IYGB - SYNOPTIC PAPER D - QUESTION 1

START BY FINDING THE CO-ORDINATES OF A & B

$$\left.\begin{array}{l} y = x^2 - 6x + 5 \\ y = -4x^2 + 24x - 20 \end{array}\right\} \Rightarrow x^2 - 6x + 5 = -4x^2 + 24x - 20$$

$$\Rightarrow 5x^2 - 30x + 25 = 0$$

$$\Rightarrow x^2 - 6x + 5 = 0$$

$$\Rightarrow (x-1)(x-5) = 0$$

$$\Rightarrow x = \begin{cases} 1 \\ 5 \end{cases}$$

VERIFY BY FACTORIZATION YIELDS

$$y = x^2 - 6x + 5$$
$$y = (x-1)(x-5)$$

$$y = -4x^2 + 24x - 20$$
$$y = -4(x^2 - 6x + 5)$$
$$y = -4(x-1)(x-5)$$

AREA BELOW THE x AXIS

$$\int_1^5 x^2 - 6x + 5 \, dx = \left[\frac{1}{3}x^3 - 3x^2 + 5x\right]_1^5$$

$$= \left(\frac{125}{3} - 75 + 25\right) - \left(\frac{1}{3} - 3 + 5\right)$$

$$= -\frac{25}{3} - \frac{7}{3}$$

$$= -\frac{32}{3} \quad \left(\text{AREA } \frac{32}{3} \text{ BELOW THE } x \text{ AXIS}\right)$$

AREA ABOVE THE x AXIS IS 4 TIMES AS LARGE AS THE OTHER CURVE IS STRETCHED VERTICALLY BY SCALE FACTOR OF 4

$$\therefore \text{TOTAL AREA} = \frac{32}{3} \times 5 = \frac{160}{3}$$

IYGB - SYNOPTIC PAPER D - QUESTION 1

ALTERNATIVE BY ACTUALLY CALCULATING THE AREA ABOVE THE x AXIS

$$\int_1^5 -4x^2+24x-20\,dx = \left[-\tfrac{4}{3}x^3+12x^2-20x\right]_1^5$$

$$= \left(-\tfrac{500}{3}+300-100\right)-\left(-\tfrac{4}{3}+12-20\right)$$

$$= \tfrac{100}{3}-\left(-\tfrac{28}{3}\right)$$

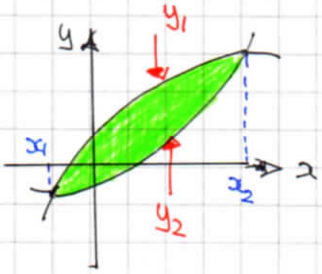
$$= \tfrac{128}{3}$$

$\therefore$ TOTAL AREA $= \frac{32}{3}+\frac{128}{3} = \frac{160}{3}$ // AS BEFORE

ALTERNATIVE

CALCULATING THE AREA IN "ONE GO"

USING AREA $= \int_{x_1}^{x_2} \left[y_1(x)-y_2(x)\right]\,dx$



$$\text{TOTAL AREA} = \int_1^5 \left(-4x^2+24x-20\right)-\left(x^2-6x+5\right)\,dx$$

$$= \int_1^5 -5x^2+30x-25\,dx$$

$$= \left[-\tfrac{5}{3}x^3+15x^2-25x\right]_1^5$$

$$= \left(-\tfrac{625}{3}+375-125\right)-\left(-\tfrac{5}{3}+15-25\right)$$

$$= \tfrac{125}{3}-\left(-\tfrac{35}{3}\right)$$

$$= \tfrac{160}{3}$$

IYGB - SYNOPTIC PAPER D - QUESTION 2

a)

$$f(x) < 10$$

$$|x-80| < 10$$

"THE DIFFERENCE OF x FROM 80 IS LESS THAN 10"

$$\Longrightarrow 70 < x < 90$$

b) USING PART (a)

$$\Longrightarrow f(1.2^n) < 10 \ , \ n \in \mathbb{N}$$

$$\Longrightarrow 70 < 1.2^n < 90$$

$$\Longrightarrow \log 70 < \log(1.2)^n < \log 90$$

$$\Longrightarrow \log 70 < n\log(1.2) < \log 90$$

$$\Longrightarrow \frac{\log 70}{\log(1.2)} < n < \frac{\log 90}{\log(1.2)}$$

$$\Longrightarrow 23.30\ldots < n < 24.68\ldots$$

$$\Longrightarrow n = 24$$

IYGB - SYNOPTIC PAPER D - QUESTION 3

BY INSPECTION

$$2x^2 - xy - y^2 = (2x+y)(x-y)$$

BY THE QUADRATIC FORMULA – TREAT x AS "THE VARIABLE"

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$
$b = -y$
$c = -y^2$

$$x = \frac{y \pm \sqrt{y^2 - 4 \times 2 \times (-y^2)}}{4}$$

$$x = \frac{y \pm \sqrt{9y^2}}{4} = \begin{cases} \frac{y+3y}{4} = y \\ \frac{y-3y}{4} = -\frac{1}{2}y \end{cases}$$

EITHER $x = y \implies x - y = 0$

OR $x = -\frac{1}{2}y$

$2x = -y \implies 2x + y = 0$

$$\therefore (x-y)(2x+y)$$ AS BEFORE

BY COMPLETING THE SQUARE TREATING y AS A VARIABLE

$$2x^2 - xy - y^2 = -\left[y^2 + xy - 2x^2\right]$$

$$= -\left[\left(y + \tfrac{1}{2}x\right)^2 - \tfrac{1}{4}x^2 - 2x^2\right]$$

$$= -\left[\left(y + \tfrac{1}{2}x\right)^2 - \tfrac{9}{4}x^2\right]$$

$$= \tfrac{9}{4}x^2 - \left(y + \tfrac{1}{2}x\right)^2$$

IYGB - SYNOPTIC PAPER D - QUESTION 3

$$= \left(\frac{3}{2}x\right)^2 - \left(y + \frac{1}{2}x\right)^2 \qquad \leftarrow A^2 - B^2 \equiv (A+B)(A-B)$$

