IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper V

Difficulty Rating: 4.1725/0.7661

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 22 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The quadratic function f is given, in terms of three non zero constants a, b and c, by

$$f(x) \equiv ax^2 + bx + c, \; x \in \mathbb{R}.$$

When $f(x)$ is divided by $(x-1)$ the remainder is 1.

When $f(x)$ is divided by $(x-2)$ the remainder is 2.

When $f(x)$ is divided by $(x+2)$ the remainder is 70.

Determine the value of each of the constants a, b and c. **(7)**

Question 2

a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$16^{\frac{1}{2}} + 16^{-\frac{3}{4}}.$$ **(3)**

b) Solve the equation

$$x^{-\frac{2}{3}} = 64.$$ **(2)**

c) Simplify fully

$$\left(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}}\right)^2.$$ **(3)**

Detailed workings must be shown in this question if full credit is to be given

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Question 3

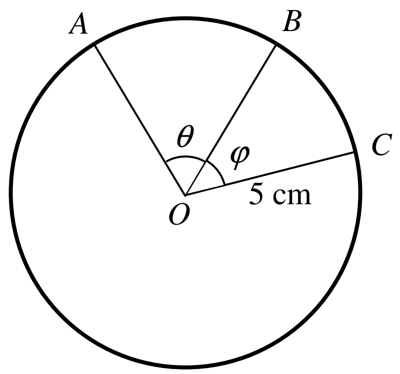
a) Find the binomial expansion of

$$(5+10x)^4 .$$ **(4)**

b) Hence, by using the answer of part (a) with a suitable value of x find the exact value of 1005^4.

You may not give the answer in standard index form. **(4)**

Question 4



The figure above shows a circle of radius 5 cm, centred at O.

The points A, B and C lie on the circumference of the circle. The angles AOB and BOC are denoted by θ and φ, respectively.

The **sum** of the areas of the sectors AOB and BOC is 20 cm^2.

The length of the arc AB is 3.5 cm **greater** than the length of the arc BC .

Determine the value of θ and the value of φ. **(6)**

Question 5

Use integration by parts find an exact value in terms of e, for the integral

$$\int_1^{e} (\ln x)^2 \, dx .$$ **(8)**

Question 6

The curve C_1, with equation $y = f(x)$, undergoes 3 transformations in the order given below.

1. A translation of 2 units, in the positive x direction.
2. A reflection about the y axis.
3. A translation of 1 unit in the positive y direction.

The resulting curve C_2 has equation

$$y = \frac{x^2 + 3x + 3}{x^2 + 4x + 5}, \quad x \in \mathbb{R}.$$

Determine an equation for C_1, giving the answer in the form $y = g(x)$, where $g(x)$ is a single simplified fraction. **(8)**

Question 7

Solve the following system of simultaneous equations

$$\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$$

$$x + y = 10.$$ **(6)**

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Question 8

The sum of the first eight terms of an arithmetic series is 124.

The sum of its first twenty terms of is 910.

The series has k terms.

Given the last term of the series is 193 find the value of k. **(7)**

Question 9

A polynomial $p(x)$ is defined as

$$p(x) \equiv \left(x^2 - 2x - 4\right)^2 - 10\left(x^2 - 2x - 4\right), \quad x \in \mathbb{R}.$$

The equation $p(x) = k$, where k is a constant, is satisfied by $x = -2$.

Determine the other three values of x that satisfy the equation $p(x) = k$. **(7)**

Question 10

The point $P(12,9)$ lies on the circle with equation

$$(x+3)^2 + (y-1)^2 = 289.$$

a) Find an equation of the normal to the circle at P. **(5)**

b) Determine the coordinates of the point Q, where the normal to the circle at P intersects the circle again. **(2)**

Question 11

A sequence u_1, u_2, u_3, u_4, … is given by

$$u_n = 3^n + (-2)^n .$$

Find an expression for u_{n+2}, as a recurrence relation of the form,

$$u_{n+2} = Au_{n+1} + Bu_n \, , \; u_1 = C \, , \; u_2 = D$$

where A, B, C and D are constants to be found. **(8)**

Question 12

The functions f and g are given by

$$f : x \mapsto x^2 \, , \; x \in \mathbb{R} .$$

$$g : x \mapsto 2x + 1 \, , \; x \in \mathbb{R} .$$

a) Solve the equation

$$fg(x) = gf(x) .$$ **(5)**

b) Find the inverse function of g. **(2)**

The function h is defined on a suitable domain such so that

$$ghf(x) = 3 - 2x^2 \, , \; x \in \mathbb{R} .$$

c) Determine an equation of h. **(5)**

Question 13

A curve has equation

$$x^3 + y = xy.$$

The straight line with equation $y + 3x + 1 = 0$ meets this curve at the point A.

