IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper S

Difficulty Rating: 4.8900/1.8018

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

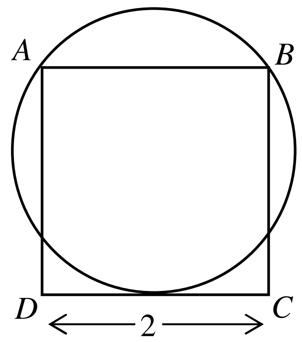
This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 17 questions in this question paper.
The total mark for this paper is 150.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows a square $ABCD$ of side length 2 units.

The vertices A and B lie on the circumference of a circle while the side DC is a tangent to the same circle.

Determine the radius of this circle. **(4)**

Question 2

Prove by **contradiction** that $\log_{10} 5$ is an irrational number. **(5)**

Question 3

The curve C has equation

$$y = \ln(1+\cos x), \ x \in \mathbb{R}, \ -\pi < x < \pi .$$

Show clearly that

$$\frac{d^4 y}{dx^4} + e^{-y}\left(\frac{dy}{dx}\right)^2 + 2e^{-2y} = 0 .$$ **(8)**

Question 4

A curve C is given parametrically by

$$x = t^2 - p^2, \qquad y = 2tp,$$

where t and p are real parameters.

The parameters t and p are related by the equation

$$p^2 = 2t^2 - 1.$$

Show that a Cartesian equation for C is

$$y^2 = 4(x-1)(2x-1).$$ **(6)**

Question 5

The function f is defined below.

$$f(x) \equiv \frac{e^{\sin x \cos x} + 1}{e^{\sin x \cos x} - 1}, \; x \in \mathbb{R}.$$

Prove that f is odd. **(6)**

Question 6

The Fibonacci sequence is given by the recurrence formula

$$u_{n+2} = u_{n+1} + u_n, \quad u_1 = 1, \; u_2 = 1.$$

It is further given that in this sequence **the ratio of consecutive terms** converges to a limit ϕ, known as the *Golden Ratio*.

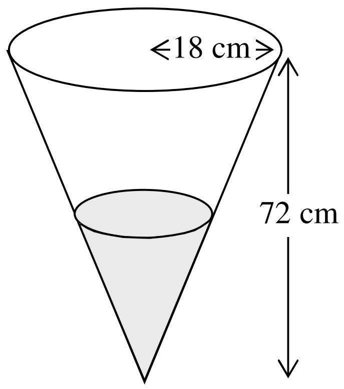
Show, by using the above recurrence formula, that $\phi = \frac{1}{2}\left(1 + \sqrt{5}\right)$. **(7)**

Question 7

The first three terms of a geometric progression are the respective 7^{th} term, 4^{th} term, and 2^{nd} term of an arithmetic progression.

Determine the common ratio of the geometric progression. **(9)**

Question 8



A container is in the shape of hollow inverted right circular cone of height 72 cm and radius 18 cm.

The container, which is initially empty, is placed, with its axis vertical, under a tap where water is flowing in at the constant rate of $k \text{ cm}^3 \text{s}^{-1}$.

The rate at which the height of the water in the container is rising 12.5 **minutes** after it was placed under the tap is $\frac{2}{75} \text{ cm s}^{-1}$.

Determine the value of k.

$\left[\text{volume of a cone of radius } r \text{ and height } h \text{ is given by } \frac{1}{3}\pi r^2 h\right]$ **(10)**

Question 9

The function $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = \frac{2xy(y+1)}{\sin^2\left(x + \frac{1}{6}\pi\right)},$$

subject to the condition $y = 1$ at $x = 0$.

Find the exact value of y when $x = \frac{\pi}{12}$. **(11)**

Question 10

$$S = 1 + \frac{2}{4} + \frac{2 \cdot 3}{4 \cdot 8} + \frac{2 \cdot 3 \cdot 4}{4 \cdot 8 \cdot 12} + \frac{2 \cdot 3 \cdot 4 \cdot 5}{4 \cdot 8 \cdot 12 \cdot 16} + \ldots$$

By considering a suitable binomial series, or other wise, find the sum to infinity of S.

(7)

Question 11

Use trigonometric algebra to find the solution of the following simultaneous equations, in the intervals $0 \le x < 2\pi$, $0 \le y < 2\pi$.

$$4\cos y = 3 - 2\sin x \qquad \text{and} \qquad 4y - 2x = \pi.$$ **(8)**

Question 12

The vertices of the triangle OAB have coordinates $A(6,-18,-6)$, $B(7,-1,3)$, where O is a fixed origin.

The point N lies on OA so that $ON : NA = 1:2$.

The point M is the midpoint of OB.

The point P is the intersection of AM and BN.

By using vector methods, or otherwise, determine the coordinates of P. **(10)**

Question 13

The curve C has equation

$$y = \left|x^2 - 16\right| + 2(x-4), \quad x \in \mathbb{R}.$$

Sketch a detailed graph of C and hence show that the area of the finite region bounded by C and the x axis, for which $y < 0$, is 32 square units.

(12)

Question 14

By using the substitution $u = \sec x + \sqrt{\tan x}$, or otherwise, find

$$\int \frac{1 + 2\sin x\sqrt{\tan x}}{2\left[1 + \cos x\sqrt{\tan x}\right]\cos x\sqrt{\tan x}}\, dx.$$ **(11)**

Question 15

The curve C has equation

$$f(x) \equiv 3x^4 + 8x^3 + 3x^2 - 12x - 6, \quad x \in \mathbb{R}.$$

The curve has a single stationary point whose x coordinate lies in the interval $[n, n+1]$, where $n \in \mathbb{Z}$.

a) Determine with full justification the value of n. **(3)**

A suitable equation is rearranged to produced three recurrence relations, each of which may be used to find the x coordinate of the stationary point of C.

These recurrence relations, all starting with $x_0 = \frac{1}{2}n$ are shown below.

(i) $x_{n+1} = \dfrac{2}{2x_n^{\,2} + 4x_n + 1}$ **(ii)** $x_{n+1} = \dfrac{1}{x_n^{\,2}} - \dfrac{1}{2x_n} - 2$ **(iii)** $x_{n+1} = \sqrt{\dfrac{2 - x_n}{4 + 2x_n}}$

b) Use a differentiation method, to investigate the result in attempting to find an approximate value for the x coordinate of the stationary point of $f(x)$, with each of these three recurrence relations.

The method must include ...

- ... whether the attempt is successful
- ... whether the convergence or divergence is a "cobweb" case or a "staircase" case.
- ... which recurrence relation converges at the fastest rate.

You may not answer part **(b)**, by simply generating sequences. **(10)**

Question 16

The point $P(x, y)$ lies on a circle with centre at $(1, 0)$ and radius 1.

Find, in exact surd form, the greatest value of $x + y$, for all the possible positions of the point P. **(11)**

Question 17

The curve C has equation

$$y+2=\left[\ln(4x+1)\right]^2, \quad x \in \mathbb{R}, \quad x \geq -\tfrac{1}{4}.$$

Sketch the graph of C and hence determine, in exact simplified form, the area of the finite region bounded by C, for which $x \geq 0$, and the coordinate axes. **(12)**