$$= \left[\frac{3}{2}x + \left(y + \frac{1}{2}x\right)\right]\left[\frac{3}{2}x - \left(y + \frac{1}{2}x\right)\right]$$

$$= (2x+y)(x-y)$$

// AS BEFORE

IYGB - SYNOPTIC PAPER D - QUESTION 4

a) PROCEED AS FOLLOWS

$$\Rightarrow (1+3x)^{-1} = 1 + \frac{-1}{1}(3x)^1 + \frac{-1(-2)}{1\times2}(3x)^2 + \frac{-1(-2)(-3)}{1\times2\times3}(3x)^3 + O(x^4)$$

$$\Rightarrow (1+3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 + O(x^4)$$

b) DIFFERENTIATING AS SUGGESTED

$$\Rightarrow \frac{d}{dx}\left[(1+3x)^{-1}\right] = \frac{d}{dx}\left[1 - 3x + 9x^2 - 27x^3 + O(x^4)\right]$$

$$\Rightarrow -3(1+3x)^{-2} = 0 - 3 + 18x - 81x^2 + O(x^3)$$

$$\Rightarrow (1+3x)^{-2} = \frac{-3}{-3} + \frac{18x}{-3} - \frac{81x^2}{-3} + O(x^3)$$

$$\Rightarrow (1+3x)^{-2} = 1 - 6x + 27x^2 + O(x^3)$$

c) USING PART (b)

$$f(x) = \frac{4+x}{(1+3x)^2} = (4+x)(1+3x)^{-2}$$

$$= (4+x)\left[1 - 6x + 27x^2 + O(x^3)\right]$$

$$= 4 - 24x + 108x^2 + O(x^3)$$

$$\; x - 6x^2 + O(x^3)$$

$$= 4 - 23x + 102x^2 + O(x^3)$$

IYGB - SYNOPTIC PAPER D - QUESTION 5

a) FORMING TWO EQUATIONS USING STANDARD FORMULAS

- $u_{16} = 6$

$\Rightarrow u_n = a + (n-1)d$

$\Rightarrow 6 = a + 15d$

- $S_{16} = 456$

$\Rightarrow S_n = \frac{n}{2}(a + L)$ u_{16}

$\Rightarrow 456 = \frac{16}{2}(a + 6)$

$\Rightarrow 456 = 8(a+6)$

$\Rightarrow 57 = a + 6$

$\Rightarrow a = 51$

&

$\Rightarrow 6 = 51 + 15d$

$\Rightarrow -45 = 15d$

$\Rightarrow d = -3$

b) USING $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 0 = \frac{k}{2}[2 \times 51 + (k-1) \times (-3)]$

$\Rightarrow 0 = \frac{k}{2}[102 - 3(k-1)]$

$\Rightarrow 0 = \frac{k}{2}[102 - 3k + 3]$

$\Rightarrow 0 = \frac{1}{2}k(105 - 3k)$

$\therefore k = \cancel{0}$ ($k \neq 0$) or 35

IYGB - SYNOPTIC PAPER D - QUESTION 6

a)

$$\boxed{\frac{dx}{dt} = -k\,x^2}$$

SQUARE

DROP

RATE

PROPORTIONAL

b) SOLVING THE DIFFERENTIAL EQUATION BY SEPARATING VARIABLES

$\Longrightarrow \quad dx = -kx^2\,dt$

$\Longrightarrow \quad -\frac{1}{x^2}\,dx = k\,dt$

$\Longrightarrow \quad \int -\frac{1}{x^2}\,dx = \int k\,dt$

$\Longrightarrow \quad \boxed{\frac{1}{x} = kt + C}$

APPLY THE CONDITION $t=0$, $x = 2.5$ (2500 CASES)

$\Longrightarrow \quad \frac{1}{2.5} = C$

$\Longrightarrow \quad C = \frac{2}{5}$

$\Longrightarrow \quad \boxed{\frac{1}{x} = kt + \frac{2}{5}}$

APPLY THE CONDITION $t=1$, $x=1.6$ (1600 CASES)

$\Longrightarrow \quad \frac{1}{1.6} = k + \frac{2}{5}$

$\Longrightarrow \quad \frac{5}{8} = k + \frac{2}{5}$

$\Longrightarrow \quad k = \frac{9}{40}$

$\Longrightarrow \quad \boxed{\frac{1}{x} = \frac{9}{40}t + \frac{2}{5}}$

IYGB - SYNOPTIC PAPER D - QUESTION 6

TIDY UP FURTHER

$$\frac{1}{x} = \frac{9t}{40} + \frac{2}{5}$$

$$\frac{1}{x} = \frac{9t}{40} + \frac{16}{40}$$

$$\frac{1}{x} = \frac{9t+16}{40}$$

$$x = \frac{40}{9t+16}$$

// AS REQUIRED

c) FINALLY WHEN $x = 0.25$ (250 CASES)

$$\Rightarrow \frac{1}{4} = \frac{40}{9t+16}$$

$$\Rightarrow 9t+16 = 160$$

$$\Rightarrow 9t = 144$$

$$\Rightarrow t = 16$$

//

IYGB - SYNOPTIC PAPER D - QUESTION 7

$$\left(125^{\frac{1}{3}} \times 5^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$$

$$= \left[\sqrt[3]{125} \times \sqrt{25} + \left(\sqrt[4]{16}\right)^3 + \sqrt[3]{64} + 49^{\frac{1}{2}}\right]^{-\frac{2}{3}}$$

$$= \left[5 \times 5 + 2^3 \times 4 + \sqrt{49}\right]^{-\frac{2}{3}}$$

$$= (25 + 8 \times 4 + 7)^{-\frac{2}{3}}$$

$$= (25 + 32 + 7)^{-\frac{2}{3}}$$

$$= 64^{-\frac{2}{3}}$$

$$= \frac{1}{64^{\frac{2}{3}}}$$

$$= \frac{1}{\left(\sqrt[3]{64}\right)^2}$$

$$= \frac{1}{4^2}$$

$$= \frac{1}{16}$$

$$a^{-m} = \frac{1}{a^m}$$

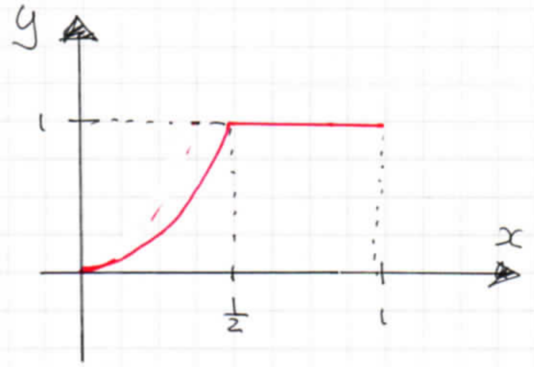
$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

