IYGB - MP2 PAPER Z - QUESTION 1

THE n^{th} TERM OF AN ARITHMETIC SERIES OF COMMON DIFFERENCE 2 IS GIVEN BY

$$u_n = a + (n-1) \times 2$$
$$u_n = a + 2(n-1)$$

HENCE WE NOW HAVE

u_3	u_6	u_{10}
$a+4$	$a+10$	$a+18$

AS THESE ARE IN GEOMETRIC PROGRESSION

$$\frac{a+10}{a+4} = \frac{a+18}{a+10} \implies (a+10)^2 = (a+4)(a+18)$$
$$\implies \cancel{a^2} + 20a + 100 = \cancel{a^2} + 22a + 72$$
$$\implies 28 = 2a$$
$$\implies a = 14$$

SO THE COMMON RATIO IS

$$r = \frac{a+10}{a+4} = \frac{24}{18} = \frac{4}{3}$$

IYGB - MP2 PAPER Z - QUESTION 2

a) ARITHMETIC SERIES

$u_n = a + (n-1)d$

$u_{11} = 2 + 10x$

GEOMETRIC SERIES

$u_n = ar^{n-1}$

$u_{11} = 2 \times x^{10}$

NOW THE SUM

$\Rightarrow (2 + 10x) + 2x^{10} = 900$

$\Rightarrow 2x^{10} + 10x = 898$

$\Rightarrow x^{10} + 5x = 449$ // AS REQUIRED

b) USING FUNCTION NOTATION

$f(x) \equiv x^{10} + 5x - 449$

$f(1.8) = -82.953... < 0$

$f(1.9) = +173.606.. > 0$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE INTERVAL $(1.8, 1.9)$ THERE IS AT LEAST ONE SOLUTION OF THE EQUATION IN $(1.8, 1.9)$ //

c) DIFFERENTIATING FIRST

$f'(x) = 10x^9 + 5$

BY THE N-R FORMULA

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = 1.8$

$x_2 = 1.8 - \dfrac{1.8^{10} + 5 \times 1.8 - 449}{10 \times 1.8^9 + 5}$

$\simeq 1.8417...$

$x_3 \approx 1.838$ //

IYGB - MP2 PAPER Z - QUESTION 3

WORK AS FOLLOWS

$$\frac{d}{dx}\left[\ln\left[\frac{1}{\sqrt{x^2+1}-x}\right]\right] = \frac{1}{\frac{1}{\sqrt{x^2+1}-x}} \times \frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}-x}\right]$$

TIDY UP AND BY THE QUOTIENT RULE.

$$= \left(\sqrt{x^2+1}-x\right) \times \frac{\cancel{\left(\sqrt{x^2+1}-x\right)\times 0} - 1 \times \frac{d}{dx}\left[(x^2+1)^{\frac{1}{2}}-x\right]}{\left(\sqrt{x^2+1}-x\right)^2}$$

$$= \frac{-\left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x - 1\right]}{\sqrt{x^2+1}-x} = \frac{-x(x^2+1)^{-\frac{1}{2}}+1}{(x^2+1)^{\frac{1}{2}}-x}$$

