = time, in years

 t = 0, V = 12

SOLVING BY SEPARATING VARIABLES

$$\Rightarrow dV = -kV^2 dt$$

$$\Rightarrow -\frac{1}{V^2} dV = k dt$$

$$\Rightarrow \int -\frac{1}{V^2} dV = \int k dt$$

$$\Rightarrow \boxed{\frac{1}{V} = kt + C}$$

APPLY CONDITION t=0, V=12

$$\Rightarrow \frac{1}{12} = C$$

$$\Rightarrow \frac{1}{V} = kt + \frac{1}{12}$$

$$\Rightarrow \dot{V} = \frac{1}{kt + \frac{1}{12}}$$

$$\Rightarrow V = \frac{12}{12kt + 1}$$

$$\Rightarrow \boxed{V = \frac{12}{at + 1}}$$

MULTIPLY TOP & BOTTOM OF THE FRACTION IN THE R.H.S BY 12

AS REQUIRED

1YGB - MP2 PAPER 2 - QUESTION 6

b) USING THE FINAL CONDITION

$$t=0 \quad V=8 \quad \leftarrow \pounds 12000 - \pounds 4000$$

$$\Rightarrow 8 = \frac{12}{2a+1}$$

$$\Rightarrow 16a + 8 = 12$$

$$\Rightarrow 16a = 4$$

$$\Rightarrow \underline{a = \frac{1}{4}}$$

REWRITING THE FORMULA

$$\Rightarrow V = \frac{12}{\frac{1}{4}t+1}$$

$$\Rightarrow V = \frac{12}{\frac{1}{4} \times 12 + 1} \quad (\text{"1" FURTHER PERIOD ...})$$

$$\Rightarrow V = 3$$

$$\therefore \underline{\pounds 3000}$$

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1YGB - MP2 PAPER 2 - QUESTION 7

a) FILL IN THE TABLE

α	$\frac{\pi}{6}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
y	3	4.1120	5.8284	8.6784	13.9282

BY THE TRAPEZIUM RULE

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(1+\sin x)^2}{\cos^2 x} dx \approx \frac{\text{"THICKNESS"}}{2} \left[\text{FIRST} + \text{LAST} + 2 \times (\text{SUM OF REST}) \right]$$
$$\approx \frac{\pi/24}{2} \left[3 + 13.9282 + 2(4.1120 + 5.8284 + 8.6784) \right]$$
$$\approx \underline{3.545}$$

b) PROCEED BY DIRECT INTEGRATION

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(1+\sin x)^2}{\cos^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1+2\sin x + \sin^2 x}{\cos^2 x} dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos x} + \tan^2 x dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + 2\tan x \sec x + (\sec^2 x - 1) dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2 x + 2\tan x \sec x - 1 dx$$

IYGB - MP2 PAPER 2 - QUESTION 7

NOW WE NOTE THAT

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \& \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

HENCE WE FINALLY HAVE

$$\begin{aligned} \dots &= \left[2 \tan x + 2 \sec x - x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\ &= \left(2 \tan \frac{\pi}{3} + 2 \sec \frac{\pi}{3} - \frac{\pi}{3} \right) - \left(2 \tan \frac{\pi}{6} + 2 \sec \frac{\pi}{6} - \frac{\pi}{6} \right) \\ &= \left(2\sqrt{3} + 4 - \frac{\pi}{3} \right) - \left(\frac{2}{\sqrt{3}} + \frac{4}{\sqrt{3}} - \frac{\pi}{6} \right) \\ &= \left(2\sqrt{3} + 4 - \frac{\pi}{3} \right) - \left(\frac{6}{\sqrt{3}} - \frac{\pi}{6} \right) \\ &= \cancel{2\sqrt{3}} + 4 - \frac{\pi}{3} - \left(\cancel{2\sqrt{3}} - \frac{\pi}{6} \right) \\ &= 4 - \frac{\pi}{6} \end{aligned}$$

-1-

1YGB - MP2 PAPER 2 - QUESTION 8

a) START BY REARRANGING THE EQUATION FOR x - THEN DIFFERENTIATE

$$\Rightarrow y = \frac{x}{y + \ln y}$$

$$\Rightarrow y^2 + y \ln y = x$$

$$\Rightarrow x = y^2 + y \ln y$$

$$\Rightarrow \frac{dx}{dx} = 2y + 1 \times \ln y + y \times \frac{1}{y}$$

$$\Rightarrow \frac{dx}{dy} = 2y + \ln y + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y + \ln y + 1}$$

