IYGB - SYNOPTIC PAPER C - QUESTION 1

a) USING STANDARD SEQUENCE/SERIES FORMULA $u_n = a + (n-1)d$

- $u_6 = 23$

$a + 5d = 23$

$a = 23 - 5d$

- $u_{15} = 50$

$a + 14d = 50$

$a = 50 - 14d$

$$23 - 5d = 50 - 14d$$

$$9d = 27$$

$$d = 3 \quad \& \quad a = 8$$

∴ THE FIRST ROW HAS 8 SEATS

b) SUMMING UP THE FIRST 20 TERMS OF AN A.P WITH $a=8$, $d=3$

$$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{20} = \frac{20}{2}\left[2 \times 8 + 19 \times 3\right]$$

$$\Rightarrow S_{20} = 10\left[16 + 57\right]$$

$$\Rightarrow S_{20} = 730$$

∴ A TOTAL OF 730 SEATS

IYGB - SYNOPTIC PAPER C - QUESTION 2

a) SETTING UP INTEGRATION BY PARTS

x	1
$2\sin\frac{1}{2}x$	$\cos\frac{1}{2}x$

$$\int x\cos\tfrac{1}{2}x \, dx = \dots$$

$$\dots = 2x\sin\tfrac{1}{2}x - \int 2\sin\tfrac{1}{2}x \, dx$$

$$= 2x\sin\tfrac{1}{2}x + 4\cos\tfrac{1}{2}x + C$$

b) USING INTEGRATION BY PARTS & PART (a)

x^2	$2x$
$-2\cos\frac{1}{2}x$	$\sin\frac{1}{2}x$

$$\int x^2\sin\tfrac{1}{2}x \, dx = \dots$$

$$= -2x^2\cos\tfrac{1}{2}x - \int -4x\cos\tfrac{1}{2}x \, dx$$

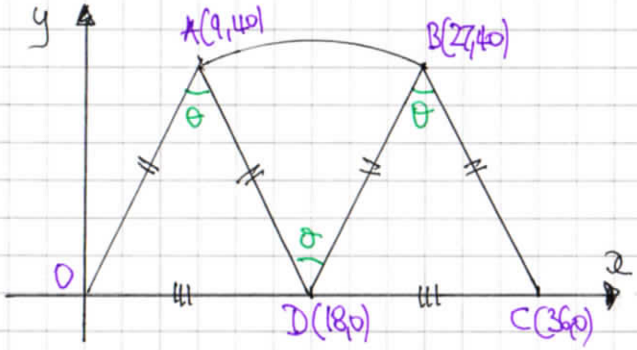
$$= -2x^2\cos\tfrac{1}{2}x + 4\int x\cos\tfrac{1}{2}x \, dx$$

$$= -2x^2\cos\tfrac{1}{2}x + 4\left[2x\sin\tfrac{1}{2}x + 4\cos\tfrac{1}{2}x\right] + C$$

$$= -2x^2\cos\tfrac{1}{2}x + 8x\sin\tfrac{1}{2}x + 16\cos\tfrac{1}{2}x + C$$

-|-

IYGB - SYNOPTIC PAPER C - QUESTION 3



a) $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$|AD| = \sqrt{(40-0)^2+(18-9)^2}$

$|AD| = \sqrt{40^2+9^2}$

$|AD| = 41\text{ m}$

b) BY THE COSINE RULE ON $\hat{OAD}$

$\hat{ADB} = \hat{OAD} = \hat{DBC}$

$18^2 = 41^2+41^2-2\times41\times41\times\cos\theta$

$324 = 1681+1681-3362\cos\theta$

$3362\cos\theta = 3038$

$\cos\theta = \frac{1519}{1681}$

$\theta \simeq 0.4426^c$

c) AREA OF THE SECTOR $\hat{DAB}$

$\text{AREA} = \text{"}\frac{1}{2}r^2\theta^c\text{"}$

$\text{AREA} = \frac{1}{2}\times41^2\times0.4426$

$\text{AREA} = 372.0295776\ldots$

AREA OF EACH OF THE TRIANGLES

$\text{AREA} = \frac{1}{2}\times41\times41\times\sin(0.4426^c)$

$\text{AREA} = 360$

$\text{TOTAL AREA} = 2\times360+372.029\ldots$

$= 1092\text{ m}^2$

IYGB - SYNOPTIC PAPER C - QUESTION 4

USING THE GIVEN SUBSTITUTION

$$\int x^5\sqrt{1-x^3}\,dx$$

$$= \int x^5 \times t \times \left(\frac{2t}{-3x^2}\,dt\right)$$

$$= \int -\frac{2}{3}\left(t^2x^3\right)\,dt$$

$$= \int -\frac{2}{3}t^2\left(1-t^2\right)\,dt$$

$$= -\frac{2}{3}\int t^2 - t^4\,dt$$

$$= -\frac{2}{3}\left[\frac{1}{3}t^3 - \frac{1}{5}t^5\right] + C$$

$$= -\frac{2}{3}\times\frac{1}{15}\left[5t^3 - 3t^5\right] + C$$

$$= -\frac{2}{45}\left[5t^3 - 3t^5\right] + C$$

$$= -\frac{2}{45}t^3\left(5-3t^2\right) + C$$

$$= -\frac{2}{45}\left(1-x^3\right)^{\frac{3}{2}}\left[5-3\left(1-x^3\right)\right] + C$$

$$= -\frac{2}{45}\left(1-x^3\right)^{\frac{3}{2}}\left(2+3x^3\right) + C$$

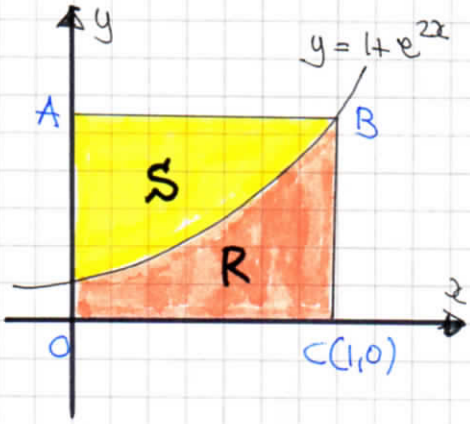
$$= -\frac{2}{45}\left(3x^3+2\right)\left(1-x^3\right)^{\frac{3}{2}} + C$$

