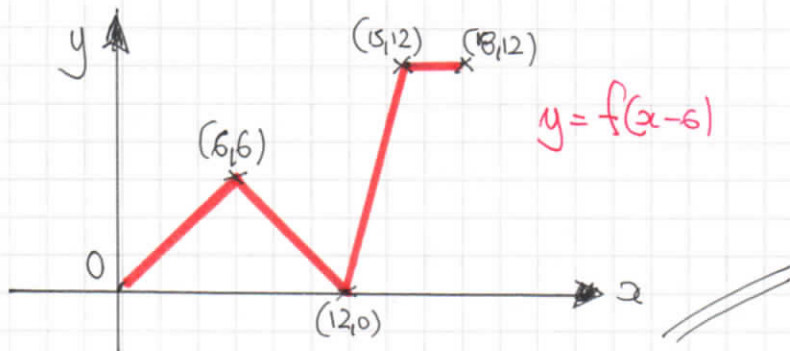
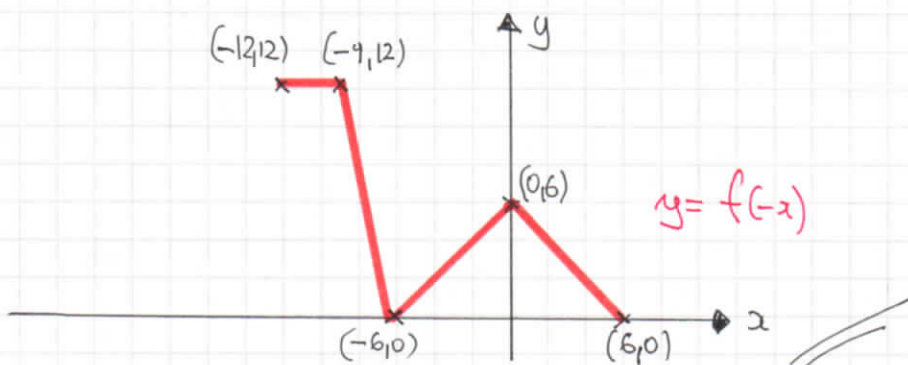


1YGB - SYNOPSIS PAPER E - QUESTION 1

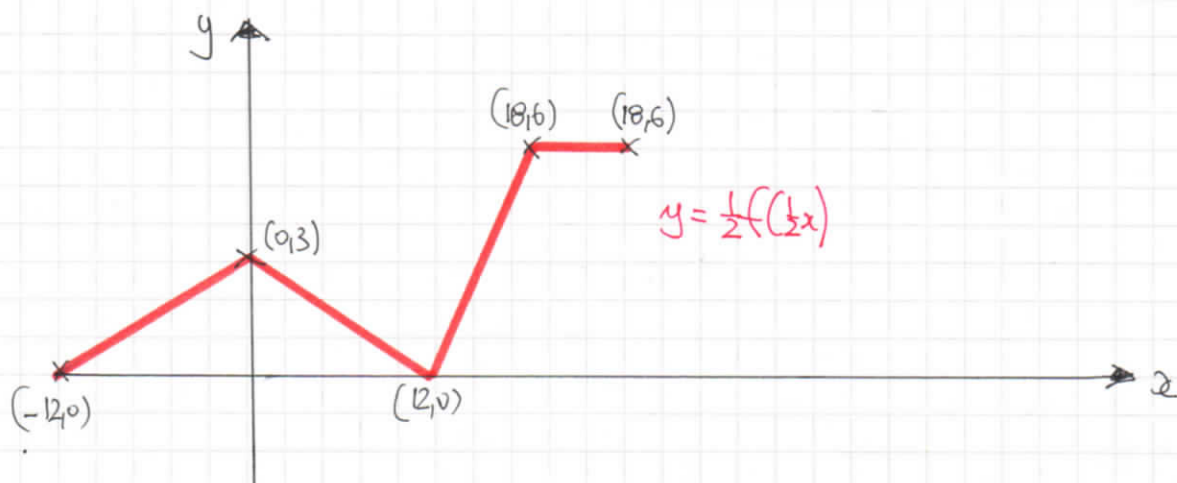
a) $y = f(x-6)$ REPRESENTS A TRANSLATION, 6 UNITS TO THE "RIGHT", IF $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$



b) $y = f(-x)$ REPRESENTS A REFLECTION IN THE y AXIS



c) $y = \frac{1}{2}f(\frac{1}{2}x)$ REPRESENTS A "VERTICAL" STRETCH BY SCALE FACTOR OF $\frac{1}{2}$, FOLLOWED BY A HORIZONTAL STRETCH BY SCALE FACTOR OF 2 (EITHER ORDER)



1YGB - SYNOPTIC PAPER E - QUESTION 2

$$f(x) = x^3 + 3x^2 - 24x + 20$$

a) BY THE FACTOR THEOREM

$$\begin{aligned} f(1) &= 1^3 + 3 \cdot 1^2 - 24 \cdot 1 + 20 \\ &= 1 + 3 - 24 + 20 \\ &= 0 \end{aligned}$$

INDEED A FACTOR

b) BY LONG DIVISION, INSPECTION OR MANIPULATION

$$f(x) = x^2(x-1) + 4x(x-1) - 20(x-1)$$

$$f(x) = (x-1)(x^2 + 4x - 20)$$

c) FROM PART (b)

ENTER $x=1$ OR

$$x^2 + 4x - 20 = 0$$

$$(x+2)^2 - 4 - 20 = 0$$

$$(x+2)^2 = 24$$

$$x+2 = \pm\sqrt{24}$$

$$x = \begin{cases} -2 + 2\sqrt{6} \\ -2 - 2\sqrt{6} \end{cases}$$

- 1 -

d) SOLVING THE EQUATION $f(x) = -8$, AND NOTING THAT $x=2$ MUST BE A REPEATED ROOT, IT $(x-2)^2$ MUST BE A FACTOR

$$\Rightarrow x^3 + 3x^2 - 24x + 20 = -8$$

$$\Rightarrow x^3 + 3x^2 - 24x + 28 = 0$$

$$\Rightarrow (x-2)^2(x+7) = 0$$

$$x = \begin{cases} 2 & \leftarrow Q \\ -7 & \leftarrow P \end{cases}$$

QUICK CHECK

$$\begin{aligned} (x+7)(x-2)^2 &= (x+7)(x^2 - 4x + 4) \\ &= x^3 - 4x^2 + 4x \\ &\quad + 7x^2 - 28x + 28 \\ &= x^3 + 3x^2 - 24x + 28 \end{aligned}$$

