IYGB - SYNOPTIC PAPER G - QUESTION 1

THE POINT $P(3,k)$ LIES ON THE CURVE

$$y = x^2 + ax - 4$$
$$k = 3^2 + a \times 3 - 4$$
$$k = 9 + 3a - 4$$
$$k = 3a + 5$$

DIFFERENTIATE & USE THE FACT THAT $\left.\frac{dy}{dx}\right|_{x=3} = 3$

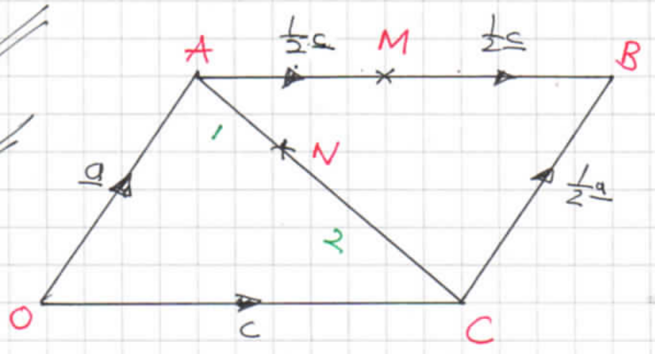
$$\frac{dy}{dx} = 2x + a$$
$$3 = 2 \times 3 + a$$
$$3 = 6 + a$$
$$a = -3$$

& USING $k = 3a + 5$

$$k = 3(-3) + 5$$
$$k = -4$$

IYGB - SYNOPTIC PAPER G - QUESTION 2

a) START BY DRAWING A DIAGRAM, AND LABEL VECTORS



- $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\underline{a} + \underline{c} = \underline{c} - \underline{a}$
- $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AC} = \frac{1}{3}(\underline{c}-\underline{a}) = \frac{1}{3}\underline{c} - \frac{1}{3}\underline{a}$
- $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \underline{a} + \frac{1}{3}\underline{c} - \frac{1}{3}\underline{a}$

  $= \frac{2}{3}\underline{a} + \frac{1}{3}\underline{c}$
- $\overrightarrow{NM} = \overrightarrow{NA} + \overrightarrow{AM} = -\overrightarrow{AN} + \overrightarrow{AM}$

  $= -\left(\frac{1}{3}\underline{c} - \frac{1}{3}\underline{a}\right) + \frac{1}{2}\underline{c} = \frac{1}{3}\underline{a} + \frac{1}{6}\underline{c}$

b) ARGUE AS FOLLOWS

$\overrightarrow{ON} = \frac{2}{3}a + \frac{1}{3}\underline{c} = \frac{1}{3}(2\underline{a} + \underline{c})$

$\overrightarrow{NM} = \frac{1}{3}\underline{a} + \frac{1}{6}\underline{c} = \frac{1}{6}(2a + \underline{c})$

AS $\overrightarrow{ON}$ & $\overrightarrow{NM}$ ARE IN THE SAME DIRECTION & SHARE THE POINT N, O, N & M MUST BE COLLINEAR

IYGB - SYNOPTIC PAPER G - QUESTION 3

EXPAND AND COMPARE COEFFICIENTS ON BOTH SIDES

$$\Rightarrow 5x^2 + Ax + 7 \equiv B(x-2)^2 + C$$
$$\Rightarrow 5x^2 + Ax + 7 \equiv B(x^2-4x+4) + C$$
$$\Rightarrow 5x^2 + Ax + 7 \equiv Bx^2 - 4Bx + 4B + C$$
$$\Rightarrow 5x^2 + Ax + 7 \equiv Bx^2 - 4Bx + (4B+C)$$

