IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper T

Difficulty Rating: 5.000/1.7143

Time: 3 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 16 questions in this question paper.
The total mark for this paper is 175.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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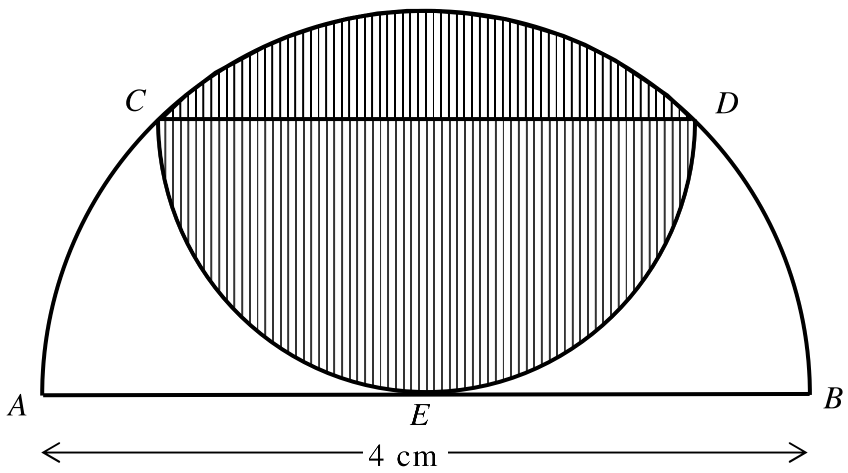
Question 1



The figure above is constructed as follows.

A semicircle with diameter AB of 4 cm is first drawn.

Then another semicircle is drawn, with its diameter CD parallel to AB.

The semicircle with CD as its diameter is circumscribed by the semicircle with AB as its diameter, as shown in the figure.

Show that the area of the shaded region is $(2\pi - 2)$ cm^2. **(6)**

Question 2

Show by a suitable algebraic method that

$$60^2 - 59^2 + 58^2 - 57^2 + \ldots + 22^2 - 21^2 = 1620.$$ **(5)**

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Question 3

$$f(x) \equiv \frac{1}{(1-5x)^2}, \quad |x| < \tfrac{1}{5}.$$

It is given that the equation

$$f(x) - (8x+3)^3 = -37x^3 - 475x^2 - 157x + 27$$

has a solution α, which is numerically small.

Find an approximate value for α. **(10)**

Question 4

A factory gets permission to dispose, at the start of every day, 600 kg of waste into a stream of water.

The running stream removes 40% of the any waste present, by the end of the day.

Determine a simplified expression for the amount of waste present in the stream at the end of the n^{th} day. **(8)**

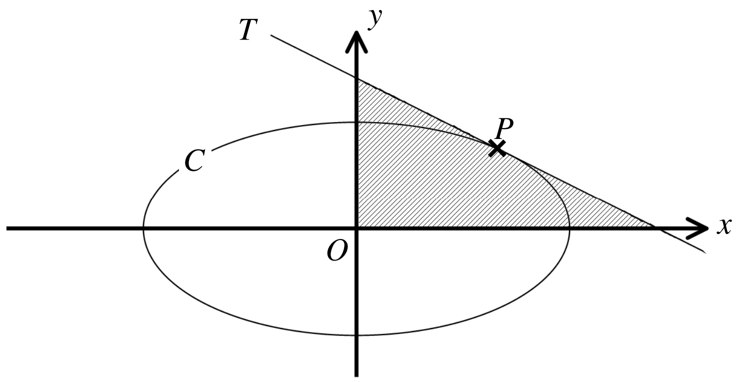
Question 5

A quartic curve C has the following equation.

$$y = x(x-4)(x+2)(x-6), \quad x \in \mathbb{R}.$$

By considering suitable transformations, show that C is even about the straight line with equation $x = 2$. **(7)**

Question 6



The figure above shows the curve C with parametric equations

$$x = 4\cos\theta, \ \ y = 3\sin\theta, \quad 0 \le \theta < 2\pi .$$

The point P lies on C where $\theta = \alpha$, where $0 < \alpha < \frac{\pi}{2}$.

The line T is a tangent to C at P.

The tangent T meets the coordinate axes at the points A and B.

The area of the triangle OAB, where O is the origin, is less than 24 square units.

Find the range of the possible values of α. **(12)**

Question 7

It is given that x, a and b are positive real numbers, with $a > b$ and $x^2 > ab$.