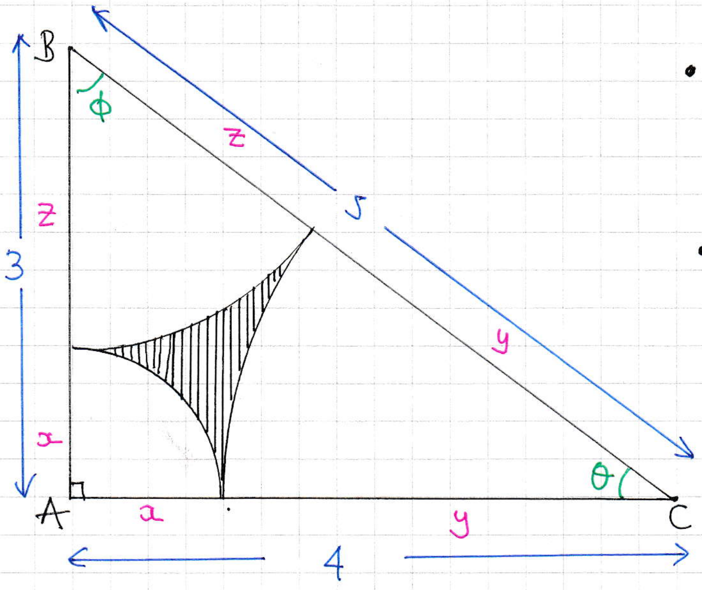
FACTORIZING FURTHER AS

$$= \frac{1 - x(x^2+1)^{-\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}\left[1 - x(x^2+1)^{-\frac{1}{2}}\right]} = \frac{1}{(x^2+1)^{\frac{1}{2}}} = \frac{1}{\sqrt{x^2+1}}$$

AS REQUIRED

IYGB – MP2 PAPER Z – QUESTION 4

LOOKING AT THE DIAGRAM



- $\tan\theta = \frac{3}{4}$

  $\theta = 0.6435^c...$

- $\phi = \pi/2 - 0.6435...$

  $\phi = 0.9273^c...$

FORMING SOME EQUATIONS WITH THE LENGTHS

$$\left.\begin{array}{l} x+y=4 \\ y+z=5 \\ z+x=3 \end{array}\right\} \text{ ADDING } \quad 2x+2y+2z=12$$

$$x+y+z=6$$

$$4+z=6$$

$$z=2,\ y=3,\ x=1$$

AREA OF THE 3 SECTORS $\left(\frac{1}{2}r^2\theta^c\right)$

$$\frac{1}{2}x^2\times\frac{\pi}{2} + \frac{1}{2}y^2\theta + \frac{1}{2}z^2\phi = \left(\frac{1}{2}\times\frac{\pi}{2}\right) + \frac{9}{2}\theta + 2\phi$$

$$= \frac{\pi}{4} + \frac{9}{2}(0.6435) + 2(0.9273)$$

$$= 5.5357...$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\frac{1}{2}\times4\times3 - 5.5357 = 0.464$$

3 s.f.

IYGB - MP2 PAPER Z - QUESTION 5

USING THE SUBSTITUTION GIVEN

$$u = \sin x + \operatorname{cosec} x$$

$$\frac{du}{dx} = \cos x - \cot x \operatorname{cosec} x$$

$$dx = \frac{1}{\cos x - \cot x \operatorname{cosec} x}\, du$$

TRANSFORMING THE INTEGRAL WE OBTAIN

$$\int \frac{\cos^3 x}{(1+\sin^2 x)\sin x}\, dx = \int \frac{\cos^3 x}{(1+\sin^2 x)\sin x} \times \frac{1}{\cos x - \cot x \operatorname{cosec} x}\, du$$

SWITCH TO SINES AND COSINES

$$= \int \frac{\cos^3 x}{(1+\sin^2 x)\sin x} \times \frac{1}{\cos x - \frac{\cos x}{\sin x} \times \frac{1}{\sin x}}\, du$$

$$= \int \frac{\cos^3 x}{(1+\sin^2 x)\sin x} \times \frac{1}{\cos x\left(1 - \frac{1}{\sin^2 x}\right)}\, du$$

MULTIPLY "TOP & BOTTOM" OF THIS FRACTION BY $\sin^2 x$

$$= \int \frac{\cos^3 x}{(1+\sin^2 x)\sin x} \times \frac{\sin^2 x}{\cos x(\sin^2 x - 1)}\, du$$

$$= \int \frac{\cos^3 x}{(1+\sin^2 x)\sin x} \times \frac{\sin^2 x}{-\cos^3 x}\, du$$

$$= \int -\frac{\sin x}{1+\sin^2 x}\, du$$

$$= \int -\frac{\sin x \operatorname{cosec} x}{1 \operatorname{cosec} x + \sin^2 x \operatorname{cosec} x}\, du$$