HENCE WE HAVE

$[x^2]$: $B = 5$ //

$[x]$: $A = -4B$

$A = -4 \times 5$

$A = -20$ //

$[x^0]$ $4B + C = 7$

$4 \times 5 + C = 7$

$C = -13$ //

IYGB - SYNOPTIC PAPER G - QUESTION 4

a) $x^2+y^2-6x+14y+33=0$

COMPLETE THE SQUARES IN x & IN y

$\Rightarrow x^2-6x+y^2+14y+33=0$

$\Rightarrow (x-3)^2-9+(y+7)^2-49+33=0$

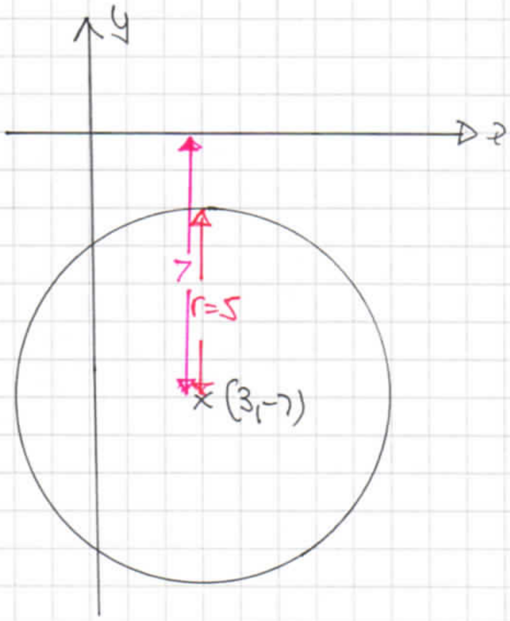
$\Rightarrow (x-3)^2+(y+7)^2=25$

$\therefore$ CENTRE AT $(3,-7)$ & RADIUS $=\sqrt{25}=5$

b) REFERRING TO THE DIAGRAM

AS $r=5<7$ (y COORDINATE OF THE CENTRE IS 7 UNITS BELOW THE x AXIS)

$\therefore$ THE CIRCLE LIES ENTIRELY BELOW THE x AXIS



c) USING THE DISTANCE FORMULA FOR THE CENTRE $(3,-7)$ & $P(6,k)$

$\Rightarrow \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} > 5$

$\Rightarrow \sqrt{[k-(-7)]^2+(6-3)^2} > 5$

$\Rightarrow \sqrt{(k+7)^2+3^2} > 5$

$\Rightarrow \sqrt{k^2+14k+49+9} > 5$

$\Rightarrow \sqrt{k^2+14k+58} > 5$

IYGB - SYNOPTIC PAPER G - QUESTION 4

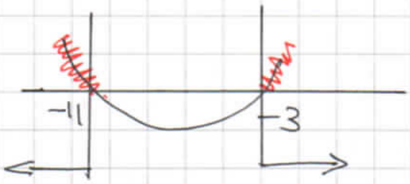
SQUARING BOTH SIDES

$\Rightarrow \quad k^2 + 14k + 58 > 25$

$\Rightarrow \quad k^2 + 14k + 33 > 0$

$\Rightarrow \quad (k+3)(k+11) > 0$

C.V = -3, -11



$\therefore \quad k < -11 \quad$ OR $\quad k > -3$

IYGB - SYNOPTIC PAPER G - QUESTION 5

a) APPROACH AS PER USUAL

$$2\cos\theta + 3\sin\theta \equiv R\cos(\theta - \alpha)$$

$$2\cos\theta + 3\sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$2\cos\theta + 3\sin\theta \equiv (R\cos\alpha)\cos\theta + (R\sin\alpha)\sin\theta$$

COMPARING

$$R\cos\alpha = 2$$
$$R\sin\alpha = 3$$

SQUARE AND ADD

$$R = \sqrt{2^2 + 3^2}$$

$$R = \sqrt{13} \quad (R > 0)$$

DIVIDING SIDE BY SIDE

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{2}$$

$$\tan\alpha = \frac{3}{2}$$

$$\alpha \simeq 0.983^c$$

$$\therefore f(\theta) \approx \sqrt{13}\cos(\theta - 0.983^c)$$

b) $f(\theta)_{MAX} = \sqrt{13}$

IT OCCURS WHEN $\cos(\theta - 0.893^c) = +1$

$$\theta - 0.893^c = 0$$

$$\theta = 0.893^c$$

c) USING PART (a) & PART (b)

$$T = g(t) = 16 + \sqrt{13}\cos\left(\frac{\pi t}{12} + 0.893^c\right)$$

$$T_{MAX} = 16 + \sqrt{13} \approx 19.6\,^{\circ}C$$

&

$$\theta = 0.893$$

$$\frac{\pi t}{12} = 0.893$$

$$t \approx 3.75 \text{ (hours)}$$

IYGB - SYNOPTIC PAPER G - QUESTION 5

d) FINALLY WE HAVE, FOR $T=17$

$$\Rightarrow 17 = 16 + 2\cos\frac{\pi t}{12} + 3\sin\frac{\pi t}{12}$$

$$\Rightarrow 1 = \sqrt{13}\cos\left(\frac{\pi t}{12} - 0.893^c\right)$$

$$\Rightarrow \cos\left(\frac{\pi t}{12} - 0.893^c\right) = \frac{1}{\sqrt{13}}$$

$$\arccos\left(\frac{1}{\sqrt{13}}\right) = 1.28976\ldots$$

$$\Rightarrow \begin{cases} \frac{\pi t}{12} - 0.893 = 1.28976 \pm 2n\pi \\ \frac{\pi t}{12} - 0.893 = 4.99342 \pm 2n\pi \end{cases} \quad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \begin{cases} \frac{\pi t}{12} = 2.27255\ldots \pm 2n\pi \\ \frac{\pi t}{12} = 5.97621\ldots \pm 2n\pi \end{cases}$$

$$\Rightarrow \begin{cases} \pi t = 27.2706\ldots \pm 24n\pi \\ \pi t = 71.7146\ldots \pm 24n\pi \end{cases}$$

$$\Rightarrow \begin{cases} t = 8.6805\ldots \pm 24n \\ t = 22.8275\ldots \pm 24n \end{cases}$$

$$\therefore t = \begin{cases} 8.681 \approx 08:41 \\ 22.828 \approx 22:50 \end{cases}$$