AS REQUIRED

$$t = \sqrt{1-x^3}$$
$$t^2 = 1-x^3$$
$$2t\frac{dt}{dx} = -3x^2$$
$$2t\,dt = -3x^2\,dx$$
$$dx = \frac{2t}{-3x^2}\,dt$$
$$x^3 = 1-t^2$$

IYGB, SYNOPTIC PAPER C - QUESTION 5

WORKING AT THE DIAGRAM



- WHEN $x=1$, $y = 1+e^2$

  I.E $B(1, 1+e^2)$

- AREA OF THE RECTANGLE OABC

  $$1 \times (1+e^2) = 1+e^2$$

NEXT FIND BY INTEGRATION THE AREA OF R

$$R = \int_0^1 1+e^{2x}\,dx = \left[x+\tfrac{1}{2}e^{2x}\right]_0^1 = \left(1+\tfrac{1}{2}e^2\right) - \left(0+\tfrac{1}{2}\right)$$

$$= \tfrac{1}{2}+\tfrac{1}{2}e^2 = \tfrac{1}{2}(1+e^2)$$

I.E HALF THE AREA OF THE RECTANGLE

$\therefore$ AREA R = AREA S

IYGB - SYN PAPER C - QUESTION 6

a) USING THE INFORMATION GIVEN

$$S_{\infty} = \frac{a}{1-r} \implies 1600 = \frac{1200}{1-r}$$

$$\implies 1600(1-r) = 1200$$

$$\implies 1-r = \frac{3}{4}$$

$$\implies \frac{1}{4} = r$$

SUMMING THE FIRST FIVE TERMS

$$S_n = \frac{a(1-r^n)}{1-r} \implies S_5 = \frac{1200(1-0.25^5)}{1-0.25}$$

$$\implies S_5 = 1600 \times \frac{1023}{1024}$$

$$\implies S_5 = 1598.4375 \approx 1598$$

b) PROCEED AS FOLLOWS

$$\sum_{r=6}^{\infty} u_r = u_6 + u_7 + u_8 + \dots$$

$$= (u_1+u_2+u_3+u_4+\dots) - (u_1+u_2+u_3+u_4+u_5)$$

$$= S_{\infty} - S_5$$

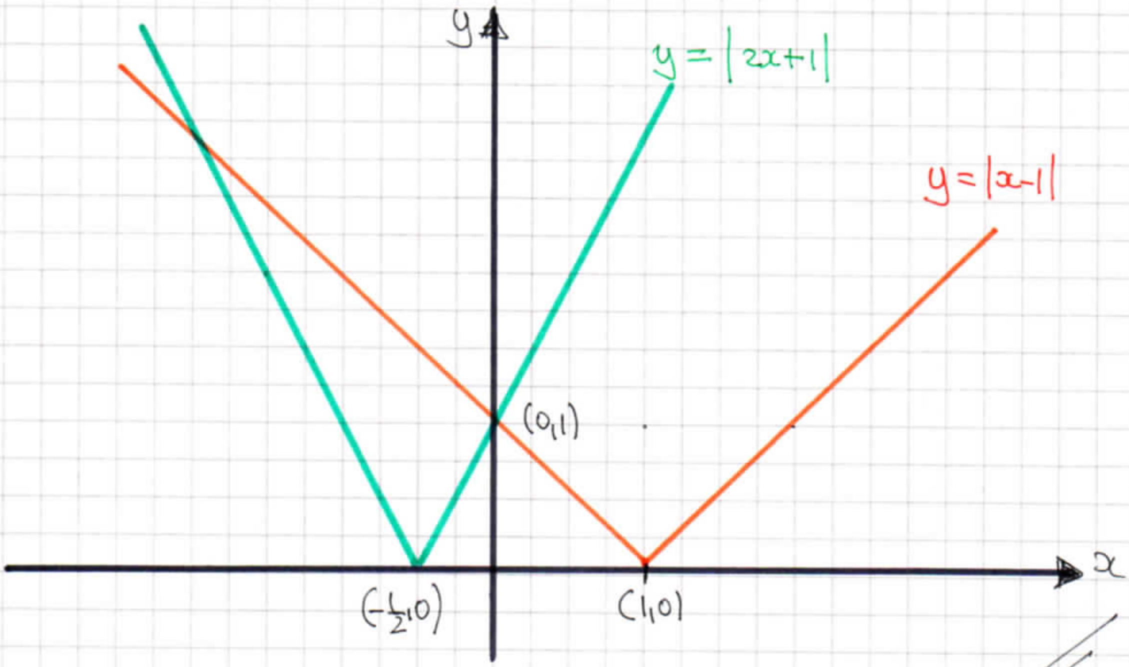
$$= 1600 - 1598.4375$$

$$= \frac{25}{16}$$

$$= 1.5625$$

IYGB - SYNOPTIC PAPER C - QUESTION 7

a)



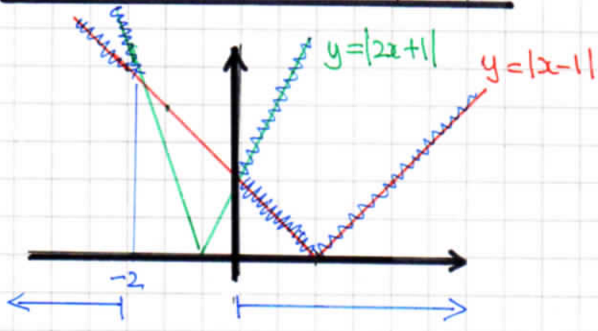
b) FINDING THE INTERSECTIONS ABOVE

$$\left.\begin{array}{l} y = |2x+1| \\ y = |x-1| \end{array}\right\} \Rightarrow |2x+1| = |x-1|$$

$$\Rightarrow 2x+1 = \left\langle \begin{array}{l} x-1 \\ 1-x \end{array}\right.$$

$$\Rightarrow x = \left\langle \begin{array}{l} -2 \\ 0 \end{array}\right.$$

LOOKING AT THE DIAGRAM



$x < -2$ OR $x > 0$

IYGB - SYNOPTIC PAPER C - QUESTION 8

START BY DIFFERENTIATION

$$y = \frac{1}{4}e^{2x-3} - 4\ln\left(\frac{x}{2}\right) = \frac{1}{4}e^{2x-3} - 4(\ln x - \ln 2)$$

$$\frac{dy}{dx} = \frac{1}{2}e^{2x-3} - \frac{4}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{2}e^{1} - 2 = \frac{1}{2}e - 2$$