IYGB - MP2 PAPER Z - QUESTION 5

$$= \int -\frac{1}{\operatorname{cosec}x + \sin x}\, du$$

$$= \int -\frac{1}{u}\, du$$

$$= -\ln|u| + C$$

$$= \ln\left|\frac{1}{u}\right| + C$$

REVERSING THE SUBSTITUTION

$$= \ln\left|\frac{1}{\sin x + \operatorname{cosec}x}\right| + C$$

$$= \ln\left|\frac{1}{\sin x + \frac{1}{\sin x}}\right| + C$$

$$= \ln\left|\frac{1\sin x}{\sin x\sin x + \frac{1}{\sin x}\sin x}\right| + C$$

$$= \ln\left|\frac{\sin x}{\sin^2 x + 1}\right| + C$$

AS REQUIRED

IYGB - MP2 PAPER Z - QUESTION 6

a) USING THE SUGGESTION GIVEN

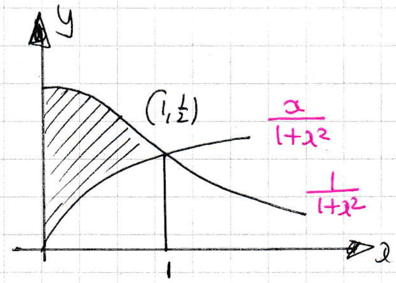
$$\Rightarrow y = \arctan x$$
$$\Rightarrow \tan y = x$$
$$\Rightarrow x = \tan y$$
$$\Rightarrow \frac{dx}{dy} = \sec^2 y$$
$$\Rightarrow \frac{dx}{dy} = 1 + \tan^2 y$$

BUT $x = \tan y$

$$\Rightarrow \frac{dx}{dy} = 1 + x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$$

AS REQUIRED

b) LOOKING AT THE DIAGRAM



REQUIRED AREA =

$$= \int_0^1 \frac{1}{1+x^2}\,dx - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \left[\arctan x - \frac{1}{2}\ln(1+x^2)\right]_0^1$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right) - \left(0 - \frac{1}{2}\ln 1\right)$$

$$= \frac{1}{4}(\pi - \ln 4)$$

IYGB – MP2 PAPER Z – QUESTION 7

a) ELIMINATE AS FOLLOWS

$$\Longrightarrow x = \tan\theta - \sec\theta$$
$$\Longrightarrow x^2 = (\tan\theta - \sec\theta)^2$$
$$\Longrightarrow x^2 = \tan^2\theta - 2\tan\theta\sec\theta + \sec^2\theta$$
$$\Longrightarrow x^2 = \tan^2\theta - 2\tan\theta\sec\theta + (1+\tan^2\theta)$$
$$\Longrightarrow x^2 = 2\tan^2\theta - 2\tan\theta\sec\theta + 1$$
$$\Longrightarrow x^2 = 2\tan\theta(\tan\theta - \sec\theta) + 1$$
$$\Longrightarrow x^2 = 2\tan\theta \times x + 1$$
$$\Longrightarrow \tan\theta = \frac{x^2-1}{2x}$$

WITH ANALOGOUS WORKINGS & THE IDENTITY $1+\cot^2\theta = \operatorname{cosec}^2\theta$

$$\cot\theta = \frac{y^2-1}{2y}$$

THUS WE FINALLY HAVE

$$\Longrightarrow \tan\theta\cot\theta = \left(\frac{x^2-1}{2x}\right)\left(\frac{y^2-1}{2y}\right)$$
$$\Longrightarrow 1 = \frac{(x^2-1)(y^2-1)}{4xy}$$
$$\Longrightarrow (x^2-1)(y^2-1) = 4xy$$