Use proof by contradiction to show that

$$\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} > 0 .$$ **(8)**

Question 8

Use an appropriate substitution followed by integration by parts to find a simplified expression for

$$\int \frac{\left[\ln\left(x^2+1\right)-2\ln x\right]\sqrt{x^2+1}}{x^4}\,dx. \qquad (12)$$

Question 9

The piecewise continuous function f is given below.

$$f(x) \equiv \begin{cases} 2x-2 & x \le 5 \\ x+3 & x > 5 \end{cases}$$

a) Determine an expression, in similar form to that of $f(x)$ above, for the inverse function, $f^{-1}(x)$. **(5)**

b) Sketch a detailed graph for the composition $ff(x)$. **(7)**

Question 10

With respect to a fixed origin, the points A and B have position vectors $10\mathbf{i}+9\mathbf{j}-6\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+10\mathbf{k}$, respectively.

The position vector of the point C has $\mathbf{i}$ component equal to 2.

The distance of C from both A and B is 12 units.

Show that one of the two possible position vectors of C is $2\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and determine the other. **(12)**

Question 11

$$\frac{dy}{dx} = \sqrt{\frac{y^4 - y^2}{x^4 - x^2}}, \quad x > 0, \; y > 0.$$

Find the solution of the above differential equation subject to the boundary condition $y = \frac{2}{\sqrt{3}}$ at $x = 2$.

Give the answer in the form $y = \frac{2x}{f(x)}$, where $f(x)$ is a function to be found. **(15)**

Question 12

A curve C is defined in the largest real domain by the equation

$$y = \log_x 2.$$

a) Sketch a detailed graph of C. **(2)**

The point P, where $x = 2$ lies on C.

The normal to C at P meets C again at the point Q.

b) Show that the x coordinate of Q is a solution of the equation

$$[1 + x\ln 4 \; - \; \ln 16] \ln x = \ln 2.$$ **(8)**

c) Use an iterative formula of the form $x_{n+1} = e^{f(x_n)}$, with a suitable starting value, to find the coordinates of Q, correct to 3 decimal places. **(4)**

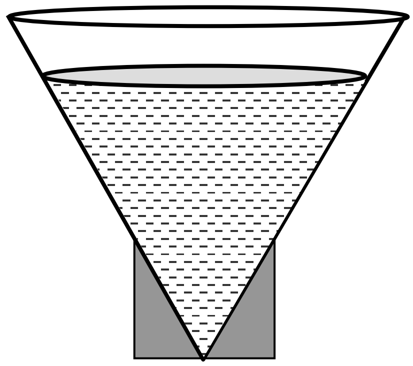
Question 13

The acute angles x and y, satisfy the following relationships.

$$2\tan x = 1 \qquad \text{and} \qquad \sin(x+y) = \frac{7}{\sqrt{50}}.$$

Determine the possible values of $\tan y$. **(12)**

Question 14



A container is in the shape of a hollow inverted right circular cone, whose ratio of its base radius to its height is $\pi : 1$.

The container is initially empty when water is begins to flow in at the constant rate k.

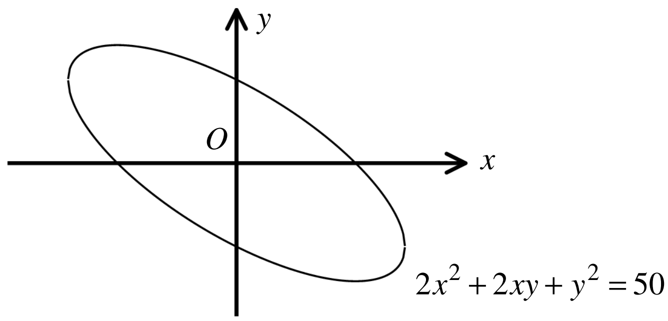
At time t, the area of the circular surface of the water in the cone is A.

Show that at time $t = T$, the rate at which A is changing is

$$2\pi\sqrt[3]{f(k,T)},$$

where $f(k,T)$ is an expression to be found. **(10)**

Question 15



The figure above shows the curve with equation

$$2x^2 + 2xy + y^2 = 50.$$

Determine the area of the finite region bounded by the x axis and the part of the curve for which $y \geq 0$.

(16)

Question 16

A curve C has equation

$$y^2 = \frac{x^2}{x-1}, \quad x \in \mathbb{R}, \quad x > 1.$$

Show that there exist exactly two tangents to C which pass through the point $(1,2)$, and find their equations.

(16)