(0.681×60) (0.828×60)

IYGB - SYNOPTIC PAPER G - QUESTION 6

PROCEED AS FOLLOWS

$$\tan 20° = \tan(2 \times 10°) =$$

$$t = \frac{2\tan 10°}{1 - \tan^2 10°}$$

$$\boxed{\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}}$$

$$t = \frac{2x}{1-x^2}$$

where $x = \tan 10°$

REARRANGING

$$\Rightarrow t(1-x^2) = 2x$$

$$\Rightarrow t - tx^2 = 2x$$

$$\Rightarrow 0 = tx^2 + 2x - t$$

$$\Rightarrow x^2 + \frac{2}{t} - 1 = 0$$

BY THE QUADRATIC FORMULA OR BY COMPLETING THE SQUARE

$$\Rightarrow \left(x+\frac{1}{t}\right)^2 - \left(\frac{1}{t}\right)^2 - 1 = 0$$

$$\Rightarrow \left(x+\frac{1}{t}\right)^2 = 1 + \frac{1}{t^2}$$

$$\Rightarrow \left(x+\frac{1}{t}\right)^2 = \frac{t^2+1}{t^2}$$

$$\Rightarrow x + \frac{1}{t} = \pm\sqrt{\frac{t^2+1}{t^2}}$$

$$\Rightarrow x = -\frac{1}{t} \pm \frac{\sqrt{t^2+1}}{t}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{t^2+1}}{t}$$

NOW $-1 - \sqrt{t^2+1} < 0$

AND $x = \tan 10° > 0$

$$\therefore x = \frac{-1+\sqrt{t^2+1}}{t}$$

$$\therefore \tan 10° = \frac{-1+\sqrt{t^2+1}}{t}$$

AS REQUIRED

IYGB – SYNOPTIC PAPER G – QUESTION 7

a) SUBSTITUTING INTO THE RECURRENCE RELATION

$\Longrightarrow a_{n+1} = p + q a_n$

$\Longrightarrow a_2 = p + q a_1 \qquad \therefore \ 220 = p + q \times 250$

$\Longrightarrow a_3 = p + q a_2 \qquad \therefore \ 196 = p + q \times 220$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} p + 250q = 220 \\ p + 220q = 196 \end{array}\right\} \Longrightarrow 30q = 24$$

$$q = \frac{4}{5}$$

$\Longrightarrow p + 250 \times \frac{4}{5} = 220$

$p + 200 = 220$

$p = 20$

b) LET THE REQUIRED LIMIT BE L

AS $n \to \infty \qquad a_n \approx a_{n+1} \longrightarrow L$

$\Longrightarrow a_{n+1} = 20 + \frac{4}{5} a_n$

$\Longrightarrow L = 20 + \frac{4}{5} L$

$\Longrightarrow 5L = 100 + 4L$

$\Longrightarrow L = 100$

INDEED IT CONVERGES TO 100

IYGB - SYNOPTIC PAPER G - QUESTION 8

$$y = \sqrt{x^2+16} \qquad x \in \mathbb{R}$$

a) LET $y = f(x)$

THEN $y = f(4x) = \sqrt{(4x)^2+16} = \sqrt{16x^2+16} = \sqrt{16(x^2+1)}$

$= \sqrt{16}\sqrt{x^2+1} = 4\sqrt{x^2+1}$

∴ HORIZONTAL STRETCH, BY SCALE FACTOR OF $\frac{1}{4}$

b) LET THE TRANSLATION BY THE VECTOR $\binom{k}{0}$ BE $y = f(x-k)$

(NOTE THAT IF k IS NEGATIVE THEN THE TRANSLATION WILL BE TO THE "LEFT")