AS REQUIRED

IYGB – MP2 PAPER Z – QUESTION 7

b) BY IMPLICIT DIFFERENTIATION OR PARAMETRIC DIFFERENTIATION

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\operatorname{cosec}^2\theta + \operatorname{cosec}\theta\cot\theta}{\sec^2\theta - \sec\theta\tan\theta}$$

$$= \frac{\operatorname{cosec}\theta(\cot\theta - \operatorname{cosec}\theta)}{\sec\theta(\sec\theta - \tan\theta)}$$

$$= \frac{\operatorname{cosec}\theta}{\sec\theta} \times \frac{y}{-x}$$

$$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} \times \left(-\frac{y}{x}\right)$$

$$= \frac{\cos\theta}{\sin\theta}\left(-\frac{y}{x}\right)$$

$$= -\frac{y\cot\theta}{x}$$

BUT IN PART (a) WE OBTAINED $\cot\theta = \frac{y^2-1}{2y}$

$$\therefore \frac{dy}{dx} = -\frac{y}{x}\left(\frac{y^2-1}{2y}\right)$$

$$= -\frac{1}{x}\left(\frac{y^2-1}{2}\right)$$

$$= \frac{1-y^2}{2x}$$

AS REQUIRED

IYGB - MP2 PAPER Z - QUESTION 8

a) $f(x) = \ln(4-2x)\ ,\ x<2$

- SET $x=0$

$y = \ln 4 = 2\ln 2$

$\therefore (0, \ln 4)$

- SET $y=0$

$0 = \ln(4-2x)$

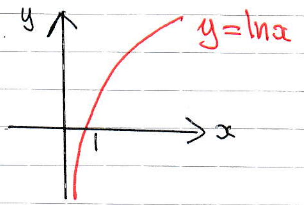
$e^0 = 4-2x$

$1 = 4-2x$

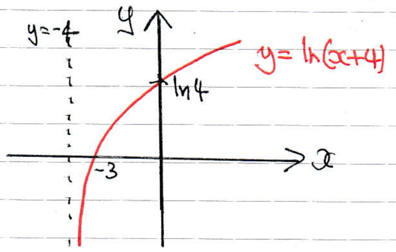
$2x = 3$

$x = \frac{3}{2}$ $\qquad \therefore \left(\frac{3}{2}, 0\right)$

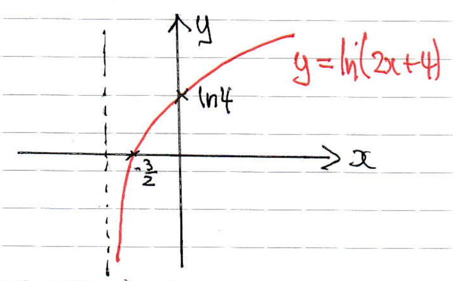
b) DESCRIBING THE TRANSFORMATION TO ALLOW SKETCH



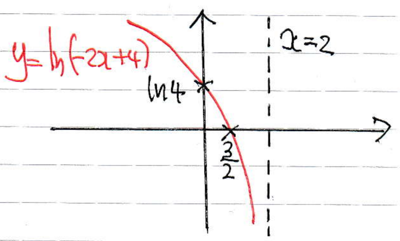
T_1 TRANSLATION, "LEFT", BY 4 UNITS

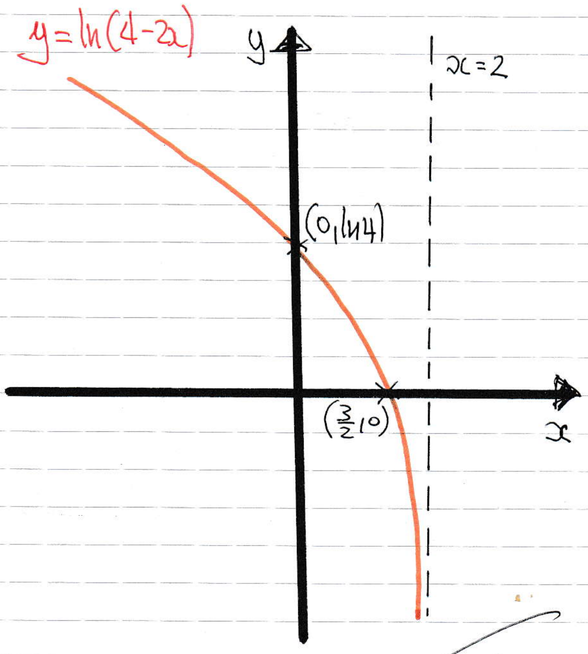


T_2 STRETCH, HORIZONTALLY, SCALE FACTOR $\frac{1}{2}$



T_3 REFLECTION IN THE y AXIS





IYGB – MP2 PAPER Z – QUESTION 8

c) USING THE STANDARD METHOD TO FIND THE INVERSE

$$f(x) = \ln(4-2x)$$
$$y = \ln(4-2x)$$
$$e^y = 4-2x$$
$$2x = 4-e^y$$
$$x = 2-\tfrac{1}{2}e^y$$

$$\therefore f^{-1}(x) = 2-\tfrac{1}{2}e^x$$

d)

	$f(x)$	$f^{-1}(x)$
DOMAIN	$x<2$	$x \in \mathbb{R}$
RANGE	$f(x) \in \mathbb{R}$	$f^{-1}(x) < 2$

$\therefore$ DOMAIN OF $f^{-1}(x)$: $x \in \mathbb{R}$

