IYGB - SYN PAPER B - QUESTION 1

PROOF BY EXHAUSTION

"THE SQUARE OF ANY INTEGER CAN NEVER BE OF THE FORM $3k+2$, $k \in \mathbb{N}$"

THE NUMBER TO BE SQUARED, SAY a, CAN TAKE ONE OF THE FOLLOWING 3 FORMS

$$a = 3m \quad , \quad a = 3m+1 \quad , \quad a = 3m+2 \quad , \quad m \in \mathbb{N}$$

- IF $a = 3m \implies a^2 = 9m^2 = 3(3m^2) = 3k$, $k \in \mathbb{N}$
- IF $a = 3m+1 \implies a^2 = 9m^2+6m+1 = 3(3m^2+2m)+1 = 3k+1$, $k \in \mathbb{N}$
- IF $a = 3m+2 \implies a^2 = 9m^2+12m+4 = 3(3m^2+4m+1)+1 = 3k+1$, $k \in \mathbb{N}$

$\therefore$ SQUARING ANY INTEGER ONLY PRODUCES INTEGERS OF THE FORM $3k$ OR $3k+1$, $k \in \mathbb{N}$

$\therefore$ IT IS NOT POSSIBLE TO HAVE A SQUARE NUMBER OF THE FORM $3k+2$, $k \in \mathbb{N}$

IYGB - SYNOPTIC PAPER B - QUESTION 2

$$p = \log_6 25 \quad \& \quad q = \log_6 2$$

a) $\log_6 200 = \log_6 (25 \times 8)$

$= \log_6 25 + \log_6 8$

$= \log_6 25 + \log_6 2^3$

$= \log_6 25 + 3\log_6 2$

$= \underline{p + 3q}$

b) $\log_6 (3.2) = \log_6 \left(\frac{32}{10}\right) = \log_6 \left(\frac{16}{5}\right)$

$= \log_6 16 - \log_6 5$

$= \log_6 2^4 - \log_6 25^{\frac{1}{2}}$

$= 4\log_6 2 - \frac{1}{2}\log_6 25$

$= \underline{4q - \frac{1}{2}p}$

c) $\log_6 75 = \log_6 (25 \times 3)$

$= \log_6 25 + \log_6 3$

$= \log_6 25 + \log_6 \left(\frac{6}{2}\right)$

$= \log_6 25 + \left[\log_6 6 - \log_6 2\right]$

$= \log_6 25 + \log_6 - \log_6 2$

$= \underline{p + 1 - q}$

IYGB - SYN PAPER B - QUESTION 3

EXPAND AND SIMPLIFY

$$\Rightarrow (2x+3)^2 - (4-x)^2 = 45$$

$$\Rightarrow 4x^2+12x+9-(16-8x+x^2) = 45$$

$$\Rightarrow 4x^2+12x+9-16+8x-x^2 = 45$$

$$\Rightarrow 3x^2+20x-7 = 45$$

$$\Rightarrow 3x^2+20x-52 = 0$$

FACTORIZING NOTING THAT 1×52, 2×26, 4×13

$$\Rightarrow (3x+26)(x-2) = 0$$

$$\Rightarrow x = \begin{cases} 2 \\ -\frac{26}{3} \end{cases}$$

ALTERNATIVE

$$\Rightarrow (2x+3)^2 - (4-x)^2 = 45$$

$$\Rightarrow [(2x+3)+(4-x)][(2x+3)-(4-x)] = 45$$

$$\Rightarrow (x+7)(3x-1) = 45$$

BY INSPECTION $x=2$ IS A SOLUTION

$$\Rightarrow 3x^2+20x-7 = 45$$

$$\Rightarrow 3x^2+20x-52 = 0$$

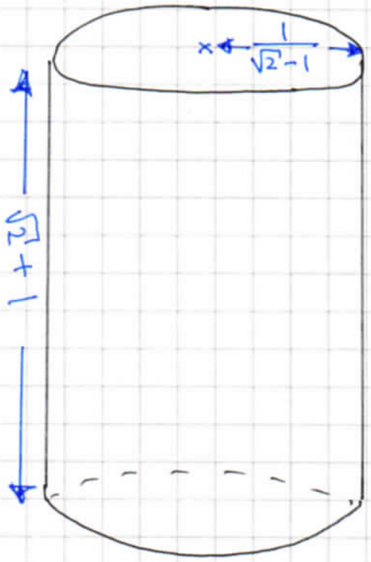
$$\Rightarrow (x-2)(3x+26) = 0$$

(FROM ABOVE)

$$\therefore x = \begin{cases} 2 \\ -\frac{26}{3} \end{cases}$$

IYGB - SYN PAPER B - QUESTION 4

$$V = \pi r^2 h$$



$$\Rightarrow V = \pi\left(\frac{1}{\sqrt{2}-1}\right)^2 \times (\sqrt{2}+1)$$

$$\Rightarrow V = \pi \times \frac{1}{(\sqrt{2}-1)^2} \times (\sqrt{2}+1)$$

$$\Rightarrow V = \frac{\pi(\sqrt{2}+1)}{(\sqrt{2}-1)^2}$$

$$\Rightarrow V = \frac{\pi(\sqrt{2}+1)}{3-2\sqrt{2}}$$

$$(\sqrt{2}-1)^2 = (\sqrt{2}-1)(\sqrt{2}-1)$$
$$= 2 - \sqrt{2} - \sqrt{2} + 1$$
$$= 3 - 2\sqrt{2}$$

OR

$$(\sqrt{2}-1)^2 = (\sqrt{2})^2 - 2\times\sqrt{2}\times 1 + 1^2$$
$$= 2 - 2\sqrt{2} + 1$$
$$= 3 - 2\sqrt{2}$$

RATIONALIZE THE DENOMINATOR

$$\Rightarrow V = \frac{\pi(\sqrt{2}+1)(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

$$\Rightarrow V = \frac{\pi[3\sqrt{2}+4+3+2\sqrt{2}]}{9+\cancel{6\sqrt{2}}-\cancel{6\sqrt{2}}-8}$$