$$y = f(x-k) = \sqrt{(x-k)^2+16}$$

NOW THIS CURVE PASSES THROUGH $(6,5)$

$\Rightarrow 5 = \sqrt{(6-k)^2+16}$

$\Rightarrow 25 = (6-k)^2+16$

$\Rightarrow 9 = (6-k)^2$

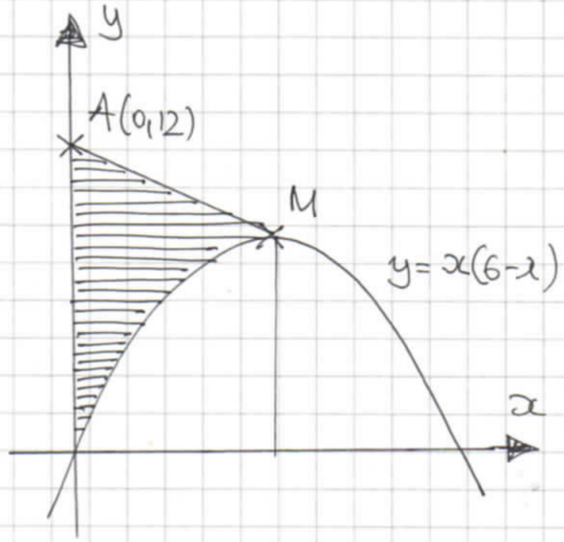
$\Rightarrow 9 = (k-6)^2$

$\Rightarrow k-6 = 3$ or -3

$\Rightarrow k = 9$ or 3

IYGB - SYNOPTIC PAPER G - QUESTION 9

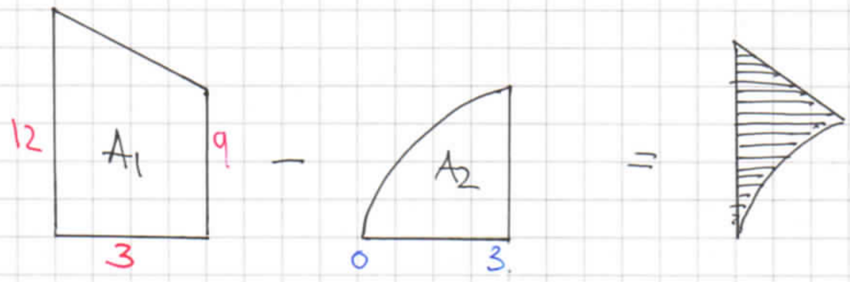
- BY INSPECTION THE CURVE MEETS THE x AXIS AT $x=0$ & $x=6$



- BY SYMMETRY THE MAXIMUM IS LOCATED AT $x=3$ (MIDPOINT)

$$y = x(6-x)$$
$$y = 3(6-3)$$
$$y = 9$$

$\therefore$ $M(3,9)$

- 

$$A_1 = \frac{9+12}{2} \times 3$$
$$A_1 = \frac{63}{2}$$

$$A_2 = \int_0^3 x(6-x)\,dx = \int_0^3 6x - x^2\,dx$$
$$= \left[3x^2 - \tfrac{1}{3}x^3\right]_0^3 = (27-9)-(0)$$
$$= 18$$

- REQUIRED AREA IS $A_1 - A_2$

$$\frac{63}{2} - 18 = \frac{27}{2}$$

IYGB - SYNOPTIC PAPER G - QUESTION 10

EXPAND UP TO x^2 IN ASCENDING POWERS OF x

$$f(x) = (2-3x)^2(1+4x)^7$$

$$f(x) = (4-12x+9x^2)\left[1 + \frac{7}{1}(4x)^1 + \frac{7\times6}{1\times2}(4x)^2 + \dots\right]$$

$$f(x) = (4-12x+9x^2)(1 + 28x + 336x^2 + \dots)$$

$9x^2$

$-336x^2$

$+1344x^2$

REQUIRED COEFFICIENT IS

$$9 - 336 + 1344 = \underline{1017}$$

IYGB - SYNOPTIC PAPER G - QUESTION 11

SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left.\begin{aligned} y &= 7^x \\ y &= 2\times 5^x \end{aligned}\right\} \Longrightarrow 7^x = 2\times 5^x$$

TAKING LOGARITHMS BASE 2

$$\Rightarrow \log_2 7^x = \log_2 (2\times 5^x)$$

$$\Rightarrow x\log_2 7 = \log_2 2 + \log_2 5^x$$

$$\Rightarrow x\log_2 7 = 1 + x\log_2 5$$

$$\Rightarrow x\log_2 7 - x\log_2 5 = 1$$

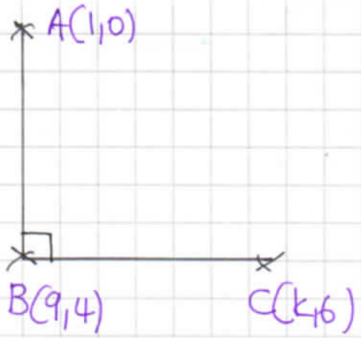
$$\Rightarrow x\left[\log_2 7 - \log_2 5\right] = 1$$