RANGE OF $f^{-1}(x)$: $f^{-1}(x) \in \mathbb{R}$ WITH $f^{-1}(x) < 2$

IYGB - MP2 PAPER Z - QUESTION 9

SEPARATE VARIABLES & INTEGRATE

$$\Rightarrow (1+x)\frac{dy}{dx} = y(1-x)$$

$$\Rightarrow (1+x)\,dy = y(1-x)\,dx$$

$$\Rightarrow \frac{1}{y}\,dy = \frac{1-x}{1+x}\,dx$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1-x}{1+x}\,dx$$

INTEGRATE THE R.H.S BY THE SUBSTITUTION $u=1+x$, OR MANIPULATION

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{2-(1+x)}{1+x}\,dx$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{2}{1+x} - \frac{1+x}{1+x}\,dx$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{2}{1+x} - 1\,dx$$

$$\Rightarrow \ln|y| = 2\ln|1+x| - x + C$$

$$\Rightarrow y = e^{2\ln|1+x| - x + C}$$

$$\Rightarrow y = e^{2\ln|1+x|} \times e^{-x} \times e^{C}$$ A

$$\Rightarrow y = e^{\ln(1+x)^2} \times e^{-x} \times A$$

$$\Rightarrow \boxed{y = Ae^{-x}(1+x)^2}$$

APPLY CONDITION $(0,1)$

$$1 = Ae^{0}\times 1^2$$

$$A = 1$$

$$\therefore \quad y = (x+1)^2 e^{-x}$$

AS REQUIRED

IYGB - MP2 PAPER Z - QUESTION 10

START BY OBTAINING THE GRADIENT FUNCTION

$$\Rightarrow y^3 + x^2 = axy$$

$$\Rightarrow \frac{d}{dx}\left[y^3 + x^2\right] = \frac{d}{dx}\left[axy\right]$$

$$\Rightarrow 3y^2\frac{dy}{dx} + 2x = ay + ax\frac{dy}{dx}$$

$$\Rightarrow (3y^2 - ax)\frac{dy}{dx} = ay - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{ay - 2x}{3y^2 - ax}$$

LOOK FOR "HORIZONTAL" TANGENTS

$$\frac{dy}{dx} = 0 \Rightarrow ay - 2x = 0$$

$$\Rightarrow ay = 2x$$

$$\Rightarrow x = \frac{ay}{2}$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow y^3 + \left(\frac{a}{2}y\right)^2 = a\left(\frac{a}{2}y\right)y$$

$$\Rightarrow y^3 + \frac{1}{4}a^2y^2 = \frac{1}{2}a^2y^2$$

$$\Rightarrow y^3 - \frac{1}{4}a^2y^2 = 0$$

$$\Rightarrow \frac{1}{4}y^2\left[4y - a^2\right] = 0$$

$$\therefore y = \begin{cases} \cancel{0} \\ \frac{1}{4}a^2 \end{cases} \qquad x = \begin{cases} \cancel{0} \\ \frac{1}{8}a^3 \end{cases}$$