FINALLY $f(-7) = (-7)^3 + 3(-7)^2 - 24(-7) + 20 \leftarrow \text{AS A CHECK}$

$$\begin{aligned} &= -343 + 147 + 168 + 20 \\ &= -8 \end{aligned}$$

\therefore $P(-7, -8)$

- 1 -

IYGB - SYNOPTIC PAPER E - QUESTION 3

$$\alpha = \ln(\sec 3y) \quad 0 < y < \frac{\pi}{6}$$

PROCEED BY THE INVERSE RULE

$$\frac{dx}{dy} = \frac{1}{\sec 3y} \times \sec 3y \tan 3y \times 3$$

$$\frac{dx}{dy} = 3 \tan 3y$$

$$\frac{dy}{dx} = \frac{1}{3 \tan 3y}$$

NOW WE MANIPULATE THE EQUATION AS FOLLOWS

$$\Rightarrow \alpha = \ln(\sec 3y)$$

$$\Rightarrow e^\alpha = \sec 3y$$

$$\Rightarrow (e^\alpha)^2 = (\sec 3y)^2$$

$$\Rightarrow e^{2\alpha} = \sec^2 3y$$

$$\Rightarrow e^{2\alpha} = 1 + \tan^2 3y$$

$$\Rightarrow e^{2\alpha} - 1 = \tan^2 3y$$

$$\Rightarrow \tan 3y = \pm \sqrt{e^{2\alpha} - 1}$$

NOW WE OBSERVE THAT

$$0 < y < \frac{\pi}{6}$$

$$0 < 3y < \frac{\pi}{2}$$

$$0 < \tan 3y < +\infty$$

$$\tan 3y > 0$$

$$\Rightarrow \tan 3y = +\sqrt{e^{2\alpha} - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3\sqrt{e^{2\alpha} - 1}}$$

1YGB - SYNOPSIS PAPER 5 - QUESTION 4

a) UNRAILING THE EXPONENTIAL GRAPH AS FOLLOWS

$$\Rightarrow W = ab^t$$

$$\Rightarrow \log W = \log(ab^t)$$

$$\Rightarrow \log W = \log a + \log b^t$$

$$\Rightarrow \log W = \log a + t \log b$$

$$\Rightarrow \log W = (\log b)t + (\log a)$$

$$Y = mX + c$$

b)

t	1	3	4	7	8	10
W	2	4	6.5	19	34	65

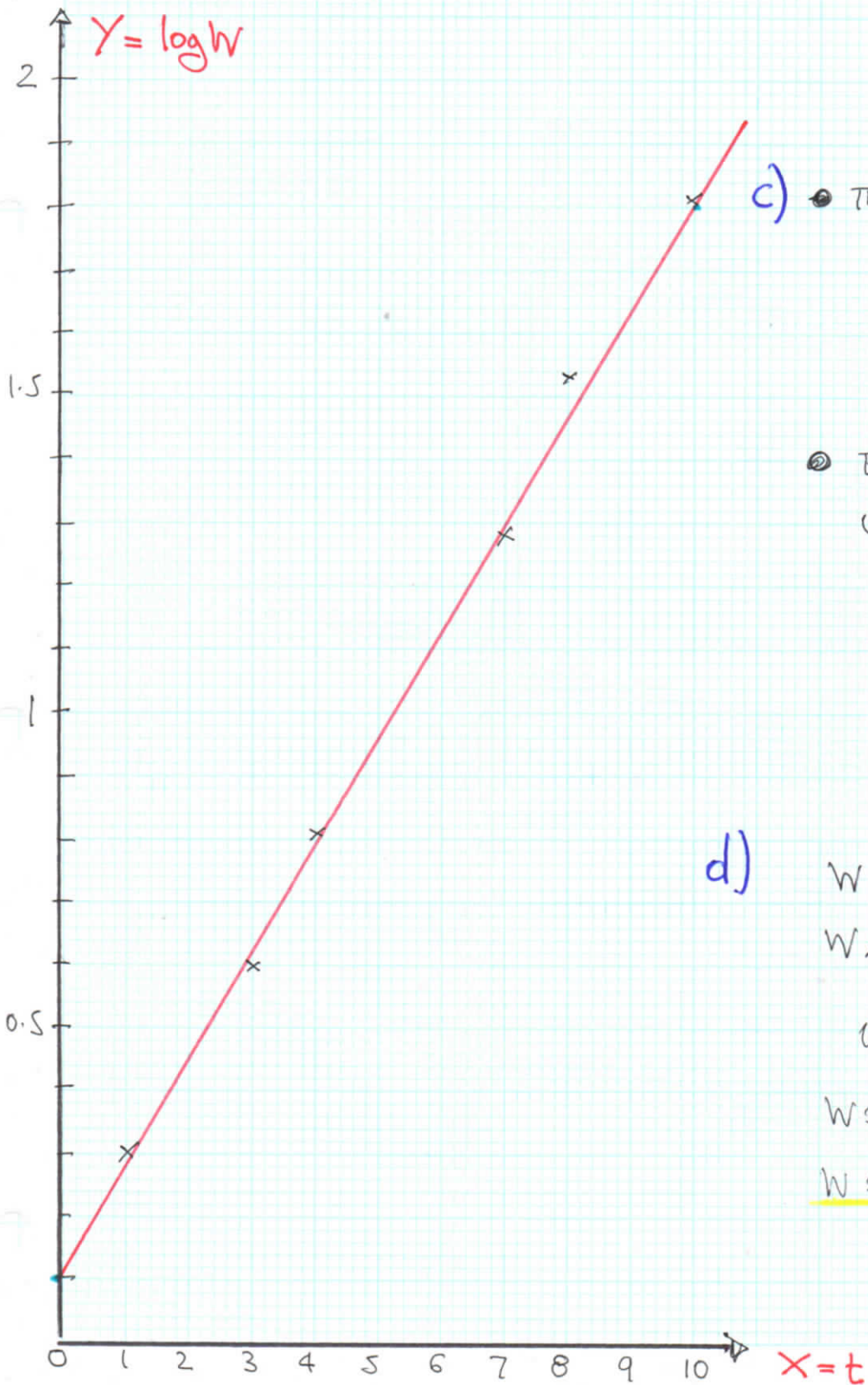
X = t	1	3	4	7	8	10
-------	---	---	---	---	---	----

Y = log w	0.30	0.60	0.81	1.28	1.53	1.81
-----------	------	------	------	------	------	------

PUTTING THESE DATA ON AN ACCURATE DIAGRAM

(SEE NEXT PAGE)