IYGB - SYNOPTIC PAPER D - QUESTION 8

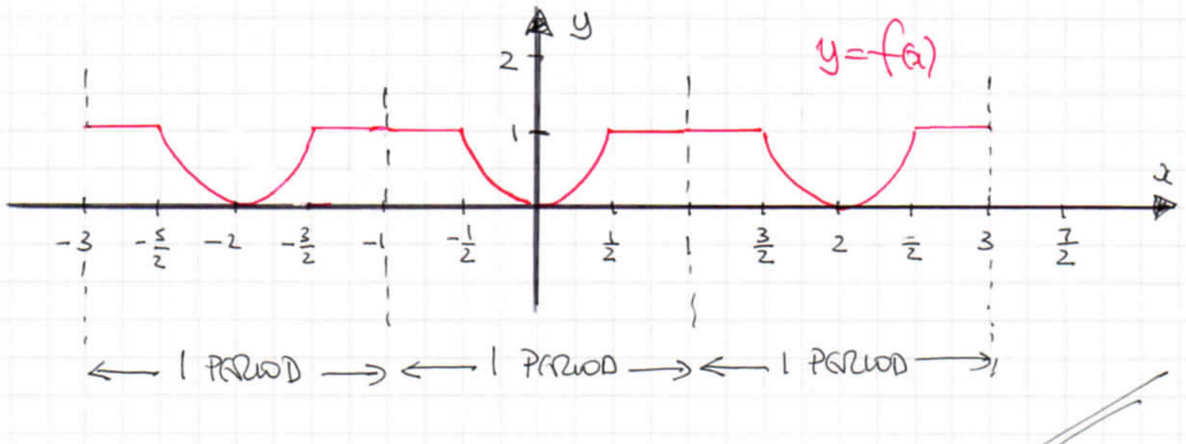
EVEN $\Rightarrow$ REFLECTION ABOUT THE y AXIS

PERIOD OF 2 $\Rightarrow$ REPEATS EVERY 2 UNITS

SKETCHING BETWEEN 0 & 1

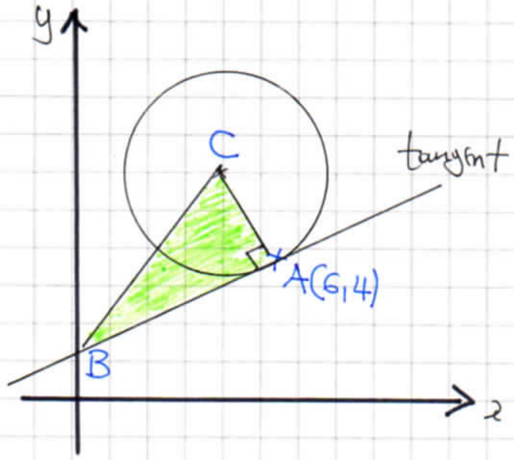


SKETCHING BETWEEN −3 & 3



IYGB - SYNOPTIC PAPER D - QUESTION 9

a) REWRITTING THE EQUATION OF THE CIRCLE, TO READ THE CENTRE



$$x^2+y^2-10x-12y+56=0$$
$$x^2-10x+y^2-12y+56=0$$
$$(x-5)^2-25+(y-6)^2-36+56=0$$
$$(x-5)^2+(y-6)^2=5$$

$\therefore C(5,6)$ & $r=\sqrt{5}$

FIND THE GRADIENT OF AC, WHERE $C(5,6)$ & $A(6,4)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-6}{6-5}=\frac{-2}{1}=-2$$

USING PERPENDICULAR GRADIENT OF $+\frac{1}{2}$ WE "OBTAIN THE TANGENT"

$$y-y_0=m(x-x_0)$$
$$y-4=\frac{1}{2}(x-6)$$
$$2y-8=x-6$$
$$2y=x+2$$

OR $y=\frac{1}{2}x+1$

b) FIND THE CO-ORDINATES OF B

WITH $x=0$ $\quad 2y=2$

$y=1$

$\therefore B(0,1)$

IYGB - SYNOPTIC PAPER D - QUESTION 9

FIND THE DISTANCE AB, WHERE A(6,4) & B(0,1)

$$\Rightarrow d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$\Rightarrow |AB| = \sqrt{(1-4)^2+(0-6)^2}$$

$$\Rightarrow |AB| = \sqrt{9+36}$$

$$\Rightarrow |AB| = \sqrt{45} = 3\sqrt{5}$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\Rightarrow \text{AREA} = \frac{1}{2}|AB||AC|$$

$$\Rightarrow \text{AREA} = \frac{1}{2} \times 3\sqrt{5} \times \sqrt{5}$$

$$\Rightarrow \text{AREA} = \frac{15}{2}$$

IYGB - SYNOPTIC PAPER D - QUESTION 10

a) STARTING WITH THE INFORMATION GIVEN

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$$\Rightarrow S_4 = 5S_2$$

$$\Rightarrow \frac{\cancel{a}(1-r^4)}{\cancel{1-r}} = 5 \times \frac{\cancel{a}(1-r^2)}{\cancel{1-r}}$$