Solving $\frac{dy}{dx} = 2$ YIELD

$$\Rightarrow 2 = \frac{1}{2y + \ln y + 1}$$

$$\Rightarrow 4y + 2 \ln y + 2 = 1$$

$$\Rightarrow 4y + 2 \ln y + 1 = 0$$

$$\Rightarrow 2 \ln y = -1 - 4y$$

$$\Rightarrow \ln y = -\frac{1}{2} - 2y$$

$$\Rightarrow \ln y = -\frac{1}{2}(4y + 1)$$

$$\therefore y = e^{-\frac{1}{2}(4y+1)} \quad \text{As required}$$

b) USING THE ITERATION FORMULA $y = e^{-\frac{1}{2}(4y+1)}$

STARTING WITH $y_1 = 0.3$

$$y_2 = 0.33287 \dots$$

$$y_3 = 0.311691 \dots$$

$$y_4 = 0.325178 \dots$$

1YGB - MP2 PAPER 2 - QUESTION 8

THE CONVERGENCE IS BY OSCILLATION BUT VERY SLOW

$$y_5 = 0.316521\dots$$

$$y_6 = 0.32205\dots$$

$$y_7 = 0.31851\dots$$

$$y_8 = 0.32077\dots$$

$$y_9 = 0.31932\dots$$

$$y_{10} = 0.32025\dots$$

$$y_{11} = 0.31965\dots$$

$$y_{12} = 0.32003\dots$$

$$y_{13} = 0.31979\dots$$

$$y_{14} = 0.31995\dots$$

$$y_{15} = 0.31985\dots$$

$$\therefore \underline{y = 0.320 \text{ (CORRECT TO 3 d.p.)}}$$

USING $y = 0.3199$ IN $\alpha = y^2 + y \ln y$ WE OBTAIN $\alpha = -0.262$

$$\therefore \underline{P(-0.262, 0.320)}$$

1YGB - MP2 PAPER 2 - QUESTION 9

a)
$$f(x) \equiv \frac{16x^2 + 3x - 2}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{3x-2}$$

$$16x^2 + 3x - 2 \equiv A(3x-2) + Bx(3x-2) + Cx^2$$

• IF $x=0$

$$-2 = -2A$$

$$A = 1$$

• IF $x = \frac{2}{3}$

$$\frac{64}{9} + 2 - 2 = C \times \frac{4}{9}$$

$$C = 16$$

• IF $x=1$

$$17 = A + B + C$$

$$17 = 1 + B + 16$$

$$B = 0$$

b)
$$\frac{1}{3x-2} = -\frac{1}{2-3x} = -(2-3x)^{-1} = -(2)^{-1} \left[1 - \frac{3}{2}x\right]^{-1}$$

$$= -\frac{1}{2} \left(1 - \frac{3}{2}x\right)^{-1}$$

$$= -\frac{1}{2} \left[1 + \frac{-1}{1} \left(-\frac{3}{2}x\right)^1 + \frac{-1(-2)}{1 \times 2} \left(-\frac{3}{2}x\right)^2 + \frac{(-1)(-2)(-3)}{1 \times 2 \times 3} \left(-\frac{3}{2}x\right)^3 + \dots \right]$$

$$= -\frac{1}{2} \left[1 + \frac{3}{2}x + \frac{9}{4}x^2 + \frac{27}{8}x^3 + \dots \right]$$

$$= -\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 - \dots$$

c) METHOD A (ONLY UP TO x^3 IS DIRECTLY AVAILABLE)

$$\frac{16x^2 + 3x - 2}{3x-2} = (-2 + 3x + 16x^2) \left[-\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + \dots \right]$$

$$= 1 + \cancel{\frac{3}{2}x} + \cancel{\frac{9}{4}x^2} + \cancel{\frac{27}{8}x^3} + \dots$$

$$- \cancel{\frac{3}{2}x} - \cancel{\frac{9}{4}x^2} - \cancel{\frac{27}{8}x^3} + \dots$$

$$- 8x^2 - 12x^3 + \dots$$

$$= \underline{1 - 8x^2 - 12x^3 + \dots}$$

1YGB - MP2 PAPER 2 - QUESTION 9

METHOD B (USING PREVIOUS PARTS)

$$\frac{16x^2 + 3x - 2}{x^2(3x-2)} = \frac{1}{x^2} + \frac{16}{3x-2}$$

$$\frac{1}{x^2} \left(\frac{16x^2 + 3x - 2}{3x-2} \right) = \frac{1}{x^2} + 16 \left(\frac{1}{3x-2} \right)$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 + 16x^2 \left(\frac{1}{3x-2} \right)$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 + 16x^2 \left[-\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + o(x^4) \right]$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 - 8x^2 - 12x^3 - 18x^4 - 27x^5 + o(x^6)$$

IYGB - MP2 PAPER R - QUESTION 10

a) STARTING FROM THE L.H.S

$$\begin{aligned}
\text{LHS} &= \sin 3\theta \\
&= \sin(2\theta + \theta) \\
&= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \quad \left. \begin{array}{l} \text{red arrow} \\ \sin(A+B) = \sin A \cos B + \cos A \sin B \end{array} \right\} \\
&= (2\sin \theta \cos \theta) \cos \theta + (1 - 2\sin^2 \theta) \sin \theta \\
&= 2\sin \theta \cos^2 \theta + \sin \theta + 2\sin^3 \theta \\
&= 2\sin \theta (1 - \sin^2 \theta) + \sin \theta + 2\sin^3 \theta \\
&= 2\sin \theta - 2\sin^3 \theta + \sin \theta + 2\sin^3 \theta \\
&= 3\sin \theta - 4\sin^3 \theta \\
&= \underline{\text{RHS}} \quad \left. \begin{array}{l} \text{red arrow} \\ \text{AS REQUIRED} \end{array} \right\}
\end{aligned}$$