$$\Rightarrow x = \frac{1}{\log_2 7 - \log_2 5}$$

// AS REQUIRED

IYGB – SYNOPTIC PAPER G – QUESTION 12

a) BY CONSIDERING GRADIENTS



$$\text{GRADIENT } AB = \frac{4-0}{9-1} = \frac{4}{8} = \frac{1}{2}$$

$$\text{GRADIENT } BC = \frac{6-4}{k-9} = \frac{2}{k-9}$$

THESE GRADIENTS MUST BE NEGATIVE RECIPROCALS OF EACH OTHER

$$\Rightarrow \frac{2}{k-9} = -2 \quad \leftarrow \text{NEGATIVE RECIPROCAL OF } \tfrac{1}{2}$$

$$\Rightarrow 2 = -2k + 18$$

$$\Rightarrow 2k = 16$$

$$\Rightarrow k = 8$$

AS REQUIRED

b) USING PART (a), GRADIENT OF BC MUST BE -2 & PASSING THROUGH $B(9,4)$

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -2(x - 9)$$

$$y - 4 = -2x + 18$$

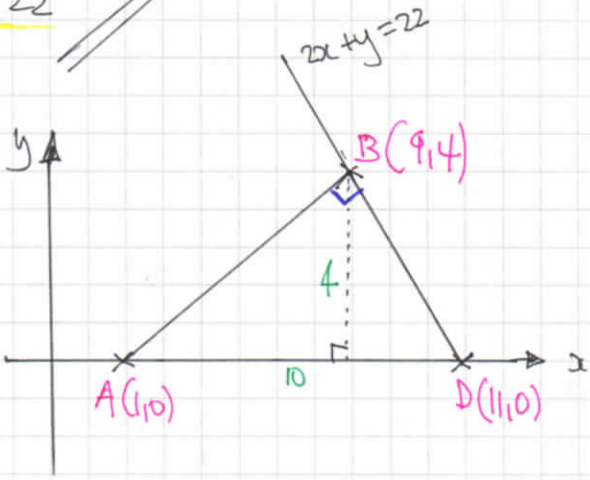
$$2x + y = 22$$

c) WHEN $y=0$

$$2x + 0 = 22$$

$$x = 11$$

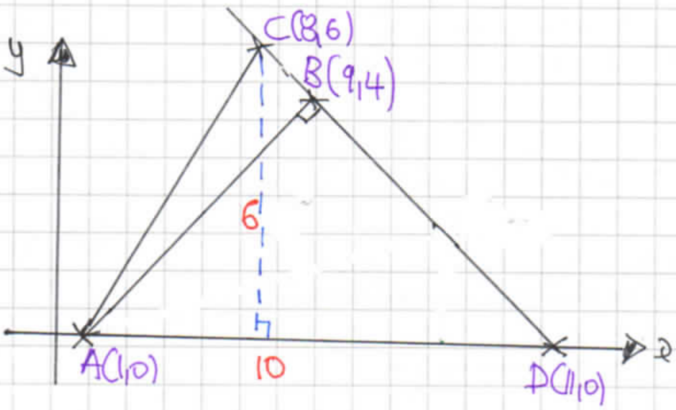
$$\therefore D(11,0)$$



$$\text{REQUIRED AREA} = \tfrac{1}{2} \times 10 \times 4 = 20$$

IYGB - SYNOPTIC PAPER G - QUESTION 12

d) LOOKING AT THE DIAGRAM BELOW



AREA OF $\overset{\Delta}{ACD} = \frac{1}{2} \times 10 \times 6 = 30$

AREA OF $\overset{\Delta}{ABD} = 20$ (PART c)

AREA OF $\overset{\Delta}{ABC} = 30 - 20 = 10$

IYGB - SYNOPTIC PAPER G - QUESTION 13

LOOKING AT THE "GEOMETRIC PATTERN"

$$u_3 \qquad u_4 \qquad u_5$$

$$x+5 \xrightarrow{\times r} 4x-1 \xrightarrow{\times r} 2x+3$$

FORMING TWO EQUATIONS

$$(x+5)r = 4x-1$$
$$(4x-1)r = 2x+3$$

DIVIDE EQUATIONS TO ELIMINATE r

$$\frac{(x+5)\cancel{r}}{(4x-1)\cancel{r}} = \frac{4x-1}{2x+3}$$

$$\Longrightarrow (4x-1)^2 = (2x+3)(x+5)$$
$$\Longrightarrow 16x^2 - 8x + 1 = 2x^2 + 13x + 15$$
$$\Longrightarrow 14x^2 - 21x - 14 = 0$$
$$\Longrightarrow 2x^2 - 3x - 2 = 0$$
$$\Longrightarrow (2x+1)(x-2)$$
$$\Longrightarrow x = \begin{cases} -\frac{1}{2} \\ 2 \end{cases}$$

USING EACH OF THE VALUES OF x FOUND

- IF $x = -\frac{1}{2}$

$$u_3 = \frac{9}{2} \quad \times -\frac{2}{3}$$
$$u_4 = -3 \quad \times -\frac{2}{3}$$
$$u_5 = 2$$

- ~~IF $x = 2$~~

~~$u_3 = 7$~~
~~$u_4 = 7$~~
~~$u_5 = 7$~~

NO G.P.