FIND THE EQUATION OF THE TANGENT - FIND THE y CO-ORDINATE OF THE POINT OF TANGENCY

$$x=2, \quad y = \frac{1}{4}e^{1} - \cancel{4\ln 1} \quad \text{IE} \quad \left(2, \frac{1}{4}e\right)$$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - \frac{1}{4}e = \left(\frac{1}{2}e - 2\right)(x-2)$$

NEXT FIND THE CO-ORDINATES OF A & B

- WHEN $x=0$

$$\Rightarrow y - \frac{1}{4}e = \left(\frac{1}{2}e-2\right)(-2)$$

$$\Rightarrow y - \frac{1}{4}e = -e + 4$$

$$\Rightarrow y = 4 - \frac{3}{4}e$$

$$\Rightarrow y = \frac{16-3e}{4}$$

- WHEN $y=0$

$$\Rightarrow 0 - \frac{1}{4}e = \left(\frac{1}{2}e-2\right)(x-2)$$

$$\Rightarrow -\frac{1}{4}e = \left(\frac{1}{2}e-2\right)x - 2\left(\frac{1}{2}e-2\right)$$

$$\Rightarrow -\frac{1}{4}e = 4 - e + \left(\frac{1}{2}e-2\right)x$$

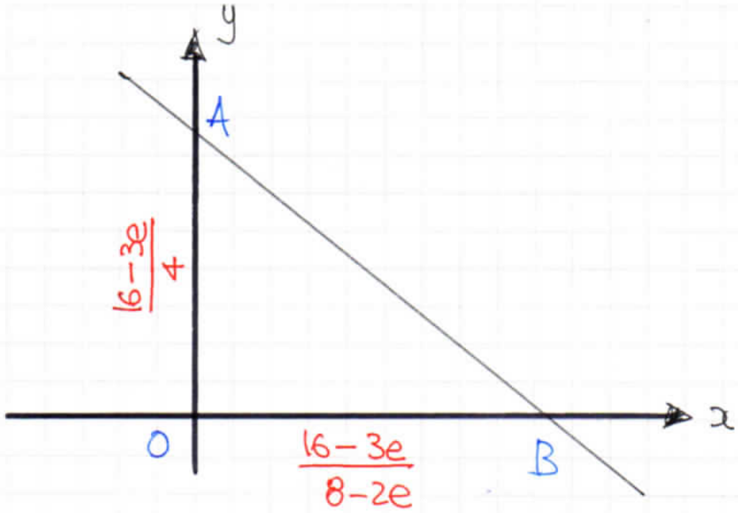
$$\Rightarrow \frac{3}{4}e - 4 = \left(\frac{1}{2}e-2\right)x \quad \times 4$$

$$\Rightarrow 3e - 16 = (2e-8)x$$

$$\Rightarrow x = \frac{3e-16}{2e-8} = \frac{16-3e}{8-2e}$$

IYGB - SYNOPTIC PAPER C - QUESTION 8

FINALLY LOOKING AT THE DIAGRAM



$$\Longrightarrow \text{AREA } \overset{\triangle}{OAB} = \frac{1}{2}|OA||OB|$$

$$= \frac{1}{2} \times \frac{16-3e}{4} \times \frac{16-3e}{8-2e}$$

$$= \frac{(16-3e)^2}{16(4-e)}$$

AS REQUIRED

IYGB - SYN PAPER C - QUESTION 9

$$y^3 - y^2 = e^x \quad , \quad \frac{dy}{dx} = \frac{6}{5}$$

- START BY REARRANGING THE EQUATION

$$\Rightarrow \ln(y^3 - y^2) = x$$

$$\Rightarrow \frac{dx}{dy} = \frac{3y^2 - 2y}{y^3 - y^2}$$

- LOOKING AT THE EQUATION $y \neq 0$, SO WE MAY DIVIDE THROUGH

$$\Rightarrow \frac{dx}{dy} = \frac{3y - 2}{y^2 - y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - y}{3y - 2}$$

- SETTING $\frac{dy}{dx} = \frac{6}{5}$

$$\Rightarrow \frac{y^2 - y}{3y - 2} = \frac{6}{5}$$

$$\Rightarrow 5y^2 - 5y = 18y - 12$$

$$\Rightarrow 5y^2 - 23y + 12 = 0$$

$$\Rightarrow (5y - 3)(y - 4)$$

$$\Rightarrow y = \begin{cases} 4 \\ \frac{3}{5} \end{cases}$$

IYGB – SYN PAPER C – QUESTION 9

HENCE WE NOW HAVE

- $y = 4 \implies x = \ln(4^3 - 4^2)$

  $\implies x = \ln(64 - 16)$

  $\implies x = \ln 48$

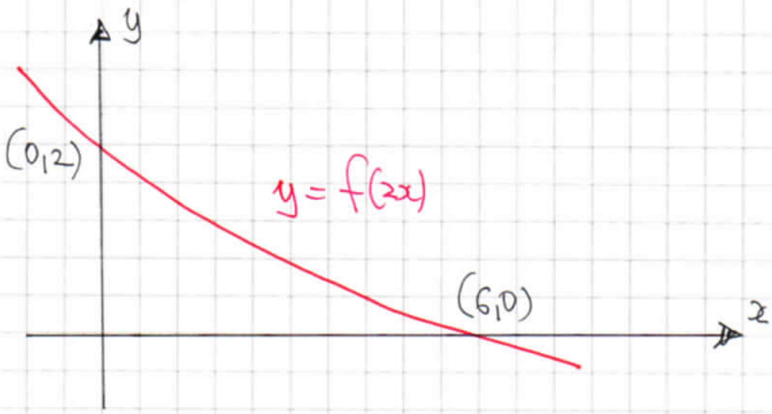
- $y = \frac{3}{5} \implies x = \ln\left[\left(\frac{3}{5}\right)^3 - \left(\frac{3}{5}\right)^2\right]$