$a \neq 0$, $1-r \neq 0$

$$\Rightarrow 1-r^4 = 5(1-r^2)$$

$$\Rightarrow 1-r^4 = 5-5r^2$$

$$\Rightarrow 0 = r^4 - 5r^2 + 4$$

$$\Rightarrow 0 = (r^2-4)(r^2-1)$$

$$\Rightarrow r^2 = 4, \ 1$$

$$\Rightarrow r = \cancel{2}, \ -2, \ \cancel{1}, \ \cancel{-1}$$

← TERMS ALTERNATE IN SIGN

$r \neq 0, 1, -1$

b)

$$u_n = ar^{n-1}$$

$$u_5 = ar^4$$

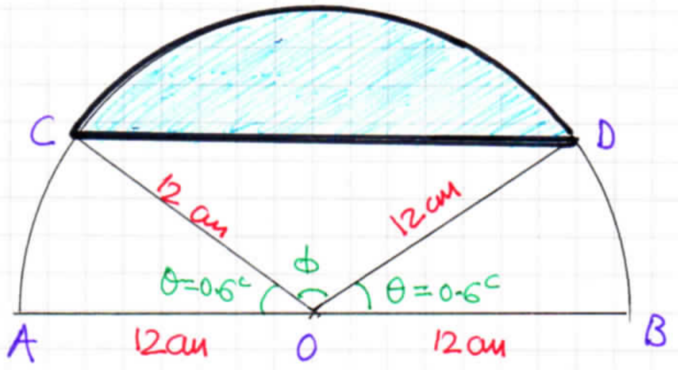
$$36 = a(-2)^4$$

$$36 = 16a$$

$$a = \frac{9}{4}$$

IYGB – SYNOPTIC PAPER D – QUESTION 11

a)



LOOKING AT THE DIAGRAM ABOVE

$$\phi = \pi - 2\theta \quad \text{(STRAIGHT LINE)}$$

$$\phi = \pi - 2 \times 0.6$$

$$\phi = 1.94159\ldots^c$$

AREA OF THE SECTOR

$$= \frac{1}{2}r^2\phi^c = \frac{1}{2} \times 12^2 \times 1.9415\ldots = 139.79\ldots$$

AREA OF THE TRIANGLE

$$= \frac{1}{2} \times 12 \times 12 \times \sin(1.9415\ldots^c) = 67.106\ldots$$

AREA OF THE SEGMENT

$$= \quad - $$

$$= 139.79\ldots - 67.106$$

$$= 72.7 \text{ cm}^2$$

(3 sf)

IYGB - SYNOPTIC PAPER D - QUESTION 11

b) ARCLENGTH $|\overset{\frown}{CD}|$

$$\overset{\frown}{CD} = r\phi^c = 12 \times 1.94159\ldots = 23.299\ldots$$

LENGTH OF THE CHORD CD, BY THE COSINE RULE

(OR SIMPLE TRIGONOMETRY)

$$|CD|^2 = |OC|^2 + |OD|^2 - 2|OC||OD|\cos\phi$$

$$|CD|^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos(1.94159\ldots)$$

$$|CD|^2 = 392.358\ldots$$

$$|CD| = 19.808\ldots$$

$\therefore$ REQUIRED PERIMETER $= 23.299\ldots + 19.808\ldots$

$= 43.1$ cm

IYGB – SYNOPTIC PAPER D – QUESTION 12

SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 4x^2 - 7x + 11 \\ y = 5x + k \end{array}\right\} \Longrightarrow 4x^2 - 7x + 11 = 5x + k$$

$$\Longrightarrow 4x^2 - 12x + (11-k) = 0$$

IF THERE ARE TWO DISTINCT POINTS OF INTERSECTION, THE DISCRIMINANT OF THE ABOVE QUADRATIC MUST BE POSITIVE

$$\Longrightarrow \text{"}b^2 - 4ac\text{"} > 0$$

$$\Longrightarrow (-12)^2 - 4 \times 4 \times (11-k) > 0$$

$$\Longrightarrow 144 - 16(11-k) > 0$$

$$\Longrightarrow 144 - 176 + 16k > 0$$

$$\Longrightarrow 16k > 32$$

$$\Longrightarrow k > 2$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER D - QUESTION 13

a) FROM THE "THIRD" ITEM ON THE INFORMATION GIVEN

$$f(x) \equiv (x-2)(x+2)g(x) + ax + b$$

NOW $f(2) = 5$

$5 = 0 + 2a + b$

$2a + b = 5$

AND $f(-2) = -11$

$-11 = 0 - 2a + b$

$-2a + b = -11$

ADDING & SUBSTRACTING YIELDS

$b = -3$ & $a = 4$

b) $f(x) = 3x^4 + px + q$

$f(2) = 5$

$3 \times 2^4 + 2p + q = 5$

$2p + q = -43$

$f(-2) = -11$

$3(-2)^4 - 2p + q = -11$

$-2p + q = -59$

ADDING

$2q = -102$

$\boxed{q = -51}$

& $2p + q = -43$

$2p - 51 = 43$

$2p = 8$

$\boxed{p = 4}$

IYGB - SYNOPTIC PAPER D - QUESTION 13

HENCE WE HAVE

$$f(x) \equiv (x-2)(x+2)g(x) + ax + b$$

$$3x^4 + px + q \equiv (x^2-4)g(x) + 4x - 3$$

$$3x^4 + \cancel{4x} - 51 \equiv (x^2-4)g(x) + \cancel{4x} - 3$$

$$3x^4 - 48 \equiv (x^2-4)g(x)$$

$$3(x^4 - 16) \equiv (x^2-4)g(x)$$

$$3(x^2-4)(x^2+4) \equiv (x^2-4)g(x)$$

BY COMPARISON $g(x) = 3(x^2+4)$

THE ABOVE CAN ALSO BE DONE JUST AS GOOD BY LONG DIVISION

IYGB - SYNOPTIC PAPER D - QUESTION 14

USING THE SUBSTITUTION GIVEN

$$u^2 = e^x - 1 \quad \text{(IN FACT THE SUBSTITUTION IS } u = \sqrt{e^x - 1}\text{)}$$

$$2u\frac{du}{dx} = e^x$$

$$2u\,du = e^x\,dx$$

$$dx = \frac{2u}{e^x}\,du$$

LIMITS

$$x = \ln 5 \mapsto u = \sqrt{e^{\ln 5} - 1} = 2$$

$$x = \ln 2 \mapsto u = \sqrt{e^{\ln 2} - 1} = 1$$

TRANSFORMING THE INTEGRAL

$$\int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}}\,dx = \int_1^2 \frac{3e^{2x}}{\cancel{u}}\left(\frac{2\cancel{u}}{e^x}\,du\right)$$