(NOT ACTUALLY NEEDED)

NEXT LOOK FOR "VERTICAL" TANGENTS

$$\frac{dy}{dx} = \infty \Rightarrow 3y^2 - ax = 0$$

$$\Rightarrow 3y^2 = ax$$

$$\Rightarrow x = \frac{3y^2}{a}$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow y^3 + \left(\frac{3y^2}{a}\right)^2 = a\left(\frac{3y^2}{a}\right)y$$

$$\Rightarrow y^3 + \frac{9}{a^2}y^4 = 3y^3$$

$$\Rightarrow 2y^3 - \frac{9}{a^2}y^4 = 0$$

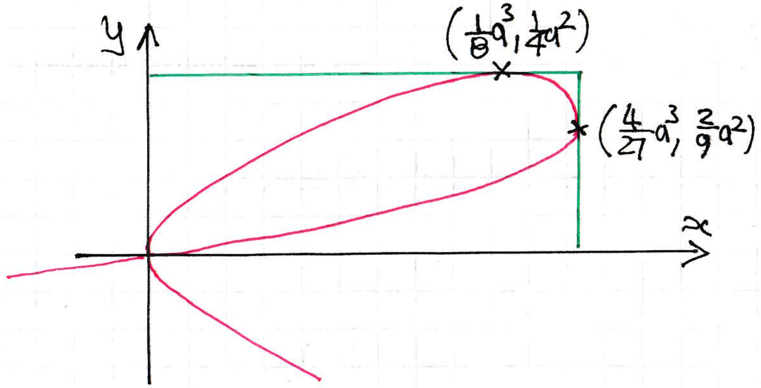
IYGB - MP2 PAPER Z - QUESTION 10

$\Rightarrow 2a^2y^3 - 9y^4 = 0$

$\Rightarrow y^3(2a^2 - 9y) = 0$

$\Rightarrow y = \begin{cases} \cancel{0} \\ \dfrac{2a^2}{9} \end{cases} \qquad \Rightarrow x = \begin{cases} \cancel{0} \\ \dfrac{3}{a}\left(\dfrac{2a^2}{9}\right)^2 = \dfrac{12a^4}{81a} = \dfrac{4}{27}a^3 \end{cases}$

- HENCE WE NOW HAVE



- AREA = 288

$\Rightarrow \frac{1}{4}a^2 \times \frac{4}{27}a^3 = 288$

$\Rightarrow \frac{1}{27}a^5 = 288$

$\Rightarrow a^5 = 27 \times 288 = 27 \times 2 \times 144 = 3^3 \times 2 \times (3 \times 2 \times 2)^2$

$\Rightarrow a^5 = 3^5 \times 2^5 = 6^5$

$\Rightarrow a = 6$

IYGB - MP2 PAPER Z - QUESTION 11

START BY EVALUATING z AT $t=4$

$$z(4) = \sqrt{4^3+8\times4^{\frac{1}{2}}+1} = \sqrt{64+16+1} = \sqrt{81} = 9$$

NEXT FORM A CHAIN OF RELATED DERIVATIVES

$$\Rightarrow \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dz} \times \frac{dz}{dt}$$

$$y = \frac{1}{(x+3)^2}$$

$$\frac{dy}{dx} = -\frac{2}{(x+3)^3}$$

$$z = (t^3+6t^{\frac{1}{2}}+1)^{\frac{1}{2}}$$

$$\frac{dz}{dt} = \frac{1}{2}(t^3+8t^{\frac{1}{2}}+1)^{-\frac{1}{2}} \times (3t^2+4t^{-\frac{1}{2}})$$