YGB - SYNOPSIS PAPER E - QUESTION 4



c) ● THE "y INTERCEPT" IS $\log a$

$$\log_{10} a \approx 0.1$$

$$a \approx 10^{0.1}$$

$$a \approx \underline{1.26}$$

● THE GRADIENT IS $\log b$

USING (0, 0.1) & (10, 1.8)

$$\log_{10} b \approx \frac{1.8 - 0.1}{10 - 0}$$

$$\log_{10} b \approx 0.17$$

$$b \approx 10^{0.17}$$

$$b \approx \underline{1.48}$$

d)

$$W = ab^t$$

$$W \approx 1.26 \times (1.48)^t$$

when $t=20$

$$W \approx 1.26 \times 1.48^{20}$$

$$W \approx \underline{3200}$$

-1-

1YGB - SYNOPSIS PAGE E - QUESTION 5

FIND THE COORDINATES OF A & C

$$\Rightarrow 3x - 2y = 24$$

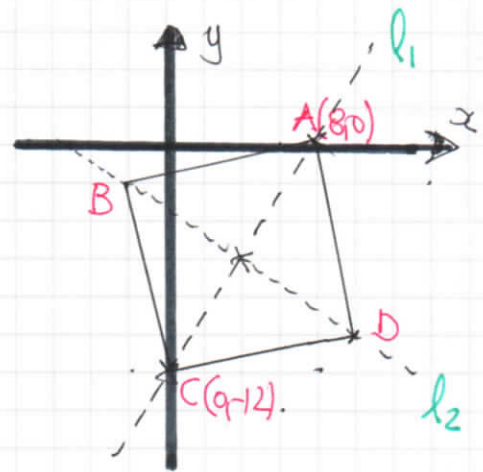
$$\Rightarrow x=0, y=-12 \quad \therefore C(0, -12)$$

$$\Rightarrow y=0, x=8 \quad \therefore A(8, 0)$$

NEXT LOOKING AT THE DIAGRAM

$$\begin{aligned} \bullet M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) &= M\left(\frac{0+8}{2}, \frac{-12+0}{2}\right) \\ &= M(4, -6) \end{aligned}$$

$$\bullet m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 0}{0 - 8} = \frac{3}{2}$$



EQUATION OF l_2 HAS GRADIENT $-\frac{2}{3}$ AND PASSES THROUGH $M(4, -6)$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y + 6 = -\frac{2}{3}(x - 4)$$

$$\Rightarrow 3y + 18 = -2x + 8$$

$$\Rightarrow \underline{2x + 3y + 10 = 0}$$

MGB - SYNOPTIC PAPER E - QUESTION 6

a) USING THE POINT (0,2)

$$\begin{aligned}y &= A \cos(x+60) \\2 &= A \cos 60 \\2 &= \frac{1}{2}A \\A &= 4\end{aligned}$$

b) SOLVING $4 \cos(x+60) = 0$

$$\Rightarrow \cos(x+60) = 0$$

$$\Rightarrow \begin{cases} x+60 = 90 \pm 360n \\ x+60 = 270 \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} x = 30^\circ \pm 360n \\ x = 210^\circ \pm 360n \end{cases}$$

$$\therefore \underline{Q(30^\circ, 0), R(210^\circ, 0), S(390^\circ, 0)}$$

- i -

IYGB - SYNOPTIC PAPER E - QUESTIONS 7

OBTAIN THE FIRST TWO DERIVATIVES

$$y = e^{-2x} + ax e^{-2x}$$

$$\frac{dy}{dx} = -2e^{-2x} + a e^{-2x} + ax(-2e^{-2x}) = e^{-2x}(-2 + a - 2ax)$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= -2e^{-2x}(a - 2 - 2ax) + e^{-2x}(-2a) \\ &= -2e^{-2x}[a - 2 - 2ax + a] \\ &= -2e^{-2x}(2a - 2ax - 2) \\ &= 4e^{-2x}(ax - a + 1)\end{aligned}$$

FOR STATIONARY WAVES $\frac{dy}{dx} = 0$

$$\Rightarrow e^{-2x}(-2 + a - 2ax) = 0$$

$$\Rightarrow -2 + a - 2ax = 0 \quad (e^{-2x} \neq 0)$$

$$\Rightarrow a - 2 = 2ax$$

$$\Rightarrow x = \frac{a-2}{2a}$$

FINALLY WE HAVE

$$\left. \frac{d^2y}{dx^2} \right|_{x = \frac{a-2}{2a}} = 4e^{-2\left(\frac{a-2}{2a}\right)} \left[a \times \frac{a-2}{2a} - a + 1 \right]$$

$$= 4e^{-\frac{a-2}{a}} \left[\frac{a-2}{2} - a + 1 \right]$$

$$= 4e^{\frac{2-a}{a}} \left[\frac{a}{2} - 1 - a + 1 \right]$$

$$= 4e^{\frac{2-a}{a}-1} \left[-\frac{a}{2} \right]$$

$$= -2ae^{\frac{2-a}{a}-1}$$

AS REQUIRED

IYGB - SYNOPSIS PAPER E - QUESTION 8

a) Proceed as follows

$$\begin{aligned}
\sin 3x &= \sin(2x+x) = \sin 2x \cos x + \cos 2x \sin x \\
&= (2\sin x \cos x) \cos x + (1-2\sin^2 x) \sin x \\
&= 2\sin x \cos^2 x + \sin x - 2\sin^3 x \\
&= 2\sin x (1-\sin^2 x) + \sin x - 2\sin^3 x \\
&= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x \\
&= 3\sin x - 4\sin^3 x
\end{aligned}$$

As required

b) Using the result of part (a)

$$\begin{aligned}
&\int \cos x (3\sin x - 4\sin^3 x)^{\frac{2}{3}} dx \\
&= \int \cos x [3\sin x - 2(2\sin^3 x)]^{\frac{2}{3}} dx \\
&= \int \cos x [3\sin x - 4\sin^3 x]^{\frac{2}{3}} dx \\
&= \int \cos x (3\sin x)^{\frac{2}{3}} dx \\
&= \int \cos x (4\sin^2 x) dx \\
&= \int 4\cos x \sin^2 x dx
\end{aligned}$$

By inspection, or using the substitution $u = \sin x$

$$= \frac{4}{3} \sin^3 x + C$$

IVGB - SYNOPTIC PAPER E - QUESTION 9

SOLVING SIMULTANEOUSLY

$$\left. \begin{aligned} y &= x^2 + 6x + 7 \\ y &= 2x + c \end{aligned} \right\} \Rightarrow x^2 + 6x + 7 = 2x + c$$
$$\Rightarrow x^2 + 4x + 7 - c = 0$$