$$= \int_1^2 6e^x\,du$$

$u^2 = e^x - 1$

$$= \int_1^2 6(u^2+1)\,du$$

$$= \int_1^2 6u^2 + 6\,du$$

$$= \left[2u^3 + 6u\right]_1^2$$

$$= (16 + 12) - (2 + 6)$$

$$= 20$$

IYGB - SYNOPTIC PAPER D - QUESTION 15

REWRITTING THE EQUATION BEFORE DIFFERENTIATION

$$y = \frac{2x^2-1-2\ln x^x}{x} = \frac{2x^2-1-2x\ln x}{x}$$

$$y = \frac{2x^2}{x} - \frac{1}{x} - \frac{2x\ln x}{x} = 2x - x^{-1} - 2\ln x$$

$(x \neq 0)$

DIFFERENTIATE WITH RESPECT TO x, TWICE

$$\Rightarrow \frac{dy}{dx} = 2 + x^{-2} - \frac{2}{x} = 2 + x^{-2} - 2x^{-1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2x^{-3} + 2x^{-2}$$

FOR POINTS OF INFLEXION $\frac{d^2y}{dx^2} = 0$

$$\Rightarrow -2x^{-3} + 2x^{-2} = 0$$

$$\Rightarrow \frac{2}{x^2} = \frac{2}{x^3}$$

$$\Rightarrow 2x^3 = 2x^2$$

$$\Rightarrow 2x^3 - 2x^2 = 0$$

$$\Rightarrow 2x^2(x-1) = 0$$

$$\Rightarrow x = \begin{cases} \cancel{0} \\ 1 \end{cases} \qquad y = \frac{2-1-2\cancel{\ln 1}}{1} = 1$$

HENCE AT $(1,1)$ THERE IS A POINT OF INFLEXION

(NO NEED TO CHECK FURTHER AS THE QUESTION ASSERTS SO)

IYGB - SYNOPTIC PAPER D - QUESTION 15

DETERMINE THE GRADIENT AT (1,1)

$$\frac{dy}{dx} = 2 + x^{-2} - 2x^{-1} = 2 + \frac{1}{x^2} - \frac{2}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 2 + 1 - 2 = 1$$

EQUATION OF THE TANGENT HAS GRADIENT 1 & PASSES THROUGH (1,1)

$$y - y_0 = m(x - x_0)$$

$$y - 1 = 1(x - 1)$$

$$y - 1 = x - 1$$

$$y = x$$

AS REQUIRED

IYGB - SYNOPTIC PAPER D - QUESTION 16

a) STARTING WITH THE GRADIENT OF AB

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4\sqrt{3}}{-3 + \sqrt{3} - 1} = \frac{3 - 4\sqrt{3}}{\sqrt{3} - 4} = \frac{4\sqrt{3} - 3}{4 - \sqrt{3}}$$

$$= \frac{(4\sqrt{3} - 3)(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})} = \frac{16\sqrt{3} + 12 - 12 - 3\sqrt{3}}{16 - 3}$$

$$= \frac{13\sqrt{3}}{13} = \sqrt{3}$$

HENCE THE EQUATION OF L IS

$$\Rightarrow y - 4\sqrt{3} = \sqrt{3}(x - 1)$$

$$\Rightarrow y - 4\sqrt{3} = \sqrt{3}x - \sqrt{3}$$

$$\Rightarrow y = \sqrt{3}x + 3\sqrt{3}$$

$$\Rightarrow y = \sqrt{3}(x + 3)$$

IE. $k = 3$

b) SETTING $y = 0$ IN THE ABOVE EQUATION

$$\Rightarrow 0 = \sqrt{3}(x + 3)$$

$$\Rightarrow x = -3$$

$\therefore$ $C(-3, 0)$, $A(1, 4\sqrt{3})$

$$\Rightarrow |AC| = \sqrt{(-3-1)^2 + (0 - 4\sqrt{3})^2}$$

$$\Rightarrow |AC| = \sqrt{16 + 48}$$

$$\Rightarrow |AC| = 8$$

IYGB - SYNOPTIC PAPER D - QUESTION 16

c)

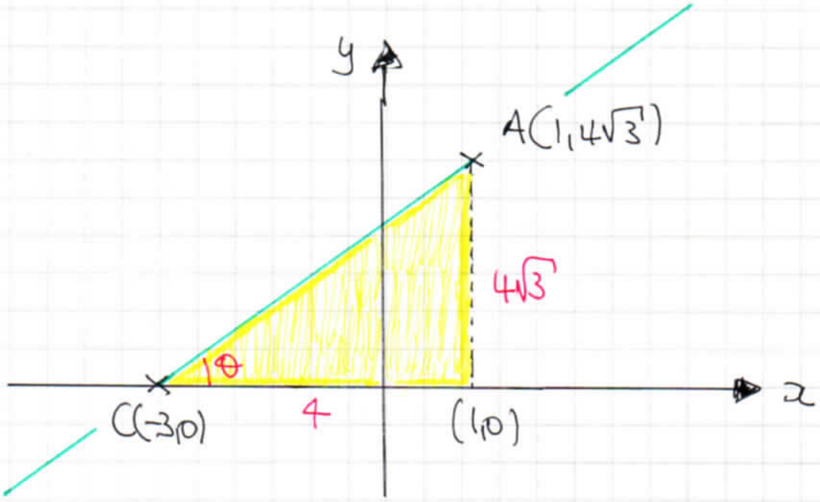
$$y = \sqrt{3}(x+3)$$

$$y = \sqrt{3}x + 3\sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60°$$

OR VIA A DIAGRAM



$$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$$

$$\theta = 60°$$

IYGB - SYNOPTIC PAPER D - QUESTION 17

a) SUBSTITUTING $P(20,60)$ INTO THE PARAMETRIC EQUATIONS

$$x = 2at$$
$$20 = 2at$$
$$\boxed{10 = at}$$

$$y = 8at - at^2$$
$$\boxed{60 = 8at - at^2}$$

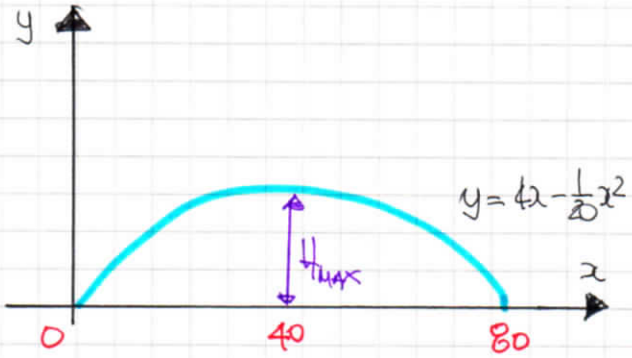
$$60 = 8 \times 10 - at^2$$
$$\boxed{at^2 = 20}$$