  $\implies x = \cancel{\ln\left[\frac{27}{125} - \frac{9}{25}\right]} < 0$

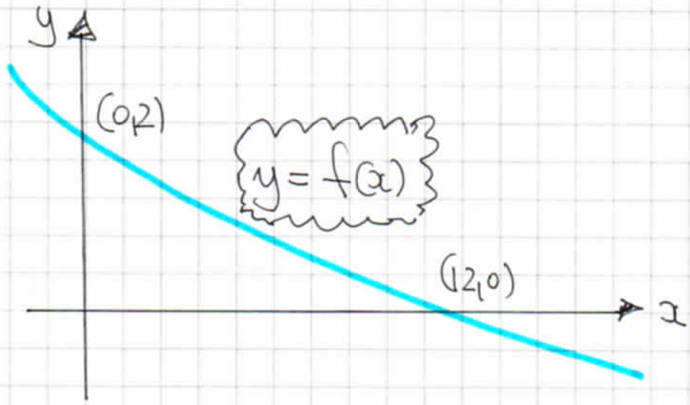
HENCE THE ONLY POINT IS $P(\ln 48, 4)$

IYGB - SYN PAPER C - QUESTION 10

a)



- f(2x) REPRESENTS A HORIZONTAL STRETCH OF SCALE FACTOR $\frac{1}{2}$ (HALVING x CO.ORDS)
- REVERSING THE TRANSFORMATION WE DOUBLE x CO.ORDS



b)

$$f(x) \longmapsto f(x-1) \longmapsto f((4x)-1)$$
$$f(4x-1)$$

REPLACE x FOR x−1

TRANSLATION, "RIGHT", BY ONE UNIT

REPLACE x FOR 4x

HORIZONTAL STRETCH, BY SCALE FACTOR $\frac{1}{4}$

ALTERNATIVE (NOT SO NATURAL)

$$f(x) \longmapsto f(4x) \longmapsto f\left(4\left(x-\tfrac{1}{4}\right)\right)$$

REPLACE x FOR 4x

HORIZONTAL STRETCH, BY SCALE FACTOR $\frac{1}{4}$

TRANSLATION, "RIGHT", BY $\frac{1}{4}$ UNIT

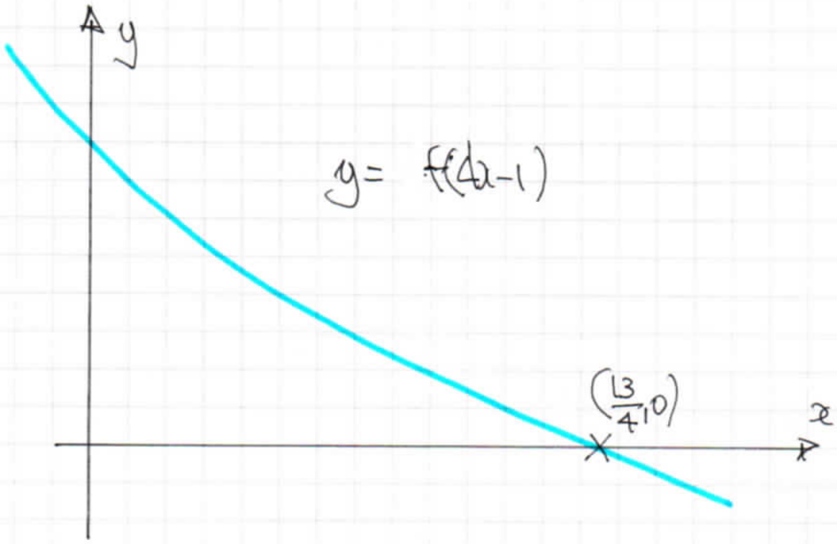
IYGB - SYN PAPER C - QUESTION 10

- NOTING THAT ONLY THE x INTERCEPT IS NEEDED

$f(x)$	$f(x-1)$	$f(4x-1)$
$(12,0)$	$(13,0)$	$(\frac{13}{4},0)$

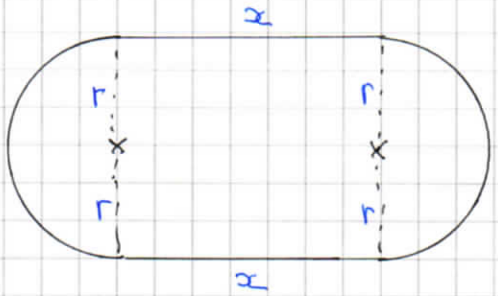
OR BY THE SECOND APPROACH

$f(x)$	$f(4x)$	$f(4x-1)$
$(12,0)$	$(3,0)$	$(3\frac{1}{4},0)$



IYGB – SYNOPTIC PAPER C – QUESTION 11

a)



"TRACK = 400 m"

$P = 400$

$2x + 2\pi r = 400$

$2xr + 2\pi r^2 = 400r$ $\times r$

$2xr = 400r - 2\pi r^2$

AREA $= 2xr + \pi r^2$

$A = (400r - 2\pi r^2) + \pi r^2$

$A = 400r - \pi r^2$

AS REQUIRED

b)

DIFFERENTIATE & SOLVE FOR ZERO

$$\frac{dA}{dr} = 400 - 2\pi r$$

FOR MIN|MAX $\frac{dA}{dr} = 0$

$0 = 400 - 2\pi r$

$2\pi r = 400$

$r = \frac{200}{\pi}$

$(\simeq 63.66)$

c)

USING THE SECOND DERIVATIVE

$$\frac{d^2A}{dr^2} = -2\pi$$

$$\left.\frac{d^2A}{dr^2}\right|_{r=\frac{200}{\pi}} = -2\pi < 0$$

INDEED $r = \frac{200}{\pi}$ MAXIMIZES A

IYGB - SYNOPTIC PAPER C - QUESTION 11

d) $A = 400r - \pi r^2$

$$A_{MAX} = 400\left(\frac{200}{\pi}\right) - \pi\left(\frac{200}{\pi}\right)^2$$

$$A_{MAX} = \frac{80000}{\pi} - \cancel{\pi}\left(\frac{40000}{\pi^{\cancel{2}}}\right)$$