b) DIFFERENTIATING THE IDENTITY W.R.T θ

$$\begin{aligned}
\frac{d}{d\theta} [\sin 3\theta] &= \frac{d}{d\theta} [3\sin \theta - 4\sin^3 \theta] \\
3\cos 3\theta &= 3\cos \theta - 12\sin^2 \theta \times \cos \theta \quad \left. \begin{array}{l} \text{red arrow} \\ \div 3 \end{array} \right\} \\
\cos 3\theta &= \cos \theta - 4\cos \theta \sin^2 \theta \\
\cos 3\theta &= \cos \theta - 4\cos \theta (1 - \cos^2 \theta) \\
\cos 3\theta &= \cos \theta - 4\cos \theta + 4\cos^3 \theta \\
\underline{\cos 3\theta} &= \underline{4\cos^3 \theta - 3\cos \theta} \quad \left. \begin{array}{l} \text{red arrow} \\ \text{AS REQUIRED} \end{array} \right\}
\end{aligned}$$

c) PROCEED AS FOLLOWS

$$\begin{aligned}
\tan 3\theta &= \frac{\sin 3\theta}{\cos 3\theta} = \frac{3\sin \theta - 4\sin^3 \theta}{4\cos^3 \theta - 3\cos \theta} = \frac{\frac{3\sin \theta}{\cos^3 \theta} - \frac{4\sin^3 \theta}{\cos^3 \theta}}{\frac{4\cos^3 \theta}{\cos^3 \theta} - \frac{3\cos \theta}{\cos^3 \theta}} \\
&= \frac{\frac{3\sin \theta}{\cos \theta} \times \frac{1}{\cos^2 \theta} - 4\tan^3 \theta}{4 - \frac{3}{\cos^2 \theta}} = \frac{3\tan \theta \sec^2 \theta - 4\tan^3 \theta}{\underline{4 - 3\sec^2 \theta}} \quad \left. \begin{array}{l} \text{red arrow} \\ \text{AS REQUIRED} \end{array} \right\}
\end{aligned}$$

14GB - MP2 PAPER R - QUESTION 10

d) USING THE IDENTITY $1 + \tan^2\theta \equiv \sec^2\theta$ WE HAVE

$$\tan 3\theta = \frac{3\tan\theta \sec^2\theta - 4\tan^3\theta}{4 - 3\sec^2\theta} = \frac{3\tan\theta(1 + \tan^2\theta) - 4\tan^3\theta}{4 - 3(1 + \tan^2\theta)}$$

$$= \frac{3\tan\theta + 3\tan^3\theta - 4\tan^3\theta}{4 - 3 - 3\tan^2\theta} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

AS REQUIRED

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IYGB - MP2 PAPER 2 - QUESTION 11

a) OBTAIN THE GRADIENT FUNCTION IN PARAMETRIC

$$\bullet \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\sin^2 t \cos t}{3\cos^2 t (-\sin t)} = - \frac{3\sin^2 t \cos t}{3\sin t \cos^2 t} = - \frac{\sin t}{\cos t}$$

$$\bullet \left. \frac{dy}{dx} \right|_{t=\theta} = - \frac{\sin \theta}{\cos \theta}$$

EQUATION OF NORMAL AT $(\cos^3 \theta, \sin^3 \theta)$ WITH GRADIENT $+\frac{\cos \theta}{\sin \theta}$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - \sin^3 \theta = \frac{\cos \theta}{\sin \theta} (x - \cos^3 \theta)$$

$$\Rightarrow y \sin \theta - \sin^4 \theta = x \cos \theta - \cos^4 \theta$$

$$\Rightarrow \cos^4 \theta - \sin^4 \theta = x \cos \theta - y \sin \theta$$

$$\Rightarrow \underbrace{(\cos^2 \theta - \sin^2 \theta)}_{\cos 2\theta} \underbrace{(\cos^2 \theta + \sin^2 \theta)}_1 = x \cos \theta - y \sin \theta$$

$$\Rightarrow \underline{\underline{x \cos \theta - y \sin \theta = \cos 2\theta}} \quad \text{As required}$$

b) When $x=0$

$$-y \sin \theta = \cos 2\theta$$

$$y = - \frac{\cos 2\theta}{\sin \theta}$$

When $y=0$

$$x \cos \theta = \cos 2\theta$$

$$x = \frac{\cos 2\theta}{\cos \theta}$$

AREA IS GIVEN BY

$$\frac{1}{2} \left| - \frac{\cos 2\theta}{\sin \theta} \times \frac{\cos 2\theta}{\cos \theta} \right| = \frac{\cos 2\theta \cos 2\theta}{2 \sin \theta \cos \theta} = \frac{\cos 2\theta \cos 2\theta}{\sin 2\theta}$$

$$= \underline{\underline{\cos^2 2\theta \cos 2\theta}}$$