$$\left.\begin{array}{l} at^2 = 20 \\ at = 10 \end{array}\right) \Longrightarrow \text{DIVIDE } t = 2$$
$$\Longrightarrow a = 5$$

b) $x = 2at \Longrightarrow 4a^2t^2 = x^2$

$$\Longrightarrow y = 4(2at) - \frac{1}{4a}(4a^2t^2)$$
$$\Longrightarrow y = 4x - \frac{1}{4a}x^2$$
$$\Longrightarrow y = 4x - \frac{1}{20}x^2$$

c) SKETCHING THE CARTESIAN PATH & CONSIDER SYMMETRY



- $y = 4x - \frac{1}{20}x^2$

  $y = \frac{1}{20}x(80 - x)$

- when $x = 40$

  $y = \frac{1}{20} \times 40 \times (80 - 40)$

  $y = 80$

- $H_{MAX} = 80\text{m}$

IYGB - SYNOPTIC PAPER D - QUESTION 18

a) USING THE STANDARD METHOD

$$5\cos\theta - 12\sin\theta \equiv R\cos(\theta+\alpha)$$
$$5\cos\theta - 12\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$
$$5\cos\theta - 12\sin\theta \equiv (R\cos\alpha)\cos\theta - (R\sin\alpha)\sin\theta$$

COMPARING & SOLVING

$$\left.\begin{array}{l} R\cos\alpha = 5 \\ R\sin\alpha = 12 \end{array}\right\}$$

$\Longrightarrow$ SQUARING & ADDING YIELDS

$$R = +\sqrt{5^2+12^2} = 13$$

$\Longrightarrow$ DIVIDING THE EQUATIONS SIDE BY SIDE

$$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{12}{5}$$
$$\alpha \approx 1.176^c$$

$$\therefore f(\theta) \approx 13\cos(\theta + 1.176^c)$$

b) FIRSTLY $f(\theta)_{MAX} = 13$

NEXT, TO GET A MAX OF 13

$$\Longrightarrow \cos(\theta + 1.176^c) = 1$$
$$\Longrightarrow \theta + 1.176^c = 0$$
$$\Longrightarrow \theta = -1.176^c \quad +2\pi$$
$$\Longrightarrow \theta = 5.107^c$$

c) USING PARTS (a) & (b)

$$\Rightarrow P = 20 + 5\cos\left(\frac{4\pi t}{25}\right) - 12\sin\left(\frac{4\pi t}{25}\right)$$
$$\Rightarrow P = 20 + 13\cos\left(\frac{4\pi t}{25} + 1.176^c\right)$$

$$\therefore P_{MAX} = 20 + 13 = 33$$

IYGB - SYNOPTIC PAPER D - QUESTION 18

AND FROM PART (b) $\implies \theta = 5.107^c$

$$\implies \frac{4\pi t}{25} = 5.107^c$$

$$\implies t \approx 10.16$$

d) SOLVING THE EQUATION $P = 15$

$$\implies 15 = 20 + 13\cos\left(\frac{4\pi t}{25} + 1.176^c\right)$$

$$\implies -5 = 13\cos\left(\frac{4\pi t}{25} + 1.176^c\right)$$

$$\implies \cos\left(\frac{4\pi t}{25} + 1.176^c\right) = -\frac{5}{13}$$

$\left[\arccos\left(-\frac{5}{13}\right) = 1.965587\ldots\right]$

$$\implies \begin{cases} \frac{4\pi t}{25} + 1.176^c = 1.9656\ldots \pm 2n\pi \\ \frac{4\pi t}{25} + 1.176^c = 4.3176\ldots \pm 2n\pi \end{cases}$$

$n = 0, 1, 2, 3, \ldots$

$$\implies \begin{cases} \frac{4\pi t}{25} = 0.78958\ldots \pm 2n\pi \\ \frac{4\pi t}{25} = \cdot\pi \pm 2n\pi \end{cases}$$

$$\implies \begin{cases} t = 1.5708 \pm \frac{25}{2}n \\ t = 6.25 \pm \frac{25}{2}n \end{cases}$$

$t =$ 1.5707 / 6.25

TIME 01:34 / 06:15

IYGB - SYNOPTIC PAPER D - QUESTION 19

I) $\boxed{6\tan x = \dfrac{2-3\sec^2 x}{\tan x - 1} \qquad 0 \leq x < 2\pi}$

$\Rightarrow 6\tan x(\tan x - 1) = 2 - 3\sec^2 x$

$\Rightarrow 6\tan^2 x - 6\tan x = 2 - 3\sec^2 x$

$\Rightarrow 6\tan^2 x - 6\tan x = 2 - 3(1+\tan^2 x)$

$\Rightarrow 6\tan^2 x - 6\tan x = -1 - 3\tan^2 x$

$\Rightarrow 9\tan^2 x - 6\tan x + 1 = 0$

$\Rightarrow (3\tan x - 1)^2 = 0$

$\Rightarrow \tan x = \frac{1}{3}$

$x = \arctan\left(\frac{1}{3}\right) \pm n\pi \qquad n = 0,1,2,3,\ldots$

$x = 0.32175\ldots \pm n\pi$

$x = \begin{cases} 0.321^c \\ 3.463^c \end{cases}$

II) METHOD A - USING THE COMPOUND ANGLE IDENTITIES

$\Rightarrow \boxed{\cos(3\theta - 60°) = \cos(3\theta + 30°)}$

$\Rightarrow \cos 3\theta \cos 60 + \sin 3\theta \sin 60 = \cos 3\theta \cos 30 - \sin 3\theta \sin 30$