$$A_{MAX} = \frac{80000}{\pi} - \frac{40000}{\pi}$$

$$A_{MAX} = \frac{40000}{\pi}$$

// AS REQUIRED

e) RETURNING TO THE CONSTRAINT WITH $r = \frac{200}{\pi}$

$$2x + 2\pi r = 400$$

$$x + \pi r = 200$$

$$x + \cancel{\pi}\left(\frac{200}{\cancel{\pi}}\right) = 200$$

$$x = 0!$$

NOT SUITABLE AS THE RESULTING TRACK, THOUGH IT WILL ENCLOSE A MAXIMUM AREA, WILL BE A CIRCLE //

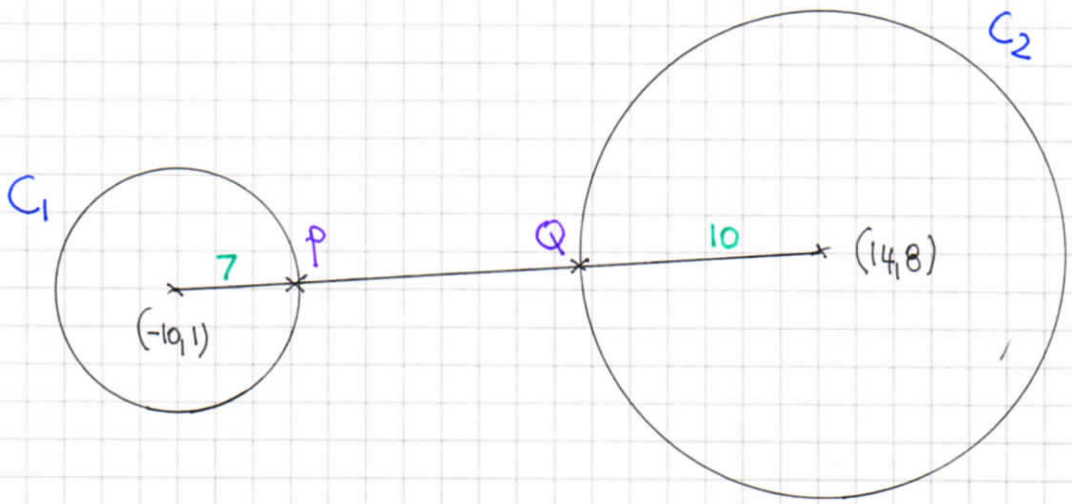
IYGB – SYN PAPER C – QUESTION 12

a) OBTAIN THE CENTRE & RADIUS BY COMPLETING THE SQUARE

$$\Rightarrow x^2+y^2+20x-2y+52=0$$
$$\Rightarrow x^2+20x+y^2-2y+52=0$$
$$\Rightarrow (x+10)^2-100+(y-1)^2-1+52=0$$
$$\Rightarrow (x+10)^2+(y-1)^2=49$$

$\therefore$ CENTRE AT $(-10,1)$ & $r=7$

b) ASSUMING THE TWO CIRCLES DO NOT MEET, BECAUSE THEN THE SHORTEST DISTANCE MUST BE ZERO



FIND THE DISTANCE BETWEEN CENTRES

$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(1-8)^2+(-10-14)^2}$$
$$=\sqrt{49+576}=25$$

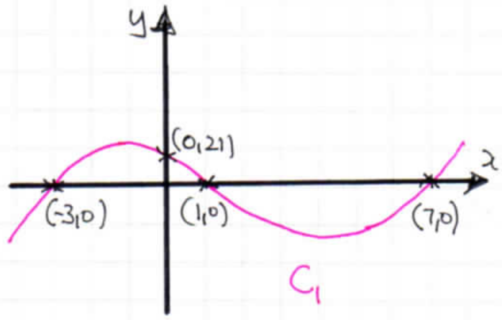
$\therefore$ REQUIRED DISTANCE $=|PQ|_{MIN}=25-7-10=8$

IYGB - SYN PAPER - QUESTION 13

a) • C_1 : $y = (x-7)(x^2+2x-3)$

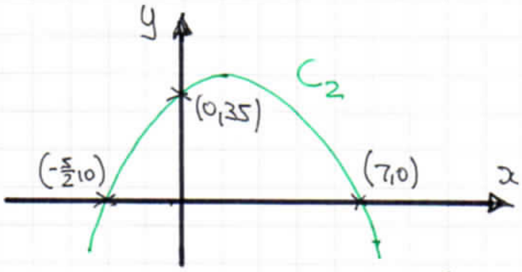
$y = (x-7)(x-1)(x+3)$

$(-3,0)$, $(1,0)$, $(7,0)$, $(0,21)$



• C_2 : $y = (2x+5)(7-x)$

$(-\frac{5}{2},0)$, $(7,0)$, $(0,35)$



b) SOLVING AS FOLLOWS

$\Rightarrow (x-7)(x^2+2x-3) = (2x+5)(7-x)$

$\Rightarrow (x-7)(x^2+2x-3) - (2x+5)(7-x) = 0$

$\Rightarrow (x-7)(x^2+2x-3) + (2x+5)(x-7) = 0$

$\Rightarrow (x-7)\left[(x^2+2x-3)+(2x+5)\right] = 0$

$\Rightarrow (x-7)(x^2+4x+2) = 0$

$\Rightarrow (x-7)\left[(x+2)^2-4+2\right] = 0$

$\Rightarrow (x-7)\left[(x+2)^2-(\sqrt{2})^2\right] = 0$

$\Rightarrow (x-7)(x+2-\sqrt{2})(x+2+\sqrt{2}) = 0$

$\Rightarrow x = \begin{cases} 7 \\ -2+\sqrt{2} \\ -2-\sqrt{2} \end{cases}$

IYGB - SYN PAPER C - QUESTION 14

PROCEED AS FOLLOWS AS THE x CO-ORDINATE IS NOT REQUIRED

- $y = 2 - 3e^{x}$

$$3e^{x} = 2 - y$$

- $y = 1 + e^{x+1} + e^{x-1}$

$$y = 1 + e^{x} \times e + e^{x} \times e^{-1}$$

$$3y = 3e^{x} \times e + 3e^{x} \times \frac{1}{e} + 3$$

$$3y = (2-y)e + (2-y) \times \frac{1}{e} + 3$$

$$3ey = e^{2}(2-y) + 2 - y + 3e$$

$$3ey = 2e^{2} - e^{2}y + 2 - y + 3e$$

$$3ey + e^{2}y + y = 2e^{2} + 3e + 2$$

$$y(e^{2} + 3e + 1) = 2e^{2} + 3e + 2$$

$$y = \frac{2e^{2} + 3e + 2}{e^{2} + 3e + 1}$$

IYGB - SYNOPTIC PAPER C - QUESTION 15

SOLVING THE EQUATION AS FOLLOWS

$$\Rightarrow \sin\left(\arcsin\tfrac{1}{4} + \arccos x\right) = 1$$

$$\Rightarrow \arcsin\left[\sin\left(\arcsin\tfrac{1}{4} + \arccos x\right)\right] = \arcsin 1 \pm 2n\pi$$