$$\frac{dz}{dt} = \frac{3t^2+4t^{-\frac{1}{2}}}{2(t^3+8t^{\frac{1}{2}}+1)^{\frac{1}{2}}}$$

$$\ln(x+3)^3 = \frac{1}{3}z$$

$$3\ln(x+3) = \frac{1}{3}z$$

$$z = 9\ln(x+3)$$

$$\frac{dz}{dx} = \frac{9}{x+3}$$

$$\frac{dx}{dz} = \frac{x+3}{9}$$

$$\Rightarrow \frac{dy}{dt} = -\frac{\cancel{2}}{(x+3)^{\cancel{3}\,2}} \times \frac{\cancel{x+3}}{9} \times \frac{3t^2+4t^{-\frac{1}{2}}}{\cancel{2}\sqrt{t^3+8\sqrt{t}+1}}$$

$$\Rightarrow \frac{dy}{dt} = -\frac{1}{9(x+3)^2} \times \frac{3t^2+4t^{-\frac{1}{2}}}{\sqrt{t^3+8\sqrt{t}+1}}$$

$$\Rightarrow \frac{dy}{dt} = -\frac{1}{9}y \times \frac{3t^2+4t^{-\frac{1}{2}}}{z}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y(3t^2+4t^{-\frac{1}{2}})}{9z}$$

IYGB - MP2 PAPER Z - QUESTION 11

NEXT WE USE $y = -e^{-2}$

- $y = e^{-2} \Longrightarrow y = \frac{1}{e^2}$

$$\Longrightarrow \frac{1}{(x+3)^2} = \frac{1}{e^2}$$

$$\Longrightarrow (x+3)^2 = e^2$$

$$\Longrightarrow x+3 = \begin{cases} e \\ -e \end{cases}$$

$$\Longrightarrow x = \begin{cases} e-3 \\ \cancel{-e-3} \end{cases} \qquad x > -3$$

- $\ln(x+3)^3 = \frac{1}{3}z \Longrightarrow z = 9\ln(x+3) \leftarrow$ FROM EARLIER

$$\Longrightarrow z = 9\ln(e-3+3)$$

$$\Longrightarrow z = 9$$

$\therefore$ WHEN $y = e^{-2}$, $x = e-3$, $z = 9$ & $t = 4$

FINALLY WE HAVE

$$\left.\frac{dy}{dt}\right|_{y=e^{-2}} = -\frac{e^{-2}(3\times 4^2 + 4\times\ )}{9\times 9}$$

$$\left.\frac{dy}{dt}\right|_{y=e^{-2}} = -\frac{50}{81e^2} \approx -0.0835$$

IYGB – MP2 PAPER Z – QUESTION 12

$$f(\theta,\phi) \equiv \sin(\theta-\phi)$$
$$g(\theta,\phi) \equiv \cos(\theta-\phi) - 2\tan\phi \sin(\theta-\phi)$$

WE ARE GIVEN THAT $f(\theta,\phi) = g(\theta,\phi)\tan\phi$

$$\Rightarrow \sin(\theta-\phi) = [\cos(\theta-\phi) - 2\tan\phi \sin(\theta-\phi)]\tan\phi$$
$$\Rightarrow \frac{\sin(\theta-\phi)}{\cos(\theta-\phi)} = \left[\frac{\cos(\theta-\phi)}{\cos(\theta-\phi)} - 2\tan\phi \frac{\sin(\theta-\phi)}{\cos(\theta-\phi)}\right]\tan\phi$$
$$\Rightarrow \tan(\theta-\phi) = [1 - 2\tan\phi \tan(\theta-\phi)]\tan\phi$$
$$\Rightarrow \tan(\theta-\phi) = \tan\phi - 2\tan^2\phi \tan(\theta-\phi)$$
$$\Rightarrow \tan(\theta-\phi) + 2\tan^2\phi\tan(\theta-\phi) = \tan\phi$$
$$\Rightarrow \tan(\theta-\phi)[1+2\tan^2\phi] = \tan\phi$$
$$\Rightarrow \left(\frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}\right)(1+2\tan^2\phi) = \tan\phi$$
$$\Rightarrow (\tan\theta - \tan\phi)(1+2\tan^2\phi) = \tan\phi(1+\tan\theta\tan\phi)$$
$$\Rightarrow \tan\theta + 2\tan^2\phi\tan\theta - \tan\phi - 2\tan^3\phi = \tan\phi + \tan^2\phi\tan\theta$$
$$\Rightarrow \tan\theta + \tan^2\phi\tan\theta = 2\tan\phi + 2\tan^3\phi$$
$$\Rightarrow \tan\theta\cancel{(1+\tan^2\phi)} = 2\tan\phi\cancel{(1+\tan^2\phi)} \qquad (1+\tan^2\phi \neq 0)$$
$$\Rightarrow \tan\theta = 2\tan\phi$$