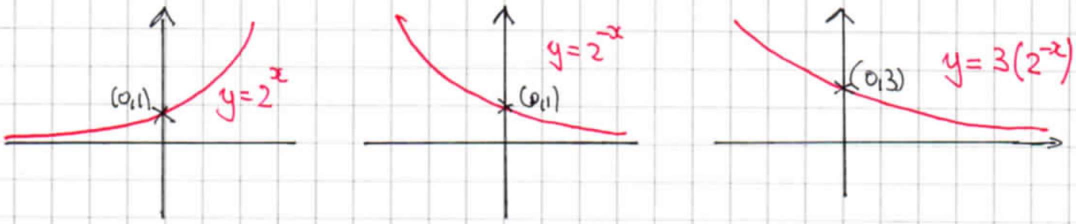
$$\Rightarrow V = \frac{\pi(7+5\sqrt{2})}{1}$$

$$\Rightarrow V = \pi(7+5\sqrt{2})$$

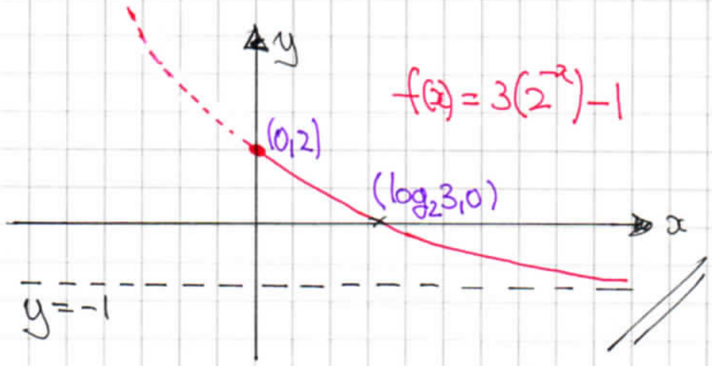
AS REQUIRED

IYGB - SYNOPTIC PAPER B - QUESTION 5

a) STARTING WITH THE GRAPH OF $y = 2^x$ & ITS TRANSFORMATIONS



HENCE TRANSLATING "DOWNWARDS" BY ONE UNIT



$$3(2^{-x}) - 1 = 0$$
$$2^{-x} = \frac{1}{3}$$
$$2^{x} = 3$$
$$x = \log_2 3$$

b) LOOKING AT THE ABOVE GRAPH

$$-1 < f(x) \leq 2$$

c)
$$f(g(x)) = f[\log_2 x] = 3\left(2^{-\log_2 x}\right) - 1$$
$$= 3\left(2^{\log_2 x^{-1}}\right) - 1$$
$$= 3\left(2^{\log_2 \left(\frac{1}{x}\right)}\right) - 1$$
$$= 3\left(\frac{1}{x}\right) - 1$$
$$= \frac{3}{x} - 1$$

IYGB - SYN PAPER B - QUESTION 6

IT IS GIVEN THAT

$u_2 + u_4 = 156$ | $u_3 + u_5 = 234$

USING $u_n = ar^{n-1}$ THE ABOVE EQUATIONS BECOME

$\Rightarrow ar + ar^3 = 156$ | $\Rightarrow ar^2 + ar^4 = 234$

$\Rightarrow ar(1+r^2) = 156$ | $\Rightarrow ar^2(1+r^2) = 234$

DIVIDE EQUATIONS SIDE BY SIDE

$$\Rightarrow \frac{\cancel{a}r^2\cancel{(1+r^2)}}{\cancel{a}r\cancel{(1+r^2)}} = \frac{234}{156} \qquad (a \neq 0 \,,\, 1+r^2 \neq 0)$$

$$\Rightarrow r = \frac{3}{2} = 1.5 \qquad (r \neq 0)$$

AND USING $ar(1+r^2) = 156$

$$\Rightarrow a \times \frac{3}{2}\left(1 + \frac{9}{4}\right) = 156$$

$$\Rightarrow \frac{39}{8}a = 156$$

$$\Rightarrow a = 32$$

IYGB - SYNOPTIC PAPER B - QUESTION 7

USING THE SUBSTITUTION GIVEN

$$\int \frac{6x^2}{2x^{\frac{3}{2}}-1}\,dx = \int \frac{6x^2}{u}\left(\frac{du}{3x^{\frac{1}{2}}}\right)$$

$u = 2x^{\frac{3}{2}}-1$

$\frac{du}{dx} = 3x^{\frac{1}{2}}$

$du = 3x^{\frac{1}{2}}\,dx$

$dx = \frac{du}{3x^{\frac{1}{2}}}$

$2x^{\frac{3}{2}} = u+1$

$$= \int \frac{6x^2}{3x^{\frac{1}{2}}u}\,du = \int \frac{2x^{\frac{3}{2}}}{u}\,du = \int \frac{u+1}{u}\,du$$

$$= \int 1 + \frac{1}{u}\,du = u + \ln|u| + C$$

$$= \left(2x^{\frac{3}{2}} - 1\right) + \ln\left|2x^{\frac{3}{2}}-1\right| + C$$

$$= 2x^{\frac{3}{2}} + \ln\left|2x^{\frac{3}{2}}-1\right| + C$$

ALTERNATIVE BY MANIPULATION/DIVISION & RECOGNITION

$$\int \frac{6x^2}{2x^{\frac{3}{2}}-1}\,dx = \int \frac{3x^{\frac{1}{2}}\left(2x^{\frac{3}{2}}-1\right) + 3x^{\frac{1}{2}}}{2x^{\frac{3}{2}}-1}\,dx$$

SPLITTING THE FRACTION

$$= \int 3x^{\frac{1}{2}} + \frac{3x^{\frac{1}{2}}}{2x^{\frac{3}{2}}-1}\,dx$$

THIS IS OF THE FORM

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

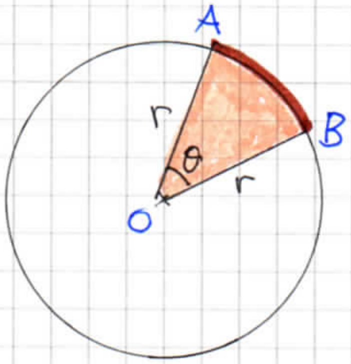
$$= x^{\frac{3}{2}} + \ln\left|2x^{\frac{3}{2}}-1\right| + C$$