$(n = 0, 1, 2, 3)$

$$\Rightarrow \arcsin\tfrac{1}{4} + \arccos x = \tfrac{\pi}{2} \pm 2n\pi$$

$$\Rightarrow \arccos x = \tfrac{\pi}{2} - \arcsin\tfrac{1}{4} \pm 2n\pi$$

BUT $\arccos x$ CAN ONLY RETURN VALUES BETWEEN 0 & π

$$\Rightarrow \arccos x = \tfrac{\pi}{2} - \arcsin\tfrac{1}{4}$$

$$\Rightarrow x = \cos\left(\tfrac{\pi}{2} - \arcsin\tfrac{1}{4}\right)$$

BUT $\cos\left(\frac{\pi}{2} - \theta\right) \equiv \sin\theta$

$$\Rightarrow x = \sin\left(\arcsin\tfrac{1}{4}\right)$$

$$\Rightarrow x = \tfrac{1}{4}$$

IYGB - SYNOPTIC PAPER C - QUESTION 16

AS THE SECTIONS ARE BOTH POLYNOMIALS THE ONLY PLACE WHERE DISCONTINUITY AND LACK OF SMOOTHNESS CAN OCCUR IS AT $x = 2$

CONTINUITY AT $x = 2$

$$ax^3 + 2 = bx^2 - 2$$
$$8a + 2 = 4b - 2$$
$$8a - 4b = -4$$
$$2a - b = -1$$

SMOOTHNESS AT $x = 2$

$$\frac{d}{dx}(ax^3+2) = \frac{d}{dx}(bx^2-2)$$
$$3ax^2 = 2bx$$
$$12a = 4b$$
$$b = 3a$$

SOLVING YIELDS

$$2a - (3a) = -1$$
$$-a = -1$$
$$a = 1 \qquad \& \qquad b = 3$$

IYGB - SYN PAPER C - QUESTION 17

PROCEED AS FOLLOWS

$$\Rightarrow \begin{cases} 3\log_8(xy) = 4\log_2 x \\ \log_2 y = 1 + \log_2 x \end{cases}$$

CHANGE OF BASE RULE

$$\log_a x \equiv \frac{\log_b x}{\log_b a}$$

$$\Rightarrow \begin{cases} 3\,\frac{\log_2(xy)}{\log_2 8} = 4\log_2 x \\ \log_2 y = \log_2 2 + \log_2 x \end{cases}$$

$$\Rightarrow \begin{cases} \cancel{3}\left[\frac{\log_2(xy)}{\cancel{3}\cancel{\log_2 2}}\right] = \log_2 x^4 \\ \log_2 y = \log_2(2x) \end{cases}$$

$$\left[\log_2 8 = \log_2 2^3 = 3\log_2 2\right]$$

$$\Rightarrow \begin{cases} \log_2(xy) = \log_2 x^4 \\ \log_2 y = \log_2 2x \end{cases}$$

$$\Rightarrow \begin{cases} xy = x^4 \\ y = 2x \end{cases}$$

SOLVE BY DIVIDING OR SUBSTITUTING

$$\Rightarrow x = \frac{x^3}{2}$$

$$\Rightarrow 2x = x^3$$

$$\Rightarrow 0 = x^3 - 2x$$

$$\Rightarrow 0 = x(x^2 - 2)$$

$$\Rightarrow 0 = x(x - \sqrt{2})(x + \sqrt{2})$$

IYGB - SYN PAPER C - QUESTION 17

$$x = \begin{cases} \cancel{0} \\ \sqrt{2} \\ \cancel{-\sqrt{2}} \end{cases} \qquad y = 2x = 2\sqrt{2}$$

$$\therefore\ (x,y) = (\sqrt{2}, 2\sqrt{2})$$

ALTERNATIVE / VARIATION

$$\Longrightarrow \begin{cases} 3\log_8(xy) = 4\log_2 x \\ \log_2 y = 1 + \log_2 x \end{cases}$$

CHANGING THE BASE AS BEFORE

$$\log_a x \equiv \frac{\log_b x}{\log_b a}$$

$$\log_8(xy) = \frac{\log_2(xy)}{\log_2 8} = \frac{\log_2(xy)}{\log_2 2^3} = \frac{\log_2(xy)}{3\cancel{\log_2 2}} = \frac{1}{3}\log_2(xy)$$

$$\Longrightarrow \begin{cases} \cancel{3} \times \frac{1}{\cancel{3}}\log_2(xy) = 4\log_2 x \\ \log_2 y = 1 + \log_2 x \end{cases}$$

$$\Longrightarrow \begin{cases} \log_2 x + \log_2 y = 4\log_2 x \\ \log_2 y = 1 + \log_2 x \end{cases}$$

$$\Longrightarrow \begin{cases} X + Y = 4X \\ Y = 1 + X \end{cases}$$

$$\Longrightarrow \begin{cases} Y = 3X \\ Y = X + 1 \end{cases}$$

$$\Longrightarrow 3X = X + 1$$

$$\Longrightarrow X = \frac{1}{2} \ \& \ Y = \frac{3}{2}$$

- $\log_2 x = \frac{1}{2}$
  $\log_2 x = \frac{1}{2}\log_2 2$
  $\log_2 x = \log_2 2^{\frac{1}{2}}$
  $x = 2^{\frac{1}{2}} = \sqrt{2}$

- $\log_2 y = \frac{3}{2}$
  $\log_2 y = \frac{3}{2}\log_2 2$
  $\log_2 y = \log_2 2^{\frac{3}{2}}$
  $y = 2^{\frac{3}{2}} = 2\sqrt{2}$

AS BEFORE

IYGB - SYNOPTIC PAPER C - QUESTION 18

a) ADD THE R.H.S & COMPARE

$$\frac{1}{t(t^2+1)} \equiv \frac{At+B}{t^2+1} + \frac{C}{t}$$

$$\Rightarrow \frac{1}{t(t^2+1)} \equiv \frac{t(At+B)+C(t^2+1)}{(t^2+1)t}$$

$$\Rightarrow \boxed{1 \equiv At^2 + Bt + Ct^2 + C}$$

$C=1 \quad B=0 \quad A+C=0$

$A=-1$

b) SOLVING BY SEPARATION OF VARIABLES

$$\Rightarrow \frac{dm}{dt} = \frac{m}{t(t^2+1)}$$

$$\Rightarrow \frac{1}{m}\,dm = \frac{1}{t(t^2+1)}\,dt$$

$$\Rightarrow \int \frac{1}{m}\,dm = \int \frac{-t}{t^2+1} + \frac{1}{t}\,dt$$

$\times 2$

$$\Rightarrow \int \frac{2}{m}\,dm = \int \frac{2}{t} - \frac{2t}{t^2+1}\,dt$$

$$\Rightarrow 2\ln m = 2\ln t - \ln(t^2+1) + \ln A$$

$$\frac{d}{dt}\left[\ln(t^2+1)\right] = \frac{1}{t^2+1}\times 2t$$

$$\Rightarrow \ln m^2 = \ln t^2 - \ln(t^2+1) + \ln A$$

$$\Rightarrow \ln m^2 = \ln\left(\frac{At^2}{t^2+1}\right)$$

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$$\Longrightarrow m^2 = \frac{At^2}{t^2+1}$$