$\Rightarrow \frac{1}{2}\cos 3\theta + \frac{\sqrt{3}}{2}\sin 3\theta = \frac{\sqrt{3}}{2}\cos 3\theta - \frac{1}{2}\sin 3\theta$

$\Rightarrow \cos 3\theta + \sqrt{3}\sin 3\theta = \sqrt{3}\cos 3\theta - \sin 3\theta$

$\Rightarrow \frac{\cos 3\theta}{\cos 3\theta} + \frac{\sqrt{3}\sin 3\theta}{\cos 3\theta} = \frac{\sqrt{3}\cos 3\theta}{\cos 3\theta} - \frac{\sin 3\theta}{\cos 3\theta}$

$\Rightarrow 1 + \sqrt{3}\tan 3\theta = \sqrt{3} - \tan 3\theta$

IYGB - SYNOPTIC PAPER D - QUESTION 19

$$\Rightarrow (\sqrt{3}+1)\tan 3\theta = \sqrt{3}-1$$

$$\Rightarrow \tan 3\theta = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow 3\theta = \arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) \pm 180n \qquad n=0,1,2,3,\dots$$

$$\Rightarrow 3\theta = 15° \pm 180n$$

$$\Rightarrow \theta = 5° \pm 60n$$

$$\Rightarrow \theta = \begin{cases} 5° \\ 65° \\ 125° \end{cases}$$

ALTERNATIVE METHOD

$$\Rightarrow \cos(3\theta - 60°) = \cos(3\theta + 30)$$

$$\Rightarrow \begin{cases} 3\theta - 60° = (3\theta + 30°) \pm 360n \\ 3\theta - 60° = 360 - (3\theta + 30°) \pm 360n \end{cases} \qquad n=0,1,2,3,\dots$$

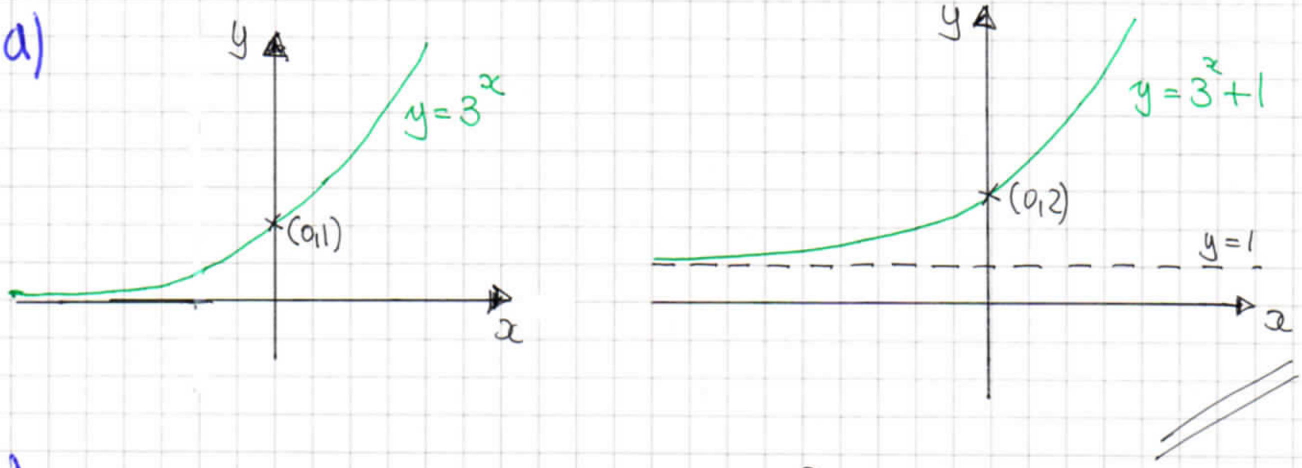
$$\Rightarrow \begin{cases} \text{INCONSISTENT} \\ 6\theta = 390° \pm 360n \end{cases}$$

$$\Rightarrow \theta = 65° \pm 60n$$

$$\Rightarrow \theta = \begin{cases} 65° \\ 5° \\ 125° \end{cases}$$

IYGB - SYNOPTIC PAPER D - QUESTION 20

a)



b) REFLECTION IN THE x AXIS : $y = -f(x)$

$$\Rightarrow y = -\left[3^x + 1\right]$$

$$\Rightarrow y = -3^x - 1$$

REFLECTION IN THE y AXIS : $y = f(-x)$

$$\Rightarrow y = -3^{-x} - 1$$

c) $3^x + 1 \longmapsto 3^x + 3 = (3^x + 1) + 2 \quad (f(x) + 2)$

TRANSLATION "UPWARDS" BY 2 UNITS

$3^x + 3 \longmapsto 3^{(x+1)} + 3 \quad (f(x+1))$

TRANSLATION, "LEFT", BY 1 UNIT

I.E TRANSLATION BY $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

d) $3^{x+1} + 3 = 3^x \times 3^1 + 3 = 3(3^x + 1)$

$$= 3f(x)$$

I.E VERTICAL STRETCH BY SCALE FACTOR 3

IYGB - SYNOPTIC PAPER D - QUESTION 21

a) BY MANIPULATION

$$\frac{4t^2}{t-1} = \frac{4t(t-1)+4(t-1)+4}{t-1} = 4t+4+\frac{4}{t-1}$$

$\therefore$ $A = B = C = 4$ //

ALTERNATIVE BY ALGEBRAIC DIVISION

$$\begin{array}{r|l} & 4t+4 \\ t-1 & 4t^2 \\ & \underline{-4t^2+4t} \\ & 4t \\ & \underline{-4t+4} \\ & 4 \end{array}$$