AS ABOVE

IYGB - SYNOPTIC PAPER B - QUESTION 8

"ARC LENGTH $= r\theta^c$"

"SECTOR AREA $= \frac{1}{2}r^2\theta^c$"



FORMING TWO EQUATIONS BASED ON THE ABOVE FORMULAE

- $r\theta = 12$
- $\frac{1}{2}r^2\theta = 48$

$$\frac{1}{2}r(r\theta) = 48$$

$$\frac{1}{2}r \times 12 = 48$$

$$6r = 48$$

$$r = 8$$

& $r\theta = 12$

$$8\theta = 12$$

$$\theta = 1.5^c$$

IYGB – SYN PAPER B – QUESTION 9

- WRITE THE EXPRESSION IN FUNCTION NOTATION FOR SIMPLICITY

$$f(x) = \frac{1}{x^2 - 2x}$$

$$f(x+h) - f(x) = \frac{1}{(x+h)^2 - 2(x+h)} - \frac{1}{x^2 - 2x}$$

$$= \frac{1}{x^2 + 2xh + h^2 - 2x - 2h} - \frac{1}{x^2 - 2x}$$

$$= \frac{\cancel{x^2} \cancel{-2x} - (\cancel{x^2} + 2xh + h^2 \cancel{-2x} - 2h)}{(x^2 + 2xh + h^2 - 2x - 2h)(x^2 - 2x)}$$

$$= \frac{-2xh - h^2 + 2h}{(x^2 + 2xh + h^2 - 2x - 2h)(x^2 - 2x)}$$

- HENCE THE DERIVATIVE NOW YIELDS

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h \to 0}\left[\frac{1}{h}\left[f(x+h) - f(x)\right]\right]$$

$$= \lim_{h \to 0}\left[\frac{1}{\cancel{h}} \times \frac{\cancel{h}(-2x - h + 2)}{(x^2 + 2xh + h^2 - 2x - 2h)(x^2 - 2x)}\right]$$

$$= \lim_{h \to 0}\left[\frac{-h + 2 - 2x}{(x^2 + 2xh + h^2 - 2x - 2h)(x^2 - 2x)}\right]$$

$$= \frac{2 - 2x}{(x^2 - 2x)(x^2 - 2x)} = \frac{2(1 - x)}{(x^2 - 2x)^2}$$

IYGB - SYNOPTIC PAPER B - QUESTION 10

REWRITE THE EQUATION BEFORE DIFFERENTIATION

$$\Rightarrow \frac{3x^2}{y} - 5y = 2(x+8)$$

$$\Rightarrow 3x^2 - 5y^2 = 2y(x+8)$$

$$\Rightarrow 3x^2 - 5y^2 = 2xy + 16y$$

DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}(3x^2) - \frac{d}{dx}(5y^2) = \frac{d}{dx}(2xy) + \frac{d}{dx}(16y)$$

$$\Rightarrow 6x - 10y\frac{dy}{dx} = \left[2y + 2x\frac{dy}{dx}\right] + 16\frac{dy}{dx}$$

FOR STATIONARY POINTS $\frac{dy}{dx} = 0$

$$\Rightarrow 6x = 2y$$

$$\Rightarrow y = 3x$$

ANY STATIONARY POINTS MUST LIE ON THE LINE $y = 3x$ - SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$$\left.\begin{array}{l} 3x^2 - 5y^2 = 2xy + 16y \\ y = 3x \end{array}\right\} \Rightarrow 3x^2 - 5(3x)^2 = 2x(3x) + 16(3x)$$

$$\Rightarrow 3x^2 - 45x^2 = 6x^2 + 48x$$

$$\Rightarrow 0 = 48x^2 + 48x$$

$$\Rightarrow 48x(x+1) = 0$$

$$\Rightarrow x = \begin{cases} 0 \\ -1 \end{cases} \quad y = \begin{cases} 0 \\ -3 \end{cases}$$

$\therefore$ ONLY POINT IS $(-1,-3)$ AS $y \neq 0$

IYGB - SYNOPTIC PAPER B - QUESTION 11

a) SOLVING BY SEPARATING VARIABLES

p = population (millions)
t = time (days)

$t=0$, $p=p_0$

$$\Rightarrow \frac{dp}{dt} = kp\cos kt$$

$$\Rightarrow dp = kp\cos kt \, dt$$

$$\Rightarrow \frac{1}{p}\,dp = k\cos kt \, dt$$

$$\Rightarrow \int \frac{1}{p}\,dp = \int k\cos kt \, dt$$

$$\Rightarrow \ln p = \sin kt + C$$

TIDY BEFORE APPLYING THE GIVEN CONDITION

$$\Rightarrow p = e^{\sin kt + C}$$

$$\Rightarrow p = e^{\sin kt} \times e^{C}$$

$$\Rightarrow p = Ae^{\sin kt} \qquad (A = e^{C})$$

$$t=0 \quad p=p_0 \Rightarrow p_0 = Ae^{0}$$
$$\Rightarrow A = p_0$$

$$\Rightarrow p = p_0 e^{\sin kt}$$

IYGB - SYNOPTIC PAPER B - QUESTION 11

b) TAKING $k=3$, THE SOLUTION BECOMES

$$\Rightarrow P = P_0 e^{\sin 3t}$$

$$\Rightarrow P_0 = P_0 e^{\sin 3t}$$

$$\Rightarrow 1 = e^{\sin 3t}$$

$$\Rightarrow \ln 1 = \sin 3t$$

$$\Rightarrow \sin 3t = 0$$

$$\Rightarrow 3t = \ldots -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$$