IF A TANGENT THE ABOVE QUADRATIC MUST HAVE REPEATED ROOTS, IT $b^2 - 4ac = 0$

$$\Rightarrow 4^2 - 4 \times 1 \times (7 - c) = 0$$

$$\Rightarrow 16 - 4(7 - c) = 0$$

$$\Rightarrow 16 - 28 + 4c = 0$$

$$\Rightarrow -12 + 4c = 0$$

$$\Rightarrow 4c = 12$$

$$\Rightarrow \underline{c = 3}$$

FINALLY IF $c = 3$ WE HAVE

$$\Rightarrow x^2 + 4x + 7 - c = 0$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow (x + 2)^2 = 0$$

$$\Rightarrow x = -2$$

q USING $y = 2x + 3$

$$y = 2(-2) + 3$$

$$y = -1$$

$$\therefore \underline{(-2, -1)}$$

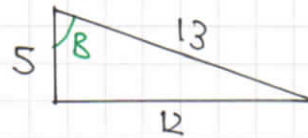
1YGB - SYNOPTIC PAPER E - QUESTION 10

USE THE INFORMATION GIVEN AS FOLLOWS

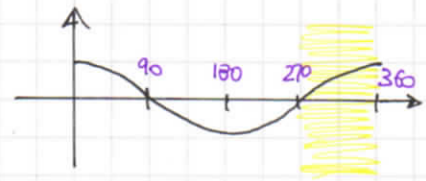
$$\cot A = -\frac{3}{4}$$

$$\tan A = -\frac{4}{3}$$

$$\cos B = \frac{5}{13}$$

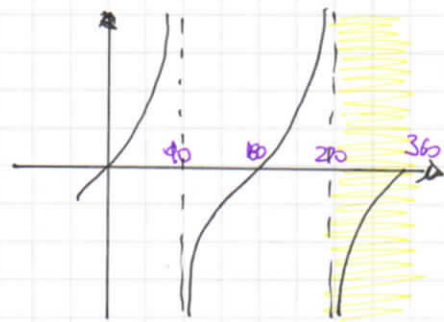


BUT "B IS REFLEX" & POSITIVE COSINE



so $270 < B < 360$ ALSO

so tangent in this range will be negative



$$\therefore \tan B = -\frac{12}{5}$$

USING THE COMPOUND TANGENT IDENTITY

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \left(-\frac{4}{3}\right)\left(-\frac{12}{5}\right)} = \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \frac{48}{15}} \\ &= \frac{-20 - 36}{15 - 48} = \frac{-56}{-33} = \frac{56}{33} \end{aligned}$$

As required

1YGB - SYNOPTIC PAPER E - QUESTION 11

a) I) SETTING $y=0$ IN EACH OF THE EQUATIONS

• $y = |3x-2|$

$0 = |3x-2|$

$0 = 3x-2$

$3x = 2$

$x = \frac{2}{3}$

∴ $A(\frac{2}{3}, 0)$

• $y = |x-5|$

$0 = |x-5|$

$0 = x-5$

$x = 5$

$B(5, 0)$

II) SETTING $x=0$ IN EACH OF THE EQUATIONS

• $y = |3x-2|$

$y = |3 \times 0 - 2|$

$y = |-2|$

$y = 2$

∴ $C(0, 2)$

• $y = |x-5|$

$y = |0-5|$

$y = |-5|$

$y = 5$

∴ $D(0, 5)$

b) FIND THE COORDINATES OF P & Q

⇒ $|3x-2| = |x-5|$

⇒ $\begin{cases} 3x-2 = x-5 \\ 3x-2 = 5-x \end{cases}$

⇒ $\begin{cases} 2x = -3 \\ 4x = 7 \end{cases}$

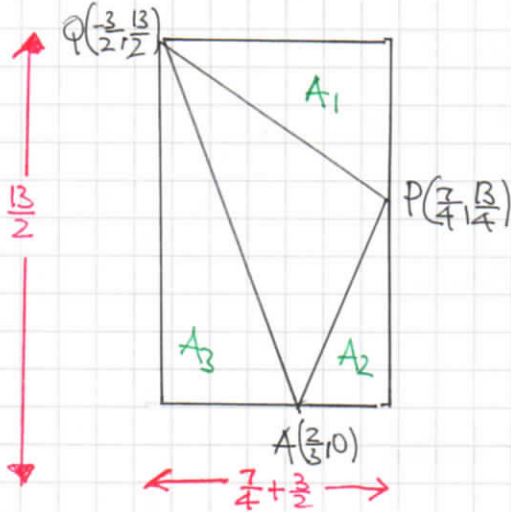
⇒ $\begin{cases} x = -\frac{3}{2} \\ x = \frac{7}{4} \end{cases}$

⇒ $y < \begin{cases} |-\frac{3}{2}-5| = |-\frac{13}{2}| = \frac{13}{2} \\ |\frac{7}{4}-5| = |-\frac{13}{4}| = \frac{13}{4} \end{cases}$

IYGB - SYNOPTIC PAPER E - QUESTION 11

$\therefore P\left(\frac{7}{4}, \frac{13}{4}\right)$ & $Q\left(-\frac{3}{2}, \frac{13}{2}\right)$

LOOKING AT THE DIAGRAM



AREA OF RECTANGLE

$$\frac{13}{2} \times \left(\frac{7}{4} + \frac{3}{2}\right) = \frac{13}{2} \times \frac{13}{4} = \frac{169}{8}$$

$$\begin{aligned} \text{AREA } A_1 &= \frac{1}{2} \times \left(\frac{13}{2} - \frac{13}{4}\right) \left(\frac{7}{4} + \frac{3}{2}\right) \\ &= \frac{1}{2} \times \frac{13}{4} \times \frac{13}{4} = \frac{169}{32} \end{aligned}$$

$$\begin{aligned} \text{AREA } A_2 &= \frac{1}{2} \times \left(\frac{7}{4} - \frac{3}{2}\right) \left(\frac{13}{4}\right) \\ &= \frac{1}{2} \times \frac{13}{12} \times \frac{13}{4} = \frac{169}{96} \end{aligned}$$

$$\begin{aligned} \text{AREA } A_3 &= \frac{1}{2} \times \left(\frac{3}{2} + \frac{3}{2}\right) \times \frac{13}{2} \\ &= \frac{1}{2} \times \frac{13}{6} \times \frac{13}{2} \\ &= \frac{169}{24} \end{aligned}$$

$$\begin{aligned} \therefore \text{RESPONSE AREA} &= \frac{169}{8} - \left(\frac{169}{32} + \frac{169}{96} + \frac{169}{24}\right) \\ &= \frac{169}{24} \end{aligned}$$

