# IYGB GCE

## Mathematics SYN

## Advanced Level

### Synoptic Paper A

Difficulty Rating: 3.4050

### Time: 3 hours

**Candidates may use any calculator allowed by the regulations of this examination.**

**Information for Candidates**

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 21 questions in this question paper.
The total mark for this paper is 200.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

**Question 1**

The points $A$ and $B$ have coordinates $(-1,5)$ and $(7,11)$, respectively.

**a)** Show that the equation of the perpendicular bisector of $AB$ is

$$4x+3y=36.$$ **(4)**

The perpendicular bisector of $AB$ crosses the coordinate axes at the points $P$ and $Q$.

**b)** Find the area of the triangle $OPQ$, where $O$ is the origin. **(3)**

---

**Question 2**

A cubic function is defined in terms of the constant $k$ as

$$f(x) \equiv x^3 + x^2 - x + k, \ x \in \mathbb{R}.$$

Given that $(x-k)$ is a factor of $f(x)$ determine the possible values of $k$. **(4)**

---

**Question 3**

**a)** Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{2}$.

**i.** $\sqrt{98}+\sqrt{2}$. **(2)**

**ii.** $\left(\sqrt{2}+3\right)\left(2-3\sqrt{2}\right)$. **(2)**

**b)** Solve the equation

$$\frac{27^t}{3^{t-1}} = 3\sqrt{3}.$$ **(3)**

***Detailed workings must be shown in this question.***

---

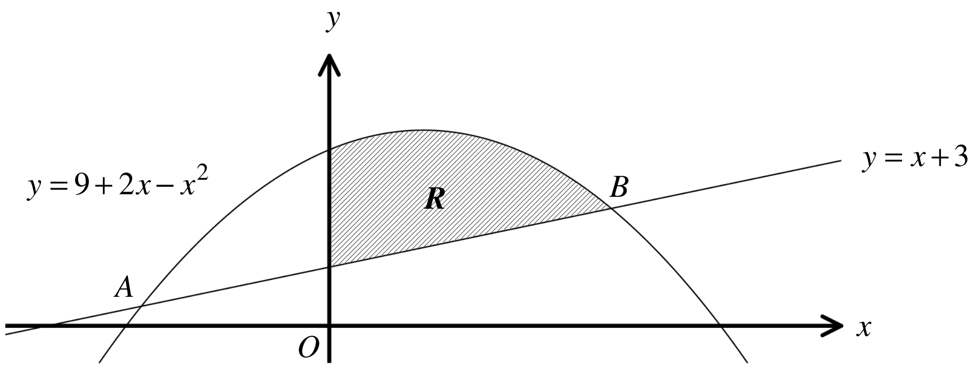
**Question 4**

Solve the following modulus inequality.

$$12-2|2x-3|\geq 7.$$ **(5)**

---

**Question 5**




The figure above shows the graph of the curve $C$ with equation

$$y=9+2x-x^2,$$

and the straight line $L$ with equation

$$y=x+3.$$

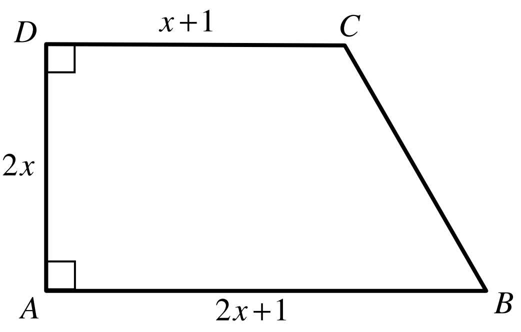
The curve meets the straight line at the points $A$ and $B$.

The finite region $R$, shown shaded in the figure, is bounded by the curve $C$, the straight line $L$ and the coordinate axes.

Show that the area of $R$ is $13.5$ square units. **(9)**

---

**Question 6**




A right angled trapezium $ABCD$ is shown in the figure above.

The trapezium has parallel sides $AB$ and $CD$ of lengths $(2x+1)$cm and $(x+1)$cm.
The height of the trapezium $AD$ is $2x$ cm.