$$\Rightarrow 3t = \pi$$

$$\Rightarrow t = \frac{\pi}{3} \text{ days}$$

$\times 24$

$$\Rightarrow t = 8\pi \text{ hours}$$

$\times 60$

$$\Rightarrow t = 480\pi \text{ minutes} \approx 1508 \text{ minutes}$$

IYGB – SYNOPTIC PAPER B – QUESTION 12

a) DIFFERENTIATE EACH OF THE PARAMETRIC EQUATIONS W.R.T t

- $x = \frac{t+3}{t+1}$

$\Rightarrow \frac{dx}{dt} = \frac{(t+1)\times 1 - (t+3)\times 1}{(t+1)^2}$

$\Rightarrow \frac{dx}{dt} = \frac{t+1-t-3}{(t+1)^2}$

$\Rightarrow \frac{dx}{dt} = \frac{-2}{(t+1)^2}$

- $y = \frac{2}{t+2}$

$\Rightarrow y = 2(t+2)^{-1}$

$\Rightarrow \frac{dy}{dt} = -2(t+2)^{-2}$

$\Rightarrow \frac{dy}{dt} = -\frac{2}{(t+2)^2}$

COMBINING TO $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\frac{2}{(t+2)^2}}{-\frac{2}{(t+1)^2}} = \frac{\cancel{-2}(t+1)^2}{\cancel{-2}(t+2)^2} = \left(\frac{t+1}{t+2}\right)^2$$

// AS REQUIRED

b) PROCEED AS FOLLOWS

- $x = \frac{t+3}{t+1}$

$\Rightarrow x = \frac{(t+2)+1}{(t+2)-1}$

$\Rightarrow x = \frac{\frac{2}{y}+1}{\frac{2}{y}-1}$

$\Rightarrow x = \frac{\frac{2}{y}y + 1y}{\frac{2}{y}y - 1y}$

- $y = \frac{2}{t+2}$

$\frac{1}{y} = \frac{t+2}{2}$

$\boxed{\frac{2}{y} = t+2}$

IYGB - SYNOPTIC PAPER B - QUESTION 12

$$\Longrightarrow \quad x = \frac{2+y}{2-y}$$

$$\Longrightarrow \quad 2x - xy = 2 + y$$

$$\Longrightarrow \quad 2x - 2 = xy + y$$

$$\Longrightarrow \quad 2(x-1) = y(x+1)$$

$$\Longrightarrow \quad y = \frac{2(x-1)}{x+1}$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER B - QUESTION 13

FIND AN EXPRESSION FOR THE COMPOSITION

$$f(x) = 3\ln(2x) \quad \& \quad g(x) = 2x^2+1$$

$$\Rightarrow y = g(f(x)) = g(3\ln 2x) = 2(3\ln 2x)^2+1$$

DIFFERENTIATE W.R.T x

$$\Rightarrow \frac{dy}{dx} = 4(3\ln 2x)^1 \times \frac{3}{2x} \times 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{36\ln 2x}{x}$$

EVALUATE AT THE GIVEN x

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=e} = \frac{36\ln(2e)}{e}$$

$$= \frac{36}{e}\left[\ln 2 + \ln e\right]$$

$$= \frac{36}{e}\left[\ln 2 + 1\right]$$

AS REQUIRED

IYGB - SYN PAPER B - QUESTION 14

START BY PARTIAL FRACTIONS

$$\frac{3x-5}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{x-1}$$

$$\boxed{3x-5 \equiv A(x-1) + Bx}$$

IF $x=1 \implies -2 = B$

$\implies B = -2$

IF $x=0 \implies -5 = -A$

$\implies A = 5$

HENCE THE INTEGRAL BECOMES

$$\implies \int_k^{2k} \frac{3x-5}{x(x-1)}\,dx = \ln 72$$

$$\implies \int_k^{2k} \frac{5}{x} - \frac{2}{x-1}\,dx = \ln 72$$

$$\implies \Big[5\ln|x| - 2\ln|x-1|\Big]_k^{2k} = \ln 72$$

$$\implies \Big[5\ln|2k| - 2\ln|2k-1|\Big] - \Big[5\ln|k| - 2\ln|k-1|\Big] = \ln 72$$

$$\implies 5\ln|2k| - 2\ln|2k-1| - 5\ln|k| + 2\ln|k-1| = \ln 72$$

$$\implies 5\ln\left|\frac{2k}{k}\right| + 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72$$

$$\implies 5\ln 2 + 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72$$

$$\implies 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72 - 5\ln 2$$

IYGB - SYN PAPER B - QUESTION 14

$$\Rightarrow \quad 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72 - \ln 32$$

$$\Rightarrow \quad 2\ln\left|\frac{k-1}{2k-1}\right| = \ln\frac{9}{4}$$

$$\Rightarrow \quad 2\ln\left|\frac{k-1}{2k-1}\right| = \ln\left(\frac{3}{2}\right)^2$$

$$\Rightarrow \quad \cancel{2}\ln\left|\frac{k-1}{2k-1}\right| = \cancel{2}\ln\frac{3}{2}$$