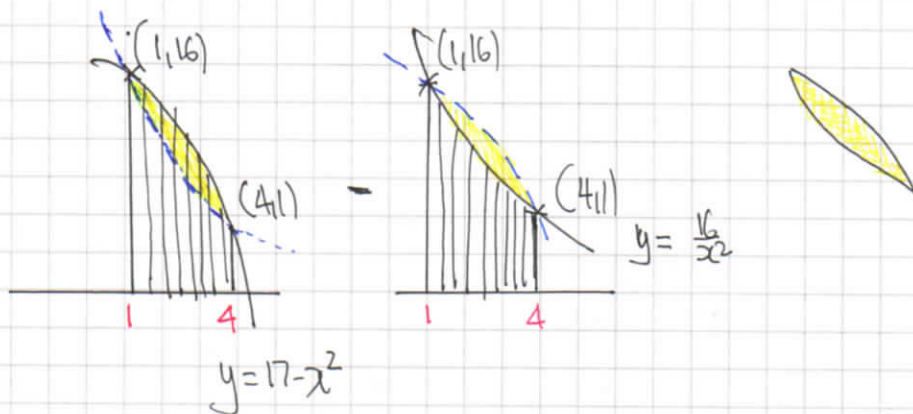
1YGB - SYNOPTIC PAPER E - QUESTION 12

START BY FINDING THE INTERSECTIONS OF THE TWO CURVES

$$\left. \begin{array}{l} y = \frac{16}{x^2} \\ y = 17 - x^2 \end{array} \right\} \Rightarrow \frac{16}{x^2} = 17 - x^2$$
$$\Rightarrow x^2 - 17 + \frac{16}{x^2} = 0$$
$$\Rightarrow x^4 - 17x^2 + 16 = 0$$
$$\Rightarrow (x^2 - 1)(x^2 - 16) = 0$$
$$\Rightarrow x^2 = \begin{cases} 1 \\ 16 \end{cases}$$
$$\Rightarrow x = \begin{cases} +1 \\ +4 \end{cases} \quad (\text{FIRST QUADRANT})$$
$$\Rightarrow y = \begin{cases} 16 \\ 1 \end{cases}$$

\therefore $(1, 16)$ & $(4, 1)$

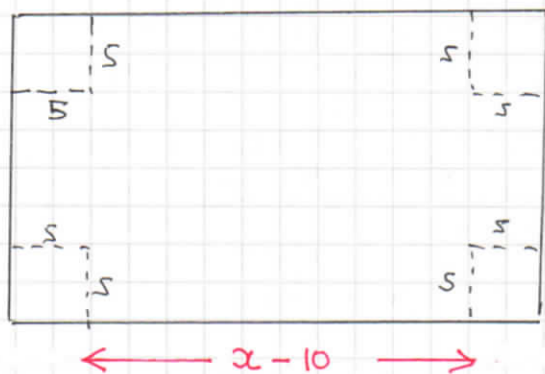
LOOKING AT THE DIAGRAM BELOW



$$\text{AREA} = \int_1^4 (17 - x^2) dx - \int_1^4 \frac{16}{x^2} dx = \int_1^4 \left(17 - x^2 - \frac{16}{x^2} \right) dx$$

1YGB - SYNOPTIC PAPER E - QUESTION 12

$$\begin{aligned} \text{AREA} &= \left[17x - \frac{1}{3}x^3 + \frac{16}{x} \right]_1^4 \\ &= \left(68 - \frac{64}{3} + 4 \right) - \left(17 - \frac{1}{3} + 16 \right) \\ &= \frac{152}{3} - \frac{98}{3} \\ &= \underline{18} \end{aligned}$$

1YGB-SYNOPSIS PAPER E - QUESTION 13a) LOOKING AT THE DIAGRAM

THE MEASUREMENTS OF THE CARD MUST BE
 x & $\frac{1200}{x}$

ALSO WE MUST HAVE

$$\frac{1200}{x} - 10 > 0 \quad \& \quad x - 10 > 0$$

$$\frac{1200}{x} > 10 \quad \& \quad x > 10$$

$$1200 > 10x$$

$$x < 120$$

FINALLY THE VOLUME MUST EXCEED 2850

$$\Rightarrow \left(\frac{1200}{x} - 10\right)(x - 10) \times 5 > 2850$$

$$\Rightarrow 10 \left(\frac{120}{x} - 1\right)(x - 10) \times 5 > 2850$$

$$\Rightarrow \left(\frac{120}{x} - 1\right)(x - 10) > 57 \quad \swarrow \div 50$$

$$\Rightarrow 120 - \frac{1200}{x} - x + 10 > 57$$

$$\Rightarrow 120x - 1200 - x^2 + 10x > 57x \quad \swarrow \times x \quad (x > 0)$$

$$\Rightarrow -x^2 + 73x - 1200 > 0$$

$$\Rightarrow \underline{x^2 - 73x + 1200 < 0}$$

AS REQUIRED

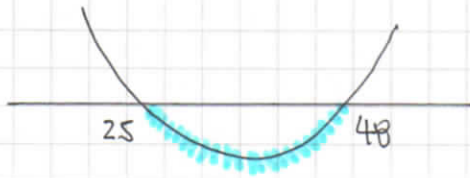
YGB - SYNOPTIC PAPER E - QUESTION 13

b) BY THE QUADRATIC FORMULA (OR FACTORIZATION)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+73 \pm \sqrt{(-73)^2 - 4 \times 1 \times 1200}}{2 \times 1}$$

$$x = \begin{cases} 48 \\ 25 \end{cases}$$



$25 < x < 48$

which also satisfies

$x > 0$ AND $x < 120$

- 1 -

INGB - SYNOPTIC PAPER E - QUESTION 14

SOLVE BY SEPARATING VARIABLES

$$\Rightarrow \frac{dy}{dx} = \frac{y^2-1}{x}$$

$$\Rightarrow \frac{1}{y^2-1} dy = \frac{1}{x} dx$$

$$\Rightarrow \int \frac{1}{(y-1)(y+1)} dy = \int \frac{1}{x} dx$$

PROCEED BY PARTIAL FRACTIONS

$$\frac{1}{(y-1)(y+1)} \equiv \frac{A}{y-1} + \frac{B}{y+1}$$

$$1 \equiv A(y+1) + B(y-1)$$

• IF $y=1$

$$1 = 2A$$

$$A = \frac{1}{2}$$

• IF $y=-1$

$$1 = -2B$$

$$B = -\frac{1}{2}$$

RETURNING TO THE O.D.T.

$$\Rightarrow \int \frac{\frac{1}{2}}{y-1} - \frac{\frac{1}{2}}{y+1} dy = \int \frac{1}{x}$$

$$\Rightarrow \int \frac{1}{y-1} - \frac{1}{y+1} dy = \int \frac{2}{x}$$

$$\Rightarrow \ln|y-1| - \ln|y+1| = 2\ln|x| + \ln A$$

$$\Rightarrow \ln \left| \frac{y-1}{y+1} \right| = \ln(Ax^2)$$

$$\Rightarrow \frac{y-1}{y+1} = Ax^2$$

APPLY CONDITION (1,2)