IYGB - SYNOPTIC PAPER G - QUESTION 13

IF $x = -\frac{1}{2}$ WE GET $r = -\frac{2}{3}$, SO USING $u_n = ar^{n-1}$

$$\Rightarrow \quad u_3 = \frac{9}{2}$$

$$\Rightarrow \quad ar^2 = \frac{9}{2}$$

$$\Rightarrow \quad a\left(-\frac{2}{3}\right)^2 = \frac{9}{2}$$

$$\Rightarrow \quad \frac{4}{9}a = \frac{9}{2}$$

$$\Rightarrow \quad a = \frac{81}{8}$$

FINALLY THE SUM TO INFINITY CAN BE FOUND

$$\Rightarrow \quad S_\infty = \frac{a}{1-r}$$

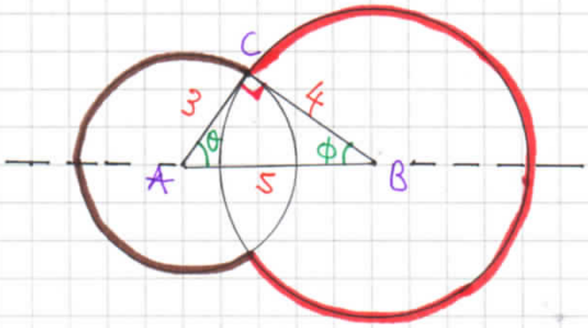
$$\Rightarrow \quad S_\infty = \frac{81/8}{1-\left(-\frac{2}{3}\right)}$$

$$\Rightarrow \quad S_\infty = \frac{81/8}{5/3}$$

$$\Rightarrow \quad S_\infty = \frac{243}{40}$$

IYGB - SYNOPTIC PAPER G - QUESTION 14

LOOKING AT THE DIAGRAM WE OBSERVE THAT $\hat{ABC}$ IS RIGHT ANGLED AS A "3-4-5" TRIANGLE



- $\sin\phi = \frac{3}{5}$

  $\phi = 0.6435^c$

- $\sin\theta = \frac{4}{5}$

  $\theta = 0.9273$

- $2\pi - 2\phi$ ←

$= 2\pi - 2\times 0.6435$

$= 4.9962^c$

- $2\pi - 2\theta$ ←

$= 2\pi - 2\times 0.9273$

$= 4.4286^c$

- "BROWN" ARC-LENGTH $=$ "$r\theta^c$" $= 3\times 4.4286 = 13.2857\ldots$

  "RED" ARC-LENGTH $=$ "$r\theta^c$" $= 4\times 4.962 = 19.9848\ldots$

$\therefore$ REQUIRED LENGTH IS $13.2857\ldots + 19.9848\ldots = 33.2705\ldots$

$\simeq 33.3$ cm

IYGB - SYNOPTIC PAPER G - QUESTION 15

USING THE SUBSTITUTION GIVEN

$$\Rightarrow u = 1 + xe^{\sin x}$$

$$\Rightarrow \frac{du}{dx} = 1\cdot e^{\sin x} + x\cdot e^{\sin x}(\cos x)$$

$$\Rightarrow \frac{du}{dx} = e^{\sin x} + xe^{\sin x}\cos x$$

$$\Rightarrow \frac{du}{dx} = e^{\sin x}(1 + x\cos x)$$

$$\Rightarrow dx = \frac{du}{e^{\sin x}(1 + x\cos x)}$$

CHANGING THE LIMITS

$x = 0 \longmapsto u = 1$

$x = \pi \longmapsto u = 1 + \pi$

TRANSFORMING THE INTEGRAL

$$\int_0^{\pi} \frac{1 + x\cos x}{x + e^{-\sin x}}\, dx = \int_1^{1+\pi} \frac{\cancel{1 + x\cos x}}{x + e^{-\sin x}} \times \frac{du}{e^{\sin x}\cancel{(1 + x\cos x)}}$$