$\therefore \frac{4t^2}{t-1} = 4t+4+\frac{4}{t-1}$

$\therefore$ $A = B = C = 4$

ALTERNATIVE BY COMPARING

$$\Rightarrow \frac{4t^2}{t-1} \equiv At + B + \frac{C}{t-1}$$

$$\Rightarrow \frac{4t^2}{t-1} \equiv \frac{At(t-1)+B(t-1)+C}{t-1}$$

$$\Rightarrow 4t^2 \equiv At^2 - At + Bt - B + C$$

$$\Rightarrow 4t^2 \equiv At^2 + (B-A)t + (C-B)$$

$\therefore$ $A = 4$ $\qquad B - A = 0 \qquad C - B = 0$

$\qquad A = B \qquad C - B = 0$

$\qquad B = 4 \qquad C = B$

IYGB - SYNOPTIC PAPER D - QUESTION 21

b) USING THE SUBSTITUTION GIVEN

$$\int_{16}^{81} \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{4}}}\,dx = \ldots$$

$$= \int_{2}^{3} \frac{1}{t^2 - t}\,(4t^3\,dt) = \int_{2}^{3} \frac{4t^3}{t(t-1)}\,dt$$

$$= \int_{2}^{3} \frac{4t^2}{t-1}\,dt$$

$t = x^{\frac{1}{4}}$

$t^2 = x^{\frac{1}{2}}$

$t^4 = x$

$x = t^4$

$\frac{dx}{dt} = 4t^3$

$dx = 4t^3\,dt$

$x = 16 \mapsto t = 2$

$x = 81 \mapsto t = 3$

USING PART (a)

$$= \int_{2}^{3} 4t + 4 + \frac{4}{t-1}\,dt$$

$$= \Big[2t^2 + 4t + 4\ln|t-1|\Big]_{2}^{3}$$

$$= (18 + 12 + 4\ln 2) - (8 + 8 + \cancel{4\ln 1})$$

$$= 14 + 4\ln 2$$

AS REQUIRED

IYGB - SYNOPTIC PAPER D - QUESTION 22

a) I) START BY OBTAINING k, USING $t=0$ $P=175$

$$\Rightarrow P = \frac{800ke^{0.25t}}{1+ke^{0.25t}}$$

$$\Rightarrow 175 = \frac{800k \times 1}{1+k \times 1}$$

$$\Rightarrow 175 = \frac{800k}{1+k}$$

$$\Rightarrow 175k + 175 = 800k$$

$$\Rightarrow 175 = 625k$$

$$\Rightarrow k = \frac{7}{25}$$

NOW USING THE EQUATION WITH THE ABOVE VALUE OF k

$$\Rightarrow P = \frac{800 \times \frac{7}{25} e^{0.25t}}{1 + \frac{7}{25} e^{0.25t}}$$

$$\Rightarrow P = \frac{224e^{0.25t}}{1 + \frac{7}{25} e^{0.25t}}$$

MULTIPLY NUMERATOR & DENOMINATOR OF THE FRACTION BY 25

$$\Rightarrow P = \frac{5600 e^{0.25t}}{25 + 7e^{0.25t}}$$

$$\Rightarrow 560 = \frac{5600 e^{0.25t}}{25 + 7e^{0.25t}}$$

$\div 560$

$$\Rightarrow 1 = \frac{10e^{0.25t}}{25 + 7e^{0.25t}}$$

IYGB - SYNOPTIC PAPER D - QUESTION 22

$\Rightarrow 25 + 7e^{0.25t} = 10e^{0.25t}$

$\Rightarrow 25 = 3e^{0.25t}$

$\Rightarrow \frac{25}{3} = e^{\frac{1}{4}t}$

$\Rightarrow \ln\frac{25}{3} = \frac{1}{4}t$

$\Rightarrow t = 4\ln\frac{25}{3} \approx 8.48$

II) REWRITING THE FORMULA FOR SIMPLICITY

$$\Rightarrow P = \frac{5600e^{0.25t}}{25 + 7e^{0.25t}}$$

$$\Rightarrow P = \frac{5600e^{0.25t}e^{-0.25t}}{25e^{-0.25t} + 7e^{0.25t}e^{-0.25t}}$$

$$\Rightarrow \boxed{P = \frac{5600}{25e^{-0.25t} + 7}}$$

NOW AS $t \to \infty$ $e^{-0.25t} \to 0 \Rightarrow P \to \frac{5600}{7}$

$\Rightarrow P \to 800$

$\therefore$ THE POPULATION TENDS TO 800

b) USING THE EXPRESSION IN THE "RED BOX", ABOUT

$$P = \frac{5600}{25e^{-0.25t}+7} = 5600\left(7+25e^{-0.25t}\right)^{-1}$$

IYGB - SYNOPTIC PAPER D - QUESTION 22

$$\Longrightarrow \frac{dP}{dt} = -5600\left(7+25e^{-0.25t}\right)^{-2} \times 25e^{-0.25t} \times \left(-\frac{1}{4}\right)$$

$$\Longrightarrow \frac{dP}{dt} = 35000e^{-0.25t}\left(7+25e^{-0.25t}\right)^{-2}$$

NOW REARRANGING THE FORMULA FOR P, FROM EARLIER

$$25e^{-0.25t} + 7 = \frac{5600}{P}$$

$$25e^{-0.25t} = \frac{5600}{P} - 7$$

$$\Longrightarrow \frac{dP}{dt} = 1400 \times 25e^{-0.25t} \times \left(7+25e^{-0.25t}\right)^{-2}$$

$$\Longrightarrow \frac{dP}{dt} = 1400 \times \left(\frac{5600}{P} - 7\right)\left(\frac{5600}{P}\right)^{-2}$$

$$\Longrightarrow \frac{dP}{dt} = 1400 \times \left(\frac{5600}{P} - 7\right)\left(\frac{P^2}{5600^2}\right)$$

$$\Longrightarrow \frac{dP}{dt} = \frac{P^2}{22400}\left(\frac{5600}{P} - 7\right)$$

$$\Longrightarrow \frac{dP}{dt} = \frac{P}{4} - \frac{P^2}{3200}$$

$$\Longrightarrow \frac{dP}{dt} = \frac{800P - P^2}{3200}$$

$$\Longrightarrow \frac{dP}{dt} = \frac{P(800-P)}{3200}$$

AS REQUIRED