$$\frac{2-1}{2+1} = A \times 1^2$$

$$\frac{1}{3} = A$$

$$\therefore \frac{y-1}{y+1} = \frac{1}{3}x^2$$

REARRANGING

$$\Rightarrow \frac{y-1}{y+1} = \frac{x^2}{3}$$

$$\Rightarrow 3y-3 = x^2y+x^2$$

$$\Rightarrow 3y-yx^2 = 3+x^2$$

$$\Rightarrow y(3-x^2) = 3+x^2$$

$$\Rightarrow y = \frac{3+x^2}{3-x^2}$$

AS REQUIRED

YGB-SYNOPSIS PAPER E - QUESTION 15

a) FORMING AN EQUATION

$$\Rightarrow u_4 - u_1 = 5(u_3 - u_2)$$

$$\Rightarrow ar^3 - a = 5(ar^2 - ar)$$

$$\Rightarrow ar^3 - a = 5ar^2 - 5ar$$

$$\Rightarrow ar^3 - 5ar^2 + 5ar - a = 0$$

$$\Rightarrow \underline{r^3 - 5r^2 + 5r - 1 = 0}$$

$$u_4 = ar^{4-1}$$

AS REQUIRED

b) BY INSPECTION $r=1$, IS A SOLUTION

$$\Rightarrow (r-1)(r^2 + Ar + 1) = 0$$

$$\begin{aligned} r - Ar &= 5r \\ -Ar &= 4r \\ A &= -4 \end{aligned}$$

$$\Rightarrow (r-1)(r^2 - 4r + 1) = 0$$

↓

$$\Rightarrow r^2 - 4r + 1 = 0$$

$$\Rightarrow (r-2)^2 - 2^2 + 1 = 0$$

$$\Rightarrow (r-2)^2 = 3$$

$$\Rightarrow r-2 = \pm\sqrt{3}$$

$$\Rightarrow r = 2 \pm \sqrt{3}$$

$$\therefore r = \begin{cases} 2 + \sqrt{3} \\ 2 - \sqrt{3} \end{cases}$$

1YGB - SYNOPTIC PAPER E - QUESTION 15

c) THE COMMON RATIO IS $2 - \sqrt{3}$, AS THIS IS THE ONLY VALUE OF r WHICH PRODUCES A SUM TO INFINITY ($-1 < r < 1$)

$$\Rightarrow \sum_{n=0}^{\infty} r^n = \frac{a}{1-r}$$

$$\Rightarrow \sqrt{6} + \sqrt{2} = \frac{a}{1 - (2 - \sqrt{3})}$$

$$\Rightarrow \sqrt{6} + \sqrt{2} = \frac{a}{-1 + \sqrt{3}}$$

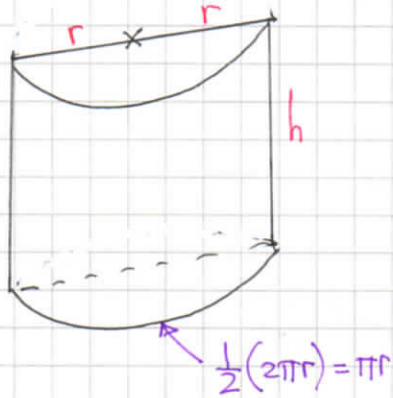
$$\Rightarrow a = (\sqrt{6} + \sqrt{2})(-1 + \sqrt{3})$$

$$\Rightarrow a = \cancel{-\sqrt{6}} + \sqrt{6} - \sqrt{2} + \sqrt{6}$$

$$\Rightarrow a = 3\sqrt{6} - \sqrt{2}$$

$$\Rightarrow a = \underline{2\sqrt{2}}$$

1YGB - SYNOPSIS PAPER E - QUESTION 16



● LET THE RADIUS BE r & THE VERTICAL HEIGHT h

● CONSTRAINT ON SURFACE AREA

$$\Rightarrow \pi r^2 + \pi r h + 2r h = \sqrt[3]{27\pi^4}$$

$$\Rightarrow \pi r h + 2r h = \sqrt[3]{27\pi^4} - \pi r^2$$

$$\Rightarrow r h (\pi + 2) = 3\pi^{4/3} - \pi r^2$$

$$\Rightarrow r h = \frac{3\pi^{4/3} - \pi r^2}{\pi + 2}$$

● NOW LOOKING AT THE VOLUME

$$\Rightarrow V = \frac{1}{2} (\pi r^2 h) = \frac{1}{2} \pi r (r h) = \frac{1}{2} \pi r \left(\frac{3\pi^{4/3} - \pi r^2}{\pi + 2} \right)$$

$$\Rightarrow V = \frac{\pi r (3\pi^{4/3} - \pi r^2)}{2(\pi + 2)}$$

$$\Rightarrow V = \frac{\pi}{2(\pi + 2)} [3\pi^{4/3} r - \pi r^3]$$

● DIFFERENTIATE & SOLVE FOR ZERO

$$\Rightarrow \frac{dV}{dr} = \frac{\pi}{2(\pi + 2)} [3\pi^{1/3} - 3\pi r^2]$$

$$\Rightarrow 0 = \frac{\pi}{2(\pi + 2)} [3\pi^{1/3} - 3\pi r^2]$$

$$\Rightarrow \cancel{3\pi^{1/3}} - \cancel{3\pi} r^2 = 0$$

$$\Rightarrow \pi^{1/3} = \pi r^2$$

IYGB - SYNOPSIS PAPER E - QUESTION 16

$$\Rightarrow r^2 = \pi^{-\frac{2}{3}}$$

$$\Rightarrow r^2 = \frac{1}{\pi^{\frac{2}{3}}}$$

$$\Rightarrow r = + \frac{1}{\pi^{\frac{1}{3}}}$$

FINALLY TO OBTAIN THE MAXIMUM VOLUME

$$\Rightarrow V = \frac{\pi}{2(\pi+2)} \left[3\pi^{\frac{1}{3}} - \pi r^3 \right]$$

$$\Rightarrow V = \frac{\pi r}{2(\pi+2)} \left[3\pi^{\frac{1}{3}} - \pi r^2 \right]$$

$$\Rightarrow V_{\text{MAX}} = \frac{\pi}{2(\pi+2)} \left(\frac{1}{\pi^{\frac{1}{3}}} \right) \left[3\pi^{\frac{1}{3}} - \pi \times \frac{1}{\pi^{\frac{2}{3}}} \right]$$

$$\Rightarrow V_{\text{MAX}} = \frac{\pi^{\frac{2}{3}}}{2(\pi+2)} \left[3\pi^{\frac{1}{3}} - \pi^{\frac{1}{3}} \right]$$

$$\Rightarrow V_{\text{MAX}} = \frac{\pi^{\frac{2}{3}}}{2(\pi+2)} \times 2\pi^{\frac{1}{3}}$$

$$\Rightarrow V_{\text{MAX}} = \frac{2\pi}{2(\pi+2)}$$

$$\Rightarrow V_{\text{MAX}} = \frac{\pi}{\pi+2}$$

1YGB - SYNOPTIC PAPER 6 - QUESTION 17

a) SUBSTITUTE, EXPAND & TIDY

$$\begin{aligned}4x^2 + 4x + 17 &= 4 \left[\frac{1}{2}(-1 + 4\tan\theta) \right]^2 + 4 \left[\frac{1}{2}(-1 + 4\tan\theta) \right] + 17 \\&= 4 \times \frac{1}{4} (-1 + 4\tan\theta)^2 + 2(-1 + 4\tan\theta) + 17 \\&= 1 - 8\tan\theta + 16\tan^2\theta - 2 + 8\tan\theta + 17 \\&= 16 + 16\tan^2\theta \\&= 16(1 + \tan^2\theta) \\&= \underline{16\sec^2\theta} \quad \swarrow \text{AS REQUIRED}\end{aligned}$$