a) Show that the x coordinate of A lies in the interval $(-0.4, -0.3)$. **(3)**

b) If P and Q are integers, use an iterative procedure based on the formula

$$x_{n+1} = \frac{1}{2}\left[Px^3 + Qx^2 - 1\right], \; x_1 = -0.35,$$

to find the x coordinate of A, correct to 2 decimal places. **(4)**

The straight line with equation $y + 3x + 1 = 0$ meets the above mentioned curve at another point B, whose the x coordinate lies in the interval $(0.8, 0.9)$.

c) Use the Newton Raphson method **twice**, starting with $x = 0.8$, to find a better approximation for the x coordinate of B. **(6)**

Question 14

$$2\log_2 x + \log_2(x-1) - \log_2(5x+4) = 1.$$

Find the only real root of the above logarithmic equation. **(8)**

Question 15

Find the solution interval of the following modulus inequality.

$$\left||x-1| - 5\right| < 3. \qquad \textbf{(7)}$$

Question 16

$$f(x) \equiv \frac{x^3}{x^2 - 4}, \quad x \neq \pm 2.$$

a) Use a suitable substitution to show that

$$\int_{\sqrt{6}}^{\sqrt{8}} f(x) \; dx = 1 + \ln 4. \qquad (7)$$

b) Express $f(x)$ in the form

$$Ax + B + \frac{C}{x-2} + \frac{D}{x+2},$$

where A, B, C and D are constants to be found. (5)

c) Use the result part **(b)** to verify the result of part **(a)**. (4)

Question 17

The limit expression shown below represents a student's evaluation for $f'(x)$, for a specific value of x.

$$\lim_{h \to 0} \left[\frac{2(1+h)^2 + 3(1+h) - 5}{h} \right].$$

Determine an expression for $f(x)$ and once obtained, **differentiate it directly** to find the value of $f'(x)$, for the specific value of x the student was evaluating.

No credit will be given for evaluating the limit directly. (7)

Question 18

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta .$$

a) Prove the validity of the above trigonometric identity. **(5)**

b) Hence or otherwise solve the trigonometric equation

$$\arcsin x = 3\arcsin\left(\frac{1}{3}\right).$$ **(5)**

Question 19

$$f(x) \equiv \frac{\sqrt{1+6\sin^2 x}}{(1+\tan x)^2}, \quad \tan x \neq -1 .$$

By using logarithmic differentiation, or otherwise, determine the value of $f'\left(\frac{\pi}{4}\right)$. **(10)**

Question 20

A curve $y = f(x)$ satisfies the differential equation

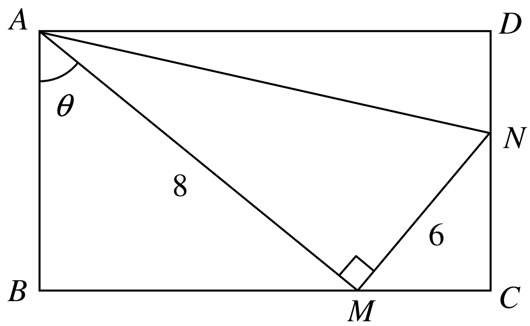
$$\frac{dy}{dx} = \frac{k(9-x)}{y}, \quad y > 0, \quad 0 \leq x \leq 9,$$

where k is a positive constant.

It is further given that $y = \frac{1}{2}$, $\frac{dy}{dx} = 2$ at $x = 1$.

Find the possible values of x when $\frac{dy}{dx} = \frac{1}{5}$. **(12)**

Question 21



The figure above shows a rectangle $ABCD$.

The points M and N lie on BC and CD respectively.

The angle AMN is $90°$, $|AM| = 8$ and $|MN| = 6$.

The angle BAM is denoted by θ.

a) Given that the perimeter of the rectangle $ABCD$ is fixed at 24 units, determine the possible value(s) of θ. **(8)**

b) Given instead that the perimeter of the rectangle $ABCD$ can vary, determine the largest possible area of the **triangle** ADN. **(5)**

Question 22

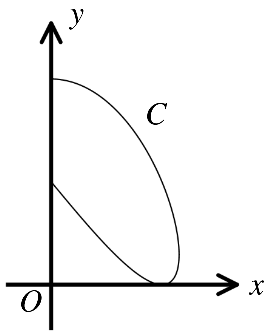


figure 1

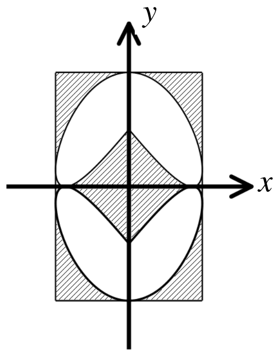


figure 2

Figure 1 above, shows the curve with parametric equations

$$x = \sin 2\theta, \ y = 1 - \sin 3\theta, \ 0 \le \theta \le \tfrac{1}{2}\pi .$$

Figure 2 above shows a glass design. It consists of the curve of figure 1, reflected successively in the x and y axis.

The resulting design fits snugly inside a rectangle, whose sides are tangents to the curve and its reflections, parallel to the coordinate axes. The region inside the 4 loops of the curve is made of clear glass while the region inside the rectangle but outside the 4 loops of C is made of yellow glass.

Determine the area of the yellow glass. **(12)**

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