Given that the area of the trapezium is 16 $\text{cm}^2$, determine the exact length of $BC$. **(7)**

---

**Question 7**

$$f(x) = \sqrt{1+\frac{1}{8}x}\,,\ |x| < 8.$$

**a)** Expand $f(x)$ as an infinite series, up and including the term in $x^2$. **(3)**

**b)** By substituting $x = 1$ in the expansion, show that

$$\sqrt{2} \approx \frac{256}{181} \quad \text{or} \quad \sqrt{2} \approx \frac{181}{128}.$$ **(4)**

---

**Question 8**

Relative to a fixed origin $O$, the points $A$, $B$ and $C$ have respective position vectors $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $5\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $7\mathbf{j}-4\mathbf{k}$.

**a)** Given that $ABCD$ is a parallelogram, determine the position vector of $D$. **(3)**

**b)** Determine the distance $AC$ and hence calculate the angle $ABC$. **(6)**

---

**Question 9**

$$\frac{x^2+3}{x-1} \equiv Ax+B+\frac{C}{x-1}.$$

**a)** Determine the value of each of the constants $A$, $B$ and $C$. **(3)**

**b)** Hence, or otherwise, evaluate

$$\int_2^4 \frac{x^2+3}{x-1}\, dx,$$

giving the answer in terms of natural logarithms. **(4)**

---

**Question 10**

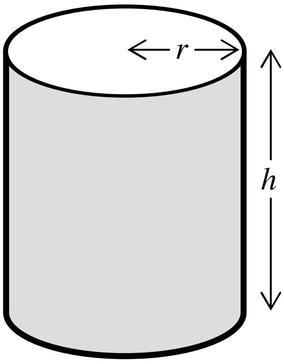
Solve the following trigonometric equation

$$\frac{\cos 2x}{1+\cos 2x} = 1-2\tan x,\ 0 \le x < 2\pi,$$

giving the answers in terms of $\pi$. **(10)**

---

**Question 11**




A hollow container, made of thin sheet metal, is in the shape of a right circular cylinder, which is **open** at one of its circular ends.

The container has radius $r$ cm, height $h$ cm and a capacity of $1500 \text{ cm}^3$.

**a)** Show that the surface area, $A \text{ cm}^2$, of the container is given by

$$A = \pi r^2 + \frac{3000}{r}. \qquad (4)$$

**b)** Determine the value of $r$ for which $A$ has a stationary value. $(4)$

**c)** Show that the value of $r$ found in part **(b)** gives the minimum value for $A$. $(2)$

**d)** Calculate, to the nearest $\text{cm}^2$, the minimum surface area of the container. $(2)$

---





I.Y.G.B. www.madasmaths.com

**Question 12**

The functions $f$ and $g$ are defined by

$$f : x \mapsto x^2 - 2x - 3, \quad x \in \mathbb{R}, \ 0 \le x \le 5 .$$

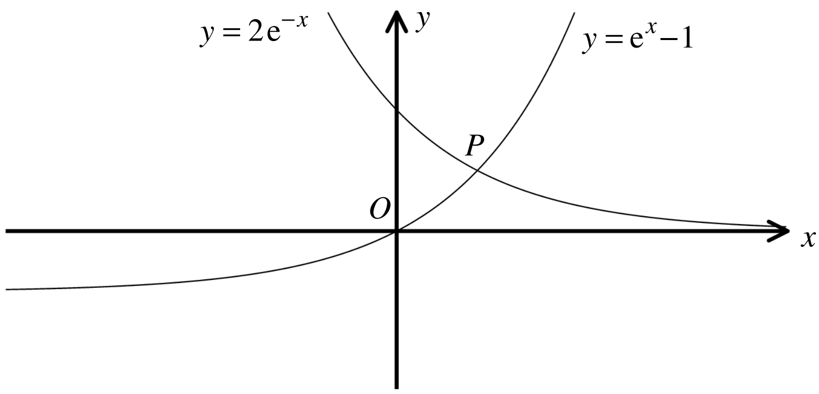
$$g : x \mapsto ax^2 + 2, \quad x \in \mathbb{R} , \ a \text{ is a real constant.}$$

**a)** Find the range of $f$. **(3)**

**b)** Determine the value of $a$, if $gf(1) = 6$. **(4)**

---

**Question 13**




The figure above shows the graphs of the curves with equations

$$y = 2e^{-x} \quad \text{and} \quad y = e^{x} - 1 .$$

The two graphs intersect at the point $P$.