$$\Longrightarrow m = \frac{kt}{\sqrt{t^2+1}} \qquad (m \geqslant 0)$$

c) APPLY THE CONDITION $t=2$, $m=10$

$$\Longrightarrow 10 = \frac{k \times 2}{\sqrt{5}}$$

$$\Longrightarrow k = 5\sqrt{5}$$

$$\Longrightarrow \boxed{m = \frac{5\sqrt{5}\,t}{\sqrt{t^2+1}}}$$

WHEN $t=4$

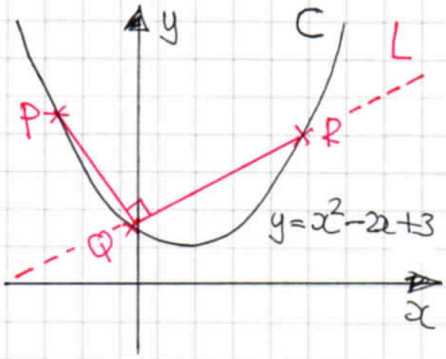
$$\Longrightarrow m = \frac{5\sqrt{5} \times 4}{\sqrt{17}} = 20\sqrt{\frac{5}{17}} \approx 10.85$$

d) LOOKING AT THE SOLUTION

$$m = \frac{5\sqrt{5}\,t}{\sqrt{t^2+1}} \qquad \text{AS } t \to \infty \qquad \frac{t}{\sqrt{t^2+1}} \longrightarrow 1$$

$$m \longrightarrow 5\sqrt{5}$$

IYGB – SYN PAPER C – QUESTION 19



START BY FINDING THE GRADIENT OF PQ

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{0-(-1)} = \frac{-3}{1} = -3$$

EQUATION OF L, LINE THROUGH Q & R

GRAD OF $L = +\frac{1}{3}$ (PERPENDICULAR TO PQ)

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 3 = \frac{1}{3}(x - 0)$$

$$\Rightarrow y = \frac{1}{3}x + 3$$

TO FIND R, WE NEED TO SOLVE SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$$\left.\begin{array}{l} C:\ y = x^2 - 2x + 3 \\ L:\ y = \frac{1}{3}x + 3 \end{array}\right\} \Rightarrow x^2 - 2x + \cancel{3} = \frac{1}{3}x + \cancel{3}$$

$$\Rightarrow x^2 - 2x = \frac{1}{3}x$$

$$\Rightarrow 3x^2 - 6x = x$$

$$\Rightarrow 3x^2 - 7x = 0$$

$$\Rightarrow x(3x - 7) = 0$$

$$\Rightarrow x = \left\langle \begin{array}{l} 0 \leftarrow Q \\ \frac{7}{3} \leftarrow P \end{array}\right.$$

$$\Rightarrow y = \frac{1}{3}\left(\frac{7}{3}\right) + 3 = \frac{7}{9} + \frac{27}{9} = \frac{34}{9}$$

$$\therefore R\left(\frac{7}{3}, \frac{34}{9}\right)$$

IYGB - SYN PAPER C - QUESTION 20

$$f(x) = 4^{ax+b}, \quad x \in \mathbb{R}$$

- **USING** $f\left(\frac{2}{3}\right) = \frac{1}{4}\sqrt[3]{4}$

$$\Longrightarrow 4^{a \times \frac{2}{3} + b} = \frac{1}{4}\sqrt[3]{4}$$

$$\Longrightarrow 4^{\frac{2}{3}a + b} = 4^{-1} \times 4^{\frac{1}{3}}$$

$$\Longrightarrow 4^{\frac{2}{3}a + b} = 4^{-\frac{2}{3}}$$

$$\Longrightarrow \frac{2}{3}a + b = -\frac{2}{3}$$

$$\Longrightarrow \boxed{b = -\frac{2}{3}a - \frac{2}{3}}$$

- **USING** $f\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{2}$

$$\Longrightarrow 4^{a \times \frac{3}{2} + b} = \frac{1}{2}\sqrt{2}$$

$$\Longrightarrow 4^{\frac{3}{2}a + b} = 2^{-1} \times 2^{\frac{1}{2}}$$

$$\Longrightarrow 4^{\frac{3}{2}a + b} = 2^{-\frac{1}{2}}$$

$$\Longrightarrow \left(2^2\right)^{\frac{3}{2}a + b} = 2^{-\frac{1}{2}}$$

$$\Longrightarrow 2^{3a + 2b} = 2^{-\frac{1}{2}}$$

$$\Longrightarrow \boxed{3a + 2b = -\frac{1}{2}}$$

- **SOLVING THE EQUATIONS SIMULTANEOUSLY GIVES**

$$\Longrightarrow 3a + 2\left(-\frac{2}{3}a - \frac{2}{3}\right) = -\frac{1}{2}$$

$$\Longrightarrow 3a - \frac{4}{3}a - \frac{4}{3} = -\frac{1}{2}$$

$$\Longrightarrow \frac{5}{3}a = \frac{5}{6}$$

$$\Longrightarrow \frac{1}{3}a = \frac{1}{6}$$

$$\Longrightarrow a = \frac{1}{2}$$

$$\& \quad b = -\frac{2}{3}\left(\frac{1}{2}\right) - \frac{2}{3} = -\frac{1}{3} - \frac{2}{3} = -1$$

$$\Longrightarrow b = -1$$

IYGB – SYNOPTIC PAPER C – QUESTION 21

a) TO FIND THE GRADIENT FUNCTION

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{6}(2\sin 2t)}{\cancel{6}(2-2\cos 2t)} = \frac{\sin 2t}{1-\cos 2t}$$

$$= \frac{2\sin t\cos t}{1-(1-2\sin^2 t)} = \frac{\cancel{2}\sin t\cos t}{\cancel{2}\sin^2 t} = \frac{\cos t}{\sin t} = \cot x$$

b) USING THE PARAMETRIC EQUATIONS WITH $t=0$ & $t=\pi$

- $t=0$ $\quad x=0,\ y=0$
- $t=\pi$ $\quad x=12\pi,\ y=0$

$\therefore |OR| = 12\pi$

c) AS THE CURVE IS SYMMETRICAL, THE HIGHEST WILL OCCUR WHEN $t=\frac{\pi}{2}$

$$\Rightarrow y = 6\left[1-\cos\left(2\times\frac{\pi}{2}\right)\right] = 12$$

$\therefore$ MAX y IS 12

d) IF $B\hat{A}O = \pi/6 \Rightarrow$ GRADIENT OF AB IS $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