b) BY SUBSTITUTION FROM PART (a)

$$\Rightarrow x = \frac{1}{2}(-1 + 4\tan\theta) = -\frac{1}{2} + 2\tan\theta$$

$$\Rightarrow \frac{dx}{d\theta} = 2\sec^2\theta$$

$$\Rightarrow \underline{dx = 2\sec^2\theta d\theta}$$

• when $x = -\frac{1}{2}$

$$-\frac{1}{2} = -\frac{1}{2} + 2\tan\theta$$

$$0 = 2\tan\theta$$

$$\underline{\theta = 0}$$

• when $x = \frac{3}{2}$

$$\frac{3}{2} = -\frac{1}{2} + 2\tan\theta$$

$$2 = 2\tan\theta$$

$$\underline{\tan\theta = 1}$$

$$\underline{\theta = \frac{\pi}{4}}$$

1YGB - SYNOPSIS PAPER E - QUESTION 17

TRANSFORMING THE INTEGRAL VIA UDS

$$\begin{aligned} \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 + 4x + 17} dx &= \int_0^{\frac{\pi}{4}} \frac{1}{16 \cancel{\sec^2 \theta}} \left(2 \cancel{\sec^2 \theta} d\theta \right) \\ &= \int_0^{\frac{\pi}{4}} \frac{1}{8} d\theta \\ &= \left[\frac{1}{8} \theta \right]_0^{\frac{\pi}{4}} \\ &= \frac{\pi}{32} - 0 \\ &= \frac{\pi}{32} \end{aligned}$$

YGB - SYNOPTIC PAPER E - QUESTION 18

a) COMPLETING THE SQUARE

$$\Rightarrow x^2 + y^2 - 14x + 33 = 0$$

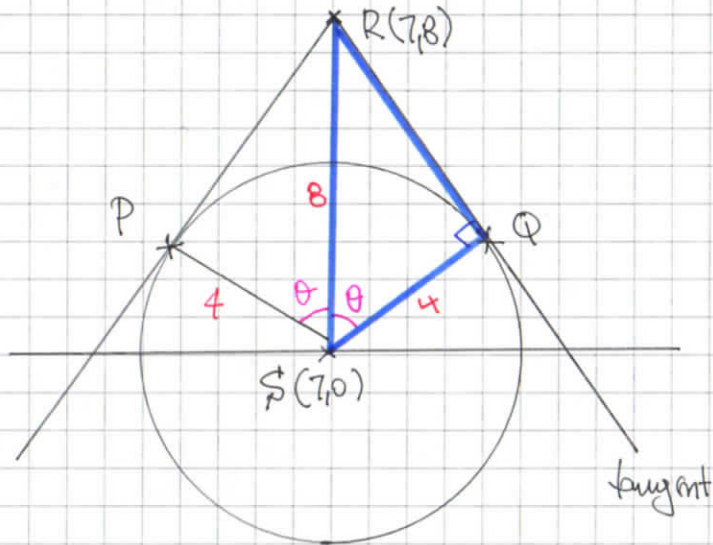
$$\Rightarrow x^2 - 14x + y^2 + 33 = 0$$

$$\Rightarrow (x-7)^2 - 49 + y^2 + 33 = 0$$

$$\Rightarrow (x-7)^2 + y^2 = 16$$

CIRCLE (7,0) & RADIUS = $\sqrt{16} = 4$

b) LOOKING AT THE DIAGRAM



USING THE RIGHT ANGLED TRIANGLE $\triangle SQR$

$$\cos \theta = \frac{4}{8} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

AREA OF SECTOR $\triangle SQR$

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} \times 4^2 \times (2 \times \frac{\pi}{3}) = \frac{16}{3} \pi$$

"'" θ IS 2θ IN THE PICTURE

AREA OF TRIANGLE $\triangle SQR$

$$A = \frac{1}{2} |RS| |SQ| \sin \frac{\pi}{3}$$

$$A = \frac{1}{2} \times 4 \times 8 \times \frac{\sqrt{3}}{2}$$

$$A = 8\sqrt{3}$$

REQUIRED AREA

2x TRIANGLE $\triangle SQR$ - SECTOR

$$= 2 \times 8\sqrt{3} - \frac{16}{3} \pi$$

$$= 16\sqrt{3} - \frac{16}{3} \pi$$

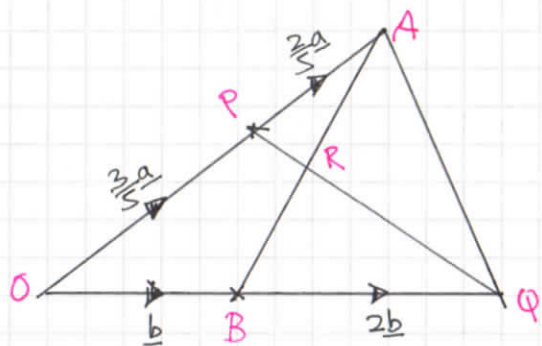
$$= \frac{16}{3} [3\sqrt{3} - \pi]$$

IS REQUIRED

— 1 —

1YGB - SYNOPTIC PAPER E - QUESTION 10

a) LOOKING AT THE DIAGRAM



$$\begin{aligned} \vec{OR} &= \vec{OA} + \vec{AR} \\ \vec{OR} &= \underline{a} + h \vec{AB} \\ \vec{OR} &= \underline{a} + h(\vec{AO} + \vec{OB}) \\ \vec{OR} &= \underline{a} + h(-\underline{a} + \underline{b}) \\ \vec{OR} &= \underline{a} - h\underline{a} + h\underline{b} \\ \vec{OR} &= \underline{(1-h)a} + h\underline{b} \end{aligned}$$

b) SIMILARLY WE HAVE

$$\begin{aligned} \vec{OR} &= \vec{OP} + \vec{PR} \\ \vec{OR} &= \vec{OP} + k \vec{PQ} \end{aligned}$$

$$\vec{OR} = \vec{OP} + k(\vec{PO} + \vec{OQ})$$

$$\vec{OR} = \frac{2}{5}\underline{a} + k(-\frac{2}{5}\underline{a} + 3\underline{b})$$

$$\vec{OR} = \frac{2}{5}\underline{a} - \frac{2}{5}k\underline{a} + 3k\underline{b}$$