Determine the exact coordinates of $P$. **(7)**

---

**Question 14**

A function $f$ is defined by

$$f(x) = 4x(x-1), \ x \in \mathbb{R}.$$

The graph of $g(x)$ is obtained by translating the graph of $f(x)$ by 1 unit in the positive $x$ direction, followed by a horizontal stretch by scale factor of $\frac{2}{3}$.

**a)** Determine a simplified equation for $g(x)$. **(4)**

The graph of $f(x)$ is obtained by translating the graph of $h(x)$ by 1 unit in the positive $x$ direction, followed by a vertical stretch by scale factor of $2$.

**b)** Determine a simplified equation for $h(x)$. **(4)**

---

**Question 15**

A circle $C$ has equation

$$x^2 + y^2 - 12x - 2y + 33 = 0.$$

**a)** Find the radius of the circle and the coordinates of its centre. **(4)**

The straight line with equation $y = x - 3$ intersects the circle at the points $P$ and $Q$, dividing the circle into two segments.

**b)** Determine the coordinates of $P$ and $Q$. **(5)**

**c)** Show that the area of the minor segment is $\pi - 2$. **(6)**

---

**Question 16**

The curve $C$ and the straight line $L$ have respective equations

$$x^2 - \frac{y^2}{2} = 1 \qquad \text{and} \qquad y = x + c,$$

where $c$ is a constant.

Show that $C$ and $L$, intersect for all values of $c$. **(8)**

---

**Question 17**

Show that a general solution of the differential equation

$$5\frac{dy}{dx} = 2y^2 - 7y + 3$$

is given by

$$y = \frac{A\mathrm{e}^x - 3}{2A\mathrm{e}^x - 1},$$

where $A$ is an arbitrary constant. **(15)**

---

**Question 18**

It is given that the angles $\theta$, $\frac{\pi}{4}$ and $\varphi$, in that order, are in arithmetic progression.

Show that

$$(\sin\theta - \sin\varphi)^2 + (\cos\theta + \cos\varphi)^2 = k,$$

where $k$ is a constant to be found. **(10)**

---

**Question 19**

A curve $C$ is given parametrically by

$$x = a + \tan t, \quad y = b + \cot^2 t, \quad 0 < t < \frac{\pi}{2},$$

where $a$ and $b$ are non zero constants.

**a)** Show that …

**i.** … $\frac{dy}{dx} = -2\cot^3 t$. **(5)**

**ii.** … a Cartesian equation of $C$ is

$$(y - b)(x - a)^2 = 1.$$ **(3)**

**b)** Given that $C$ meets the straight line with equation $y = 6x + 2$ at the points where $y = 2$ and $y = 5$, show further that $a$ is a solution of the equation

$$(a - 1)\left(12a^3 + 3a - 1\right) = 0.$$ **(8)**

**c)** Hence, state a possible value for $a$ and a possible value for $b$. **(2)**

---

**Question 20**

The curve $C$ has equation

$$y = x\sqrt{\ln x}, \quad x > 0.$$

The equation of the tangent to $C$ at the point where $x = a$ is

$$4y = bx - a,$$

where $a$ and $b$ are non zero constants.

Determine the exact value of $a$. **(14)**

---

**Question 21**

By using the substitution $\sqrt{x} = \tan\theta$, or otherwise, find a simplified expression for the following integral

$$\int \frac{1-x}{\sqrt{x}(x+1)^2}\,dx\,. \qquad \textbf{(14)}$$