BUT AB IS A TANGENT TO THE CURVE AT B $\Rightarrow \left.\frac{dy}{dx}\right|_B = \frac{\sqrt{3}}{3}$

$$\Rightarrow \cot t = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \tan t = \sqrt{3}$$

$$\Rightarrow t_B = \frac{\pi}{3}$$

IYGB – SYNOPTIC PAPER C – QUESTION 21

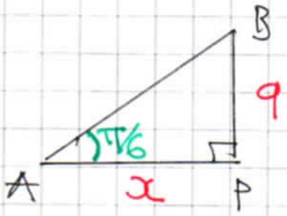
e) • AT POINT B, $t = \frac{\pi}{3} \Longrightarrow y_B = 9 \longleftarrow |BP|$

• $\tan\frac{\pi}{6} = \frac{9}{x}$

$\frac{1}{\sqrt{3}} = \frac{9}{x}$

$x = 9\sqrt{3}$

$|AP| = 9\sqrt{3}$



• AT POINT B, $t = \frac{\pi}{3} \Longrightarrow x_B = 4\pi - 3\sqrt{3}$

$\Longrightarrow |OP| = 4\pi - 3\sqrt{3}$

$\Longrightarrow |AO| = |AP| - |OP|$

$\Longrightarrow |AO| = 9\sqrt{3} - (4\pi - 3\sqrt{3})$

$\Longrightarrow |AO| = 12\sqrt{3} - 4\pi$

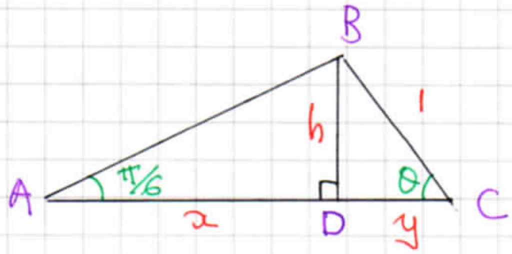
• BUT $|AO| = |RD|$ & $|OR| = 12\pi$ (FROM EARLIER)

$\Longrightarrow |AD| = 2|AO| + |OR| = 2(12\sqrt{3} - 4\pi) + 12\pi$

$\Longrightarrow |AD| = 4\pi + 24\sqrt{3}$

IYGB - SYN PAPER C - QUESTION 22

a) LOOKING AT $\hat{BDC}$



- $h = 1 \times \sin\theta$
- $y = 1 \times \cos\theta$

LOOKING AT $\hat{ABD}$

$$\Rightarrow \frac{h}{x} = \tan\frac{\pi}{6}$$

$$\Rightarrow \frac{\sin\theta}{x} = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \sqrt{3}x = 3\sin\theta$$

$$\Rightarrow x = \sqrt{3}\sin\theta$$

$$\therefore \quad x+y = \cos\theta + \sqrt{3}\sin\theta$$

$$\therefore \quad \text{AREA} = \frac{1}{2}|AC||BD|$$

$$= \frac{1}{2}(x+y)h$$

$$= \frac{1}{2}(\cos\theta + \sqrt{3}\sin\theta)\sin\theta$$

$$= \left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)\sin\theta$$

$$= \left(\sin\frac{\pi}{6}\cos\theta + \cos\frac{\pi}{6}\sin\theta\right)\sin\theta$$

$$= \sin\left(\theta + \frac{\pi}{6}\right)\sin\theta$$

IYGB - SYN PAPER C - QUESTION 22

b) LOOKING AT THE COMPOUND ANGLE IDENTITIES FOR $\cos(A \pm B)$

$$\cos\left[\theta + \left(\theta + \tfrac{\pi}{6}\right)\right] \equiv \cos\theta\cos\left(\theta + \tfrac{\pi}{6}\right) - \sin\theta\sin\left(\theta + \tfrac{\pi}{6}\right)$$

$$\cos\left[\theta - \left(\theta + \tfrac{\pi}{6}\right)\right] \equiv \cos\theta\cos\left(\theta + \tfrac{\pi}{6}\right) + \sin\theta\sin\left(\theta + \tfrac{\pi}{6}\right)$$

SUBTRACTING "UPWARDS"

$$\Rightarrow \cos\left(-\tfrac{\pi}{6}\right) - \cos\left(2\theta + \tfrac{\pi}{6}\right) \equiv 2\sin\theta\sin\left(\theta + \tfrac{\pi}{6}\right)$$

$$\Rightarrow \tfrac{1}{2}\cos\left(+\tfrac{\pi}{6}\right) - \tfrac{1}{2}\cos\left(2\theta + \tfrac{\pi}{6}\right) \equiv \sin\theta\sin\left(\theta + \tfrac{\pi}{6}\right)$$

$$\Rightarrow \sin\theta\sin\left(\theta + \tfrac{\pi}{6}\right) \equiv \frac{\sqrt{3}}{4} - \tfrac{1}{2}\cos\left(2\theta + \tfrac{\pi}{6}\right)$$

$$\Rightarrow \text{AREA} = \frac{1}{4}\left[\sqrt{3} - 2\cos\left(2\theta + \tfrac{\pi}{6}\right)\right]$$

c) MAXIMUM AREA OCCURS WHEN $\cos\left(2\theta + \frac{\pi}{6}\right) = -1$

$$\text{AREA}_{\text{MAX}} = \frac{1}{4}\left[\sqrt{3} - 2(-1)\right]$$

$$\text{AREA}_{\text{MAX}} = \frac{1}{4}\left(\sqrt{3} + 2\right)$$

AND FINALLY

$$\cos\left(2\theta + \tfrac{\pi}{6}\right) = -1$$

$$2\theta + \frac{\pi}{6} = \pi$$

$$2\theta = \frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{12}$$