$$= \int_1^{1+\pi} \frac{1}{xe^{\sin x} + 1}\, du$$

$$= \int_1^{1+\pi} \frac{1}{u}\, du$$

$$= \Big[\ln|u|\Big]_1^{1+\pi}$$

$$= \ln(1+\pi) - \cancel{\ln 1}$$

$$= \ln(1+\pi)$$

IYGB - SYNOPTIC PAPER G - QUESTION 16

$$y = x(x-2)^3 \quad , \quad x \in \mathbb{R}$$

- START BY DIFFERENTIATION (PRODUCT RULE)

$$\Rightarrow \frac{dy}{dx} = 1 \times (x-2)^3 + x \times 3(x-2)^2 \times 1$$

$$\Rightarrow \frac{dy}{dx} = (x-2)^3 + 3x(x-2)^2$$

$$\Rightarrow \frac{dy}{dx} = (x-2)^2\left[(x-2)+3x\right]$$

$$\Rightarrow \frac{dy}{dx} = (4x-2)(x-2)^2$$

$$\Rightarrow \frac{dy}{dx} = 2(2x-1)(x-2)^2$$

- NOW BY INSPECTION

$$\text{IF } x=3 \qquad \frac{dy}{dx} = 2 \times 5 \times 1^2 = 10$$

- EXPAND THE GRADIENT FUNCTION

$$\Rightarrow \frac{dy}{dx} = 2(2x-1)(x^2-4x+4)$$

$$\Rightarrow \frac{dy}{dx} = 2(2x^3-8x^2+8x-x^2+4x-4)$$

$$\Rightarrow \frac{dy}{dx} = 2(2x^3-9x^2+12x-4)$$

- SETTING EQUAL TO 10, NOTING THAT $(x-3)$ WILL BE A FACTOR OF THE RESULTING POLYNOMIAL

$$\Rightarrow 2(2x^3-9x^2+12x-4) = 10$$

$$\Rightarrow 2x^3-9x^2+12x-4 = 5$$

IYGB – SYNOPTIC PAPER G – QUESTION 16

$$\Longrightarrow 2x^3 - 9x^2 + 12x - 9 = 0$$

$$\Longrightarrow 2x^2(x-3) - 3x(x-3) + 3(x-3) = 0$$

(OR LONG DIVISION INSTEAD)

$$\Longrightarrow (x-3)(2x^2 - 3x + 3) = 0$$

$$b^2 - 4ac = (-3)^2 - 4 \times 2 \times 3$$
$$= 9 - 24$$
$$= -15 < 0$$

- ONLY SOLUTION IS $x = 3$, HENCE THERE IS ONLY ONE POINT ON THE CURVE, WHERE THE GRADIENT IS 10

IYGB - SYNOPTIC PAPER G - QUESTION 17

FORM A DIFFERENTIAL EQUATION

x = DISTANCE FROM O
t = TIME

$t=0, x=50$
$t=4, x=30$

$$\frac{dx}{dt} = k\frac{1}{x}$$

RATE; PROPORTIONALITY CONSTANT; INVERSELY PROPORTIONAL TO x

SOLVING BY SEPARATING VARIABLES

$$\Rightarrow dx = \frac{k}{x}\,dt$$

$$\Rightarrow x\,dx = k\,dt$$

$$\Rightarrow \int x\,dx = \int k\,dt$$

$$\Rightarrow \tfrac{1}{2}x^2 = kt + C$$

$$\Rightarrow x^2 = At + B$$

APPLY CONDITIONS TO FIND THE CONSTANTS

- $t=0,\ x=50 \Rightarrow 50^2 = B$

$$\Rightarrow B = 2500$$

$$\Rightarrow x^2 = At + 2500$$

IYGB - SYNOPTIC PAPER G - QUESTION 17

- $t=4,\ x=30 \implies 30^2 = A\times 4 + 2500$

$\implies 900 = 4A + 2500$

$\implies 4A = -1600$

$\implies A = -400$

$\implies x^2 = -400t + 2500$

- when $x=0 \implies 0^2 = -400t + 2500$

$\implies 400t = 2500$

$\implies 4t = 25$

$\implies t = 6.25$

IYGB - SYNOPTIC PAPER G - QUESTION 18

a) DIFFERENTIATE THE EQUATION WITH RESPECT TO x

$$\Rightarrow 2x^2 + xy - y^2 - 4x - y + 20 = 0$$

$$\Rightarrow \frac{d}{dx}(2x^2) + \frac{d}{dx}(xy) - \frac{d}{dx}(y^2) - \frac{d}{dx}(4x) - \frac{d}{dx}(y) + \frac{d}{dx}(20) = \frac{d}{dx}(0)$$

$$\Rightarrow 4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} - 4 - \frac{dy}{dx} + 0 = 0$$

$$\Rightarrow (x - 2y - 1)\frac{dy}{dx} = -4x - y + 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4x - y + 4}{x - 2y - 1}$$