$$\vec{OR} = \underline{\frac{2}{5}(1-k)a} + 3k\underline{b}$$

b) SOLVING SIMULTANEOUSLY

$$\text{I)} \Rightarrow (1-h)\underline{a} + h\underline{b} = \frac{2}{5}(1-k)\underline{a} + 3k\underline{b}$$

$$\left\{ \begin{aligned} 1-h &= \frac{2}{5}(1-k) \\ h &= 3k \end{aligned} \right\} \Rightarrow$$

$$1-3k = \frac{2}{5}(1-k)$$

$$5-15k = 2-2k$$

$$3 = 13k$$

$$k = \frac{3}{13}$$

$$h = \frac{9}{13}$$

II) BY INSPECTION

$$\vec{PR} = k \vec{PQ}$$

$$\vec{PR} = \frac{1}{6} \vec{PQ}$$

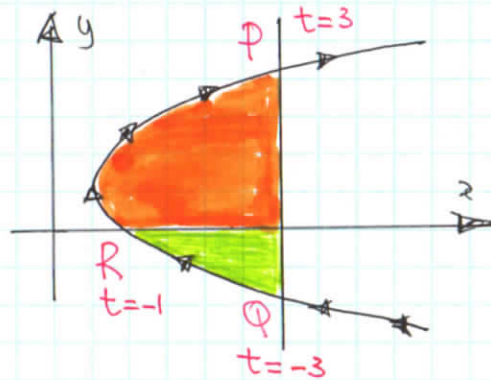
$$\frac{\vec{PR}}{\vec{PQ}} = \frac{1}{6}$$

$$\therefore \underline{1:6}$$

YGB - SYNOPSIS PART E - QUESTION 20

START BY DETERMINING THE VALUE OF t ,
AT P, Q & R (DIAGRAM)

● $y=0$
 $2t+2=0$
 $t=-1$
 ↑
 R(2,0)



● $x=10$
 $t^2+1=10$
 $t^2=9$
 $t = \begin{cases} -3 & \leftarrow Q(10, -4) \\ 3 & \leftarrow P(10, 8) \end{cases}$

INTEGRATING IN PARAMETRIC IN "ONE GO"

$$\begin{aligned} \text{AREA} &= \int_{x_1}^{x_2} y(x) dx = \int_{t_1}^{t_2} y(t) \frac{dx}{dt} dt \\ &= \int_{-3}^3 (2t+2)(2t) dt = \int_{-3}^3 4t^2 + 4t dt \end{aligned}$$

↖ R ODD
FORM (x2)

$$\begin{aligned} &= 2 \int_0^3 4t^2 dt = 8 \int_0^3 t^2 dt \\ &= \left[\frac{8}{3} t^3 \right]_0^3 = \left(\frac{8}{3} \times 27 \right) - 0 \\ &= 72 \end{aligned}$$

↖ REQUIRED

ALTERNATIVE BY SPLITTING THE AREA IN 2

"ORANGE AREA" = $\int_{-1}^3 4t^2 + 4t dt$

"GREEN AREA" = $-\int_{-1}^{-3} 4t^2 + 4t dt$

↑
 AS THIS AREA IS BELOW THE
 X-AXIS THE MINUS WILL MAKE
 IT POSITIVE

LYGB - SYNOPSIS PAPER E - QUESTION 20

HENCE THE TOTAL AREA CAN BE FOUND

$$\begin{aligned} \text{TOTAL AREA} &= \int_{-1}^3 4t^2 + 4t \, dt - \int_{-1}^{-3} 4t^2 + 4t \, dt \\ &= \int_{-1}^3 4t^2 + 4t \, dt + \int_{-3}^{-1} 4t^2 + 4t \, dt \\ &= \int_{-3}^3 4t^2 + \cancel{4t} \, dt \\ &\quad \uparrow \text{ ODD IN + SYMMETRICAL DOMAIN} \\ &\quad \text{EVEN IN + SYMMETRICAL DOMAIN} \Rightarrow \times 2 \\ &= 2 \int_{-3}^3 4t^2 \, dt \\ &= \dots \text{ AS BEFORE.} \end{aligned}$$

- 1 -

1YGB - SYNOPTIC PAPER E - QUESTION 21

a) PROCEED TO ELIMINATE THE LOGARITHMS

$$\Rightarrow \log_2(256x^2) = 1 + 2\log_2\left(\frac{1}{2}x^4\right)$$

$$\Rightarrow \log_2(256x^2) = \log_2 2 + \log_2\left(\frac{1}{2}x^4\right)^2$$

$$\Rightarrow \log_2(256x^2) = \log_2 2 + \log_2\left(\frac{1}{4}x^8\right)$$

$$\Rightarrow \log_2(256x^2) = \log_2\left[2 \times \frac{1}{4}x^8\right]$$

$$\Rightarrow \log_2[256x^2] = \log_2\left[\frac{1}{2}x^8\right]$$

EXTRACTING LOGS

$$\Rightarrow 256x^2 = \frac{1}{2}x^8$$

$$\Rightarrow \frac{1}{2}x^8 - 256x^2 = 0$$

$$\Rightarrow x^8 - 512x^2 = 0$$

$$\Rightarrow x^2(x^6 - 512) = 0$$

$$\Rightarrow x^6 - 512 = 0 \quad \left(x^2 \neq 0 \text{ BECAUSE OF THE LOGS}\right)$$

$$\Rightarrow (x^2)^3 = 512 \quad \left(512 \text{ CUBE ROOTS, BUT IT DOES NOT SQUARE ROOT}\right)$$

$$\Rightarrow x^2 = 8$$

$$\Rightarrow \underline{x = \pm \sqrt{8} = \pm 2\sqrt{2}}$$

BOTH ARE FINE

TYGB - SYNOPTIC PAPER E - QUESTION 21

b) WITH A SIMILAR METHOD TO PART (a)

$$\Rightarrow 2\log_2\left(\frac{y}{2}\right) + \log_2\sqrt{y} = 8$$

$$\Rightarrow \log_2\left(\frac{y}{2}\right)^2 + \log_2 y^{\frac{1}{2}} = 8\log_2 2$$

$$\Rightarrow \log_2\left(\frac{y^2}{4}\right) + \log_2 y^{\frac{1}{2}} = \log_2 256$$

$$\Rightarrow \log_2\left(\frac{y^2}{4} \times y^{\frac{1}{2}}\right) = \log_2 256$$

EXTRACTING FROM THE LOGS

$$\Rightarrow \frac{y^{\frac{5}{2}}}{4} = 256$$

$$\Rightarrow y^{\frac{5}{2}} = 1024$$

$$\Rightarrow \left(y^{\frac{5}{2}}\right)^{\frac{2}{5}} = 1024^{\frac{2}{5}}$$

$$\Rightarrow y = \left(\sqrt[5]{1024}\right)^2$$

$$\Rightarrow y = 4^2$$

$$\Rightarrow \underline{y = 16}$$