$$\Rightarrow \quad \frac{k-1}{2k-1} = \frac{3}{2}$$

$$\Rightarrow \quad 6k-3 = 2k-2$$

$$\Rightarrow \quad 4k = 1$$

$$\Rightarrow \quad k = \frac{1}{4}$$

IYGB - SYN PAPER B - QUESTION 15

a) $A(2,-1,4)$ $B(0,-5,10)$ $C(3,1,3)$ $D(6,7,-8)$

- PICK A POINT AT RANDOM AND CALCULATE ALL OTHER VECTORS TO THE OTHER 3 POINTS

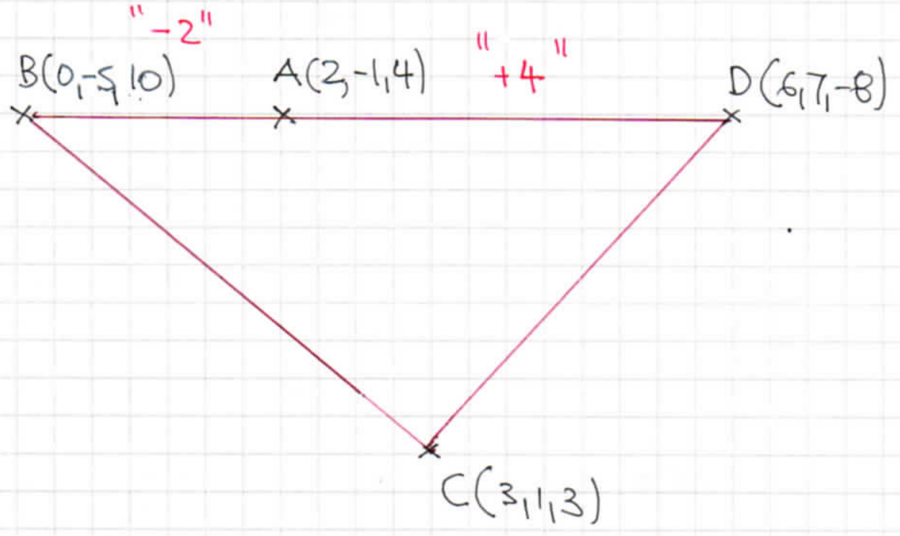
$$\vec{AB} = \underline{b} - \underline{a} = (0,-5,10)-(2,-1,4) = (-2,-4,6) = 2(-1,-2,3)$$
$$\vec{AC} = \underline{c} - \underline{a} = (3,1,3)-(2,-1,4) = (1,2,-1) = 1(1,2,-1)$$
$$\vec{AD} = \underline{d} - \underline{a} = (6,7,-8)-(2,-1,4) = (4,8,-12) = 4(1,2,-3)$$

- HENCE WE HAVE $\vec{AB}$ & $\vec{AD}$ IN "PARALLEL CONFIGURATION"

$$\vec{AB} = 2(-1,-2,3) = -2\underline{(1,2,-3)}$$
$$\vec{AD} = 4\underline{(1,2,-3)}$$

$\therefore$ A, B & D ARE COLLINEAR

b) - DRAWING A DIAGRAM



IYGB - SYN PAPER B - QUESTION 15

- THE LENGTH OF BD IS $6|1,2,-3|$ (OR COMPUTE $|\underline{d}-\underline{b}|$)

$$\Rightarrow 6\sqrt{1+4+9} = 6\sqrt{14}$$

- ALSO WE HAVE

• $|\overrightarrow{BC}| = |\underline{c}-\underline{b}| = |(3,1,3)-(0,-5,10)| = |3,6,-7|$

$$= \sqrt{9+36+49} = \sqrt{94}$$

• $|\overrightarrow{DC}| = |\underline{c}-\underline{d}| = |(3,1,3)-(6,7,-8)| = |-3,-6,11|$

$$= \sqrt{9+36+121} = \sqrt{166}$$

$\therefore$ THE SHORTEST SIDE OF THE TRIANGLE WHICH HAS THE LARGEST AREA IS $\sqrt{94}$

IYGB - SYNOPTIC PAPER B - QUESTION 16

METHOD A – USING SINES AND COSINES

LET $x = \arccos\frac{1}{\sqrt{5}} + \arccos\frac{1}{\sqrt{10}}$

$\Rightarrow x = \theta + \phi$

$\Rightarrow \cos x = \cos(\theta+\phi)$

$\Rightarrow \cos x = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\Rightarrow \cos x = \frac{1}{\sqrt{5}}\frac{1}{\sqrt{10}} - \frac{2}{\sqrt{5}}\times\frac{3}{\sqrt{10}}$

$\Rightarrow \cos x = \frac{1}{\sqrt{50}} - \frac{6}{\sqrt{50}}$

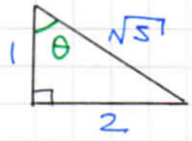
$\Rightarrow \cos x = -\frac{5}{\sqrt{50}} = -\frac{5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{3\pi}{4}$ (AS $0<\theta+\phi<\pi$)

$\therefore \arccos 5^{-\frac{1}{2}} + \arccos 10^{-\frac{1}{2}} = \frac{3\pi}{4}$

$\theta = \arccos\frac{1}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

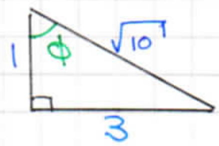


$\sin\theta = \frac{2}{\sqrt{5}}$

$\tan\theta = 2$

$\phi = \arccos\frac{1}{\sqrt{10}}$

$\cos\phi = \frac{1}{\sqrt{10}}$



$\sin\phi = \frac{3}{\sqrt{10}}$

$\tan\phi = 3$

METHOD B – USING TANGENTS

$\Rightarrow x = \theta + \phi$

$\Rightarrow \tan x = \tan(\theta+\phi)$

$\Rightarrow \tan x = \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$

$\Rightarrow \tan x = \frac{2+3}{1-2\times3} = \frac{5}{-5} = -1$

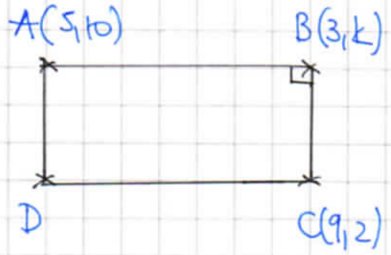
$\Rightarrow x = \frac{3\pi}{4}$ (AS $0<\theta+\phi<\pi$)

$\therefore \arccos 5^{-\frac{1}{2}} + \arccos 10^{-\frac{1}{2}} = \frac{3\pi}{4}$

AS BEFORE

IYGB - SYN PAPER B - QUESTION 17

a) LOOKING AT $\hat{ABC} = 90°$



- GRAD AB $= \frac{y_2-y_1}{x_2-x_1} = \frac{k-10}{3-5} = \frac{k-10}{-2}$

$$= \frac{10-k}{2}$$

- GRAD BC $= \frac{y_2-y_1}{x_2-x_1} = \frac{2-k}{9-3} = \frac{2-k}{6}$

- THUS $\frac{10-k}{2} = -\left(\frac{6}{2-k}\right)$ "NEGATIVE RECIPROCALS"

$$\Rightarrow \frac{10-k}{2} = \frac{-6}{2-k}$$

$$\Rightarrow (10-k)(2-k) = -12$$

$$\Rightarrow (10-k)(2-k) + 12 = 0$$

AS REQUIRED

SOLVING THE ABOVE QUADRATIC

$$\Rightarrow 20 - 10k - 2k + k^2 + 12 = 0$$

$$\Rightarrow k^2 - 12k + 32 = 0$$

$$\Rightarrow (k-4)(k-8) = 0$$

$$\therefore k = \begin{cases} 4 \\ 8 \end{cases}$$

b)(I) TRYING $k=4$ FIRST WITH $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$$|AB| = \sqrt{(4-10)^2 + (3-5)^2} = \sqrt{36+4} = \sqrt{40}$$

$$|BC| = \sqrt{(2-4)^2 + (9-3)^2} = \sqrt{4+36} = \sqrt{40}$$

IYGB - SYN PAPER B - QUESTION 17

$\therefore$ THE REQUIRED VALUE OF k IS 4 AND THE AREA OF THE SQUARE WILL BE $\sqrt{40} \times \sqrt{40} = 40$