MULTIPLY "TOP & BOTTOM" BY -1

$$\Rightarrow \frac{dy}{dx} = \frac{4x + y - 4}{2y - x + 1}$$

// AS REQUIRED

b) SOLVING $\frac{dy}{dx} = 0$

$$\Rightarrow 4x + y - 4 = 0$$

$$\Rightarrow y = 4 - 4x$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow 2x^2 + x(4 - 4x) - (4 - 4x)^2 - 4x - (4 - 4x) + 20 = 0$$

$$\Rightarrow 2x^2 + 4x - 4x^2 - (16 - 32x + 16x^2) - 4x - 4 + 4x + 20 = 0$$

$$\Rightarrow 2x^2 + 4x - 4x^2 - 16 + 32x - 16x^2 - 4 + 20 = 0$$

$$\Rightarrow -18x^2 + 36x = 0$$

$$\Rightarrow -18x(x - 2) = 0$$

$$x = \begin{cases} 0 \\ 2 \end{cases} \qquad y = \begin{cases} 4 - 4 \times 0 = 4 \\ 4 - 4 \times 2 = -4 \end{cases}$$

$\therefore (0, 4)$ & $(2, -4)$ //

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c) STARTING FROM

$$\Rightarrow 4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} - 4 - \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}(4x) + \frac{d}{dx}(y) + \frac{d}{dx}\left(x\frac{dy}{dx}\right) - 2\frac{d}{dx}\left(y\frac{dy}{dx}\right) - \frac{d}{dx}(4) - \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(0)$$

$$\Rightarrow 4 + \frac{dy}{dx} + \left[1\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right] - 2\left[\frac{dy}{dx}\frac{dy}{dx} + y\frac{d^2y}{dx^2}\right] - 0 - \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow 4 + \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 - 2y\frac{d^2y}{dx^2} - \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow 4 + 2\frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^2 + (x - 2y - 1)\frac{d^2y}{dx^2} = 0$$

// AS REQUIRED

d) CHECKING $(0,4)$, $\frac{dy}{dx} = 0$

$$4 + 0 - 0 + (0 - 8 - 1)\frac{d^2y}{dx^2} = 0$$

$$4 = 9\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{4}{9} > 0$$

$\therefore$ $(0,4)$ IS A LOCAL MIN

CHECKING $(2,-4)$ $\frac{dy}{dx} = 0$

$$4 + 0 - 0 + (2 + 8 - 1)\frac{d^2y}{dx^2} = 0$$

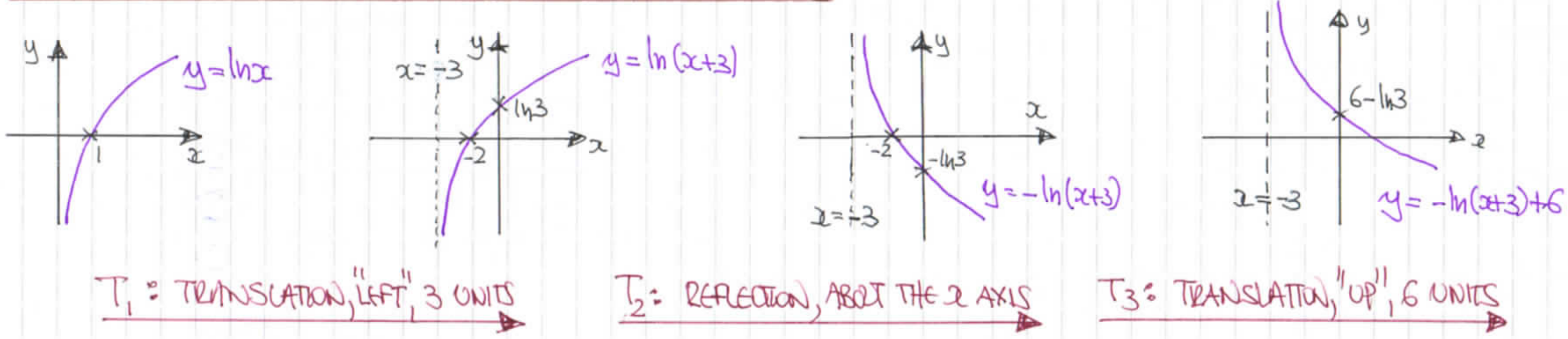
$$9\frac{d^2y}{dx^2} = -4$$

$$\frac{d^2y}{dx^2} = -\frac{4}{9} < 0$$

$\therefore$ $(2,-4)$ IS A LOCAL MAX

IYGB - SYNOPTIC PAPER G - QUESTION 19

a) SKETCHING & DESCRIBING, STAGE BY STAGE