AS REQUIRED

IYGB - MP2 PAPER Z - QUESTION 12

ALTERNATIVE BY EXPANDING INTO SINES & COSINES

$$\Rightarrow \sin(\theta-\phi) = \left[\cos(\theta-\phi) - 2\tan\phi\sin(\theta-\phi)\right]\tan\phi$$

$$\Rightarrow \sin(\theta-\phi) = \left[\cos(\theta-\phi) - \frac{2\sin\phi}{\cos\phi}\sin(\theta-\phi)\right]\tan\phi$$

$$\Rightarrow \sin(\theta-\phi) = \frac{1}{\cos\phi}\left[\cos\phi\cdot\cos(\theta-\phi) - 2\sin\phi\sin(\theta-\phi)\right]\tan\phi$$

$$\Rightarrow \sin(\theta-\phi) = \frac{\tan\phi}{\cos\phi}\left[\cos\phi\cos(\theta-\phi) - \sin\phi\sin(\theta-\phi) - \sin\phi\sin(\theta-\phi)\right]$$

$$\Rightarrow \sin(\theta-\phi) = \frac{\sin\phi}{\cos^2\phi}\left[\cos\left[\phi+(\theta-\phi)\right] - \sin\phi\sin(\theta-\phi)\right]$$

$$\Rightarrow \sin(\theta-\phi) = \frac{\sin\phi}{\cos^2\phi}\left[\cos\theta - \sin\phi\sin(\theta-\phi)\right]$$

$$\Rightarrow \cos^2\phi\sin(\theta-\phi) = \sin\phi\cos\theta - \sin^2\phi\sin(\theta-\phi)$$

$$\Rightarrow \sin(\theta-\phi)\cos^2\phi + \sin(\theta-\phi)\sin^2\phi = \sin\phi\cos\theta$$

$$\Rightarrow \sin(\theta-\phi)\left[\cancel{\cos^2\phi+\sin^2\phi}\right] = \sin\phi\cos\theta$$

$$\Rightarrow \sin\theta\cos\phi - \cos\theta\sin\phi = \sin\phi\cos\theta$$

$$\Rightarrow \sin\theta\cos\phi = 2\sin\phi\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{2\sin\phi}{\cos\phi}$$

$$\Rightarrow \tan\theta = 2\tan\phi$$

// AS BEFORE

IYGB - MP2 PAPER Z - QUESTION 13

MANIPULATING INTO A BINOMIAL

$$(0.9)^{0.9} = (1-x)^{\frac{9}{10}} \qquad \text{WITH } x = 0.1$$

$$= 1 + \frac{\frac{9}{10}}{1}(-x) + O(x^2)$$

$$= 1 - \frac{9}{10}x + O(x^2)$$

NOW LET $x = \frac{1}{10}$

$$\therefore \quad 0.9^{0.9} \simeq 1 - \frac{9}{10}\left(\frac{1}{10}\right)$$

$$\simeq 1 - \frac{9}{100}$$

$$\simeq \frac{91}{100}$$

$$\simeq 0.91$$