(II) BY INSPECTION

$$A(5,10) \xrightarrow{+6,\ -2} D(11,8) \qquad\qquad B(3,4) \xrightarrow{+6,\ -2} C(9,2)$$

IYGB - SYN PAPER B - QUESTION 18

$$y = ax^3 + bx^2 + cx + d$$

DIFFERENTIATE WITH RESPECT TO x

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

LOOKING FOR STATIONARY POINTS

$$\frac{dy}{dx} = 0$$

$$3ax^2 + 2bx + c = 0$$

FOR 2 "DISTINCT ANSWERS"

"$B^2 - 4AC$" > 0

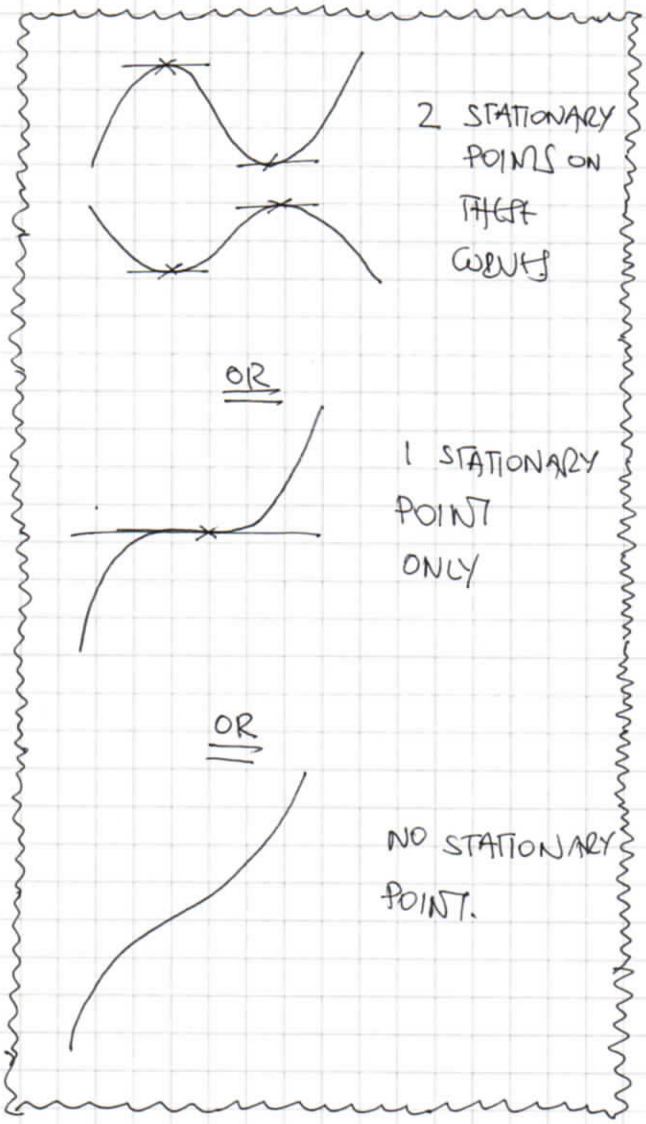
$$\Rightarrow (2b)^2 - 4(3a)c > 0$$

$$\Rightarrow 4b^2 - 12ac > 0$$

$$\Rightarrow 4b^2 > 12ac$$

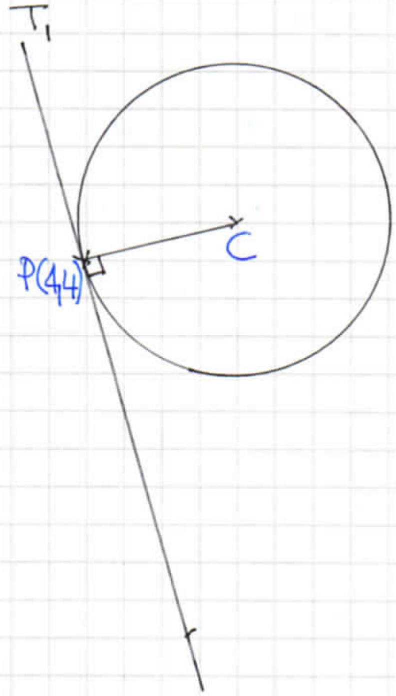
$$\Rightarrow b^2 > 3ac$$

AS REQUIRED



IYGB – SYN PAPER B – QUESTION 19

a) **OBTAIN THE PARTICULARS OF THE CIRCLE**



$\Rightarrow x^2 + y^2 - 4x - 6y + 8 = 0$

$\Rightarrow x^2 - 4x + y^2 - 6y + 8 = 0$

$\Rightarrow (x-2)^2 - 4 + (y+3)^2 - 9 + 8 = 0$

$\Rightarrow (x-2)^2 + (y-3)^2 = 5$

$\therefore$ CENTRE $C(2,3)$, $r = \sqrt{5}$

GRADIENT CP

$$m_{CP} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-4}{2-4} = \frac{-1}{-2} = \frac{1}{2}$$

EQUATION OF T_1, WITH GRADIENT -2 PASSING THROUGH $P(4,4)$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 4 = -2(x-4)$

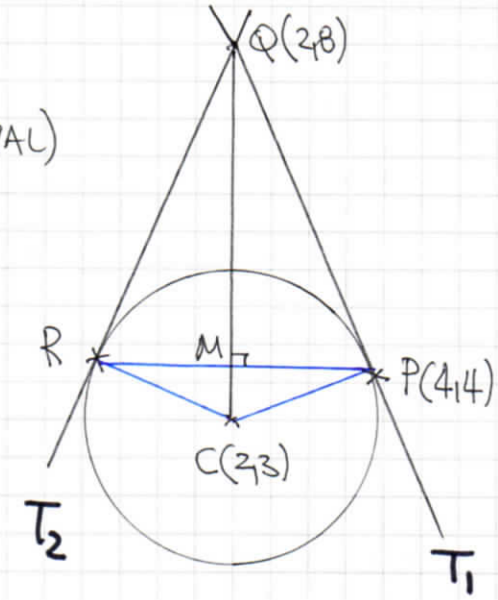
$\Rightarrow y - 4 = -2x + 8$

$\Rightarrow y = -2x + 12$ //

P.T.O

IYGB - SYN PAPER B - QUESTION 19

b) I) LOOKING AT THE DIAGRAM IT IS IMPORTANT TO NOTICE THAT C IS "VERTICALLY BELOW" Q



$\Rightarrow$ QC $\perp$ RP (RP HORIZONTAL)

&

M IS THE MIDPOINT OF RP

$\Rightarrow$ BY INSPECTION $M(2,4)$

$\Rightarrow$ BY INSPECTION $R(0,4)$

$$\left[\begin{pmatrix}4\\4\end{pmatrix} \xrightarrow[+0]{-2} \begin{pmatrix}2\\4\end{pmatrix} \xrightarrow[+0]{-2} \begin{pmatrix}0\\4\end{pmatrix}\right]$$

II) USING $R(0,4)$ & $Q(2,8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{2-0} = \frac{4}{2} = 2$$

EQUATION OF T_2 IS $y = 2x + 4$ ← FROM $(0,4)$

OR $y - y_0 = m(x - x_0)$

$$y - 4 = 2(x - 0)$$

$$y - 4 = 2x$$

$$y = 2x + 4$$

IYGB - SYNOPTIC PAPER B - QUESTION 20

a) STARTING FROM THE COMPOUND ANGLE IDENTITIES

$$\Rightarrow \left.\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \\ \sin(A-B) &= \sin A\cos B - \cos A\sin B \end{aligned}\right\} \text{ADDING}$$

$$\Rightarrow \sin(A+B) + \sin(A-B) = 2\sin A\cos B$$

NOW LET IN THE L.H.S OF THE ABOVE EXPRESSION

$$A+B = P \qquad \text{OR} \qquad P = A+B$$
$$A-B = Q \qquad\qquad\quad Q = A-B$$

ADDING THE ABOVE

$$\Rightarrow 2A = P+Q$$
$$\Rightarrow A = \frac{P+Q}{2}$$

SUBTRACTING THE ABOVE

$$2B = P-Q$$
$$B = \frac{P-Q}{2}$$

HENCE WE OBTAIN

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$$
$$\sin P + \sin Q = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

AS REQUIRED

b) USING PART (a) WITH $P=7x$ & $Q=x$

$$\Rightarrow \sin 7x + \sin x = 0$$
$$\Rightarrow 2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right) = 0$$
$$\Rightarrow \cancel{2}\sin(4x)\cos(3x) = 0$$

EITHER $\sin 4x = 0$ OR $\cos 3x = 0$

IYGB - SYNOPTIC PAPER B - QUESTION 20

$\sin 4x = 0$

$\arcsin 0 = 0$

$$\begin{cases} 4x = 0 \pm 2n\pi \\ 4x = \pi \pm 2n\pi \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\begin{cases} x = 0 \pm \frac{n\pi}{2} \\ x = \frac{\pi}{4} \pm \frac{n\pi}{2} \end{cases}$$

$\cos 3x = 0$

$\arccos 0 = \frac{\pi}{2}$

$$\begin{cases} 3x = \frac{\pi}{2} \pm 2n\pi \\ 3x = \frac{3\pi}{2} \pm 2n\pi \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\begin{cases} x = \frac{\pi}{6} \pm \frac{2n\pi}{3} \\ x = \frac{\pi}{2} \pm \frac{2n\pi}{3} \end{cases}$$

$x = 0, \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}$

$x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}$

ALTERNATIVE FOR PART (b) WITHOUT USING PART (a)

$\Longrightarrow \sin 7x + \sin x = 0$

$\Longrightarrow \sin 7x = -\sin x$

$\Longrightarrow \sin 7x = \sin(-x)$

$$\Longrightarrow \begin{cases} 7x = -x \pm 2n\pi \\ 7x = \pi - (-x) \pm 2n\pi \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\Longrightarrow \begin{cases} 8x = 0 \pm 2n\pi \\ 6x = \pi \pm 2n\pi \end{cases}$$

$$\Longrightarrow \begin{cases} x = 0 \pm \frac{n\pi}{4} \\ x = \frac{\pi}{6} \pm \frac{2n\pi}{3} \end{cases}$$

WHICH YIELDS THE SAME SOLUTIONS AS BEFORE

IYGB - SYNOPTIC PAPER B - QUESTION 21

a) EXPAND $(1+bx)^n$ IN GENERAL FORM UP TO x^3

$$(1+bx)^n = 1 + \frac{n}{1}(bx)^1 + \frac{n(n-1)}{1\times 2}(bx)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(bx)^3 + \dots$$

$$(1+bx)^n = 1 + \underbrace{nb}_{-6}x + \underbrace{\tfrac{1}{2}n(n-1)b^2}_{27}x^2 + \underbrace{\tfrac{1}{6}n(n-1)(n-2)b^3}_{\text{REQUIRED IN (b)}}x^3 + \dots$$

SOLVING SIMULTANEOUSLY

$$\begin{cases} nb = -6 \\ \frac{1}{2}n(n-1)b^2 = 27 \end{cases} \Rightarrow \begin{cases} nb = -6 \\ n(n-1)b^2 = 54 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} n^2b^2 = 36 \\ n^2(n-1)b^2 = 54n \end{cases} \Rightarrow$$

$$\Rightarrow 36(n-1) = 54n$$

$$\Rightarrow 36n - 36 = 54n$$

$$\Rightarrow -36 = 18n$$

$$n = -2$$

$$\& \quad b = 3 \qquad (nb = -6)$$

b) $[x^3]: \ \frac{1}{6}n(n-1)(n-2)b^3 = \frac{1}{6}(-2)(-3)(-4)\times 3^3 = -108$

c) VALID FOR

$$|bx| < 1$$

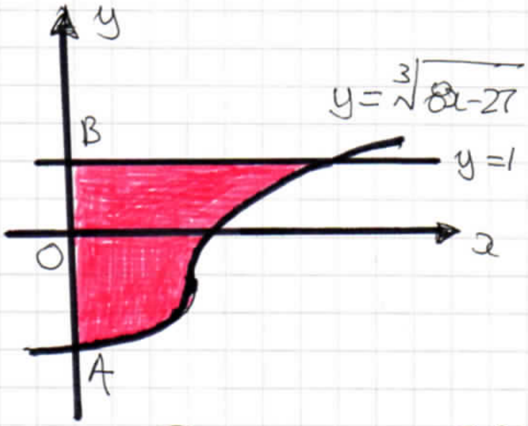
$$|3x| < 1$$

$$|x| < \frac{1}{3}$$

$$\therefore -\frac{1}{3} < x < \frac{1}{3}$$

IYGB - SYNOPTIC PAPER B - QUESTION 22

LOOKING AT THE DIAGRAM IT BECOMES OBVIOUS THAT THE INTEGRATION NEEDED IS WITH RESPECT TO y



- $x=0 \implies y=\sqrt[3]{-27}$

$\implies y=-3$

$\implies A(0,-3)$

- $\text{AREA} = \int_{y_1}^{y_2} x(y)\, dy$

$\text{AREA} = \int_{-3}^{1} \frac{1}{8}y^3+\frac{27}{8}\, dy$

$\text{AREA} = \left[\frac{1}{32}y^4+\frac{27}{8}y\right]_{-3}^{1}$

$\text{AREA} = \left(\frac{1}{32}+\frac{27}{8}\right)-\left(\frac{81}{32}-\frac{81}{8}\right)$

$\text{AREA} = \frac{109}{32}-\left(-\frac{243}{32}\right)$

$\text{AREA} = 11$

AS REQUIRED

$y=\sqrt[3]{8x-27}$

$y^3=8x-27$

$y^3+27=8x$

$x=\frac{1}{8}(y^3+27)$

$x=\frac{1}{8}y^3+\frac{27}{8}$

IYGB - SYNOPTIC PAPER B - QUESTION 23

a) $y = \arcsin 2x$

$\sin y = 2x$

$x = \frac{1}{2}\sin y$

$\frac{dx}{dy} = \frac{1}{2}\cos y$

$\frac{dy}{dx} = \frac{1}{\frac{1}{2}\cos y}$

$$\boxed{\begin{aligned} \text{NOW } \cos^2 y + \sin^2 y &= 1 \\ \cos^2 y &= 1 - \sin^2 y \\ \cos y &= \pm\sqrt{1-\sin^2 y} \end{aligned}}$$

BUT $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \Rightarrow \cos y \geq 0$

$\Rightarrow \cos y = +\sqrt{1-\sin^2 y}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1}{2}\sqrt{1-\sin^2 y}}$

BUT $\sin y = 2x$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-(2x)^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$

AS REQUIRED

IYGB - SYNOPTIC PAPER B - QUESTION 23

b) REWRITE & DIFFERENTIATE

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} = 2(1-4x^2)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}\times 2(1-4x^2)^{-\frac{3}{2}}(-8x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{8x}{(1-4x^2)^{3/2}}$$

$(A = 8)$

c) DIFFERENTIATE BY THE QUOTIENT RULE

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{(1-4x^2)^{\frac{3}{2}}\times 8 - 8x\times\frac{3}{2}(1-4x^2)^{\frac{1}{2}}\times(-8x)}{\left[(1-4x^2)^{\frac{3}{2}}\right]^2}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{8(1-4x^2)^{\frac{3}{2}} + 96x^2(1-4x^2)^{\frac{1}{2}}}{(1-4x^2)^3}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{8(1-4x^2)^{\frac{1}{2}}\left[(1-4x^2)^1 + 12x^2\right]}{(1-4x^2)^3}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{8\cancel{(1-4x^2)^{\frac{1}{2}}}(1+8x^2)}{(1-4x^2)^{\cancel{3}\ 5/2}}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{8(1+8x^2)}{(1-4x^2)^{5/2}}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{64x^2+8}{(1-4x^2)^{5/2}}$$

$B = 64$
$C = 8$

IYGB – SYN PAPER B – QUESTION 24

PROCEED BY REARRANGING AS FOLLOWS

$$\Rightarrow e^2 - e^{3x} - 1 = \left(\frac{e^{x+1}}{e^{-2x}}\right)^2$$

$$\Rightarrow e^2 - e^{3x} - 1 = \left(e^{3x+1}\right)^2$$

$$\Rightarrow e^2 - e^{3x} - 1 = e^{6x+2}$$

$$\Rightarrow e^2 - e^{6x+2} = e^{3x} + 1$$

$$\Rightarrow e^2\left(1 - e^{6x}\right) = 1 + e^{3x}$$

NOW THE L.H.S "HIDES" A DIFFERENCE OF SQUARES

$$\Rightarrow e^2\left(1 - e^{3x}\right)\left(1 + e^{3x}\right) = 1 + e^{3x}$$

$$\Rightarrow e^2\left(1 - e^{3x}\right) = 1$$

AS $e^{3x} + 1 \neq 0$

$$\Rightarrow 1 - e^{3x} = \frac{1}{e^2}$$

$$\Rightarrow 1 - \frac{1}{e^2} = e^{3x}$$

$$\Rightarrow 3x = \ln\left(1 - e^{-2}\right)$$

$$\Rightarrow x = \frac{1}{3}\ln\left(1 - e^{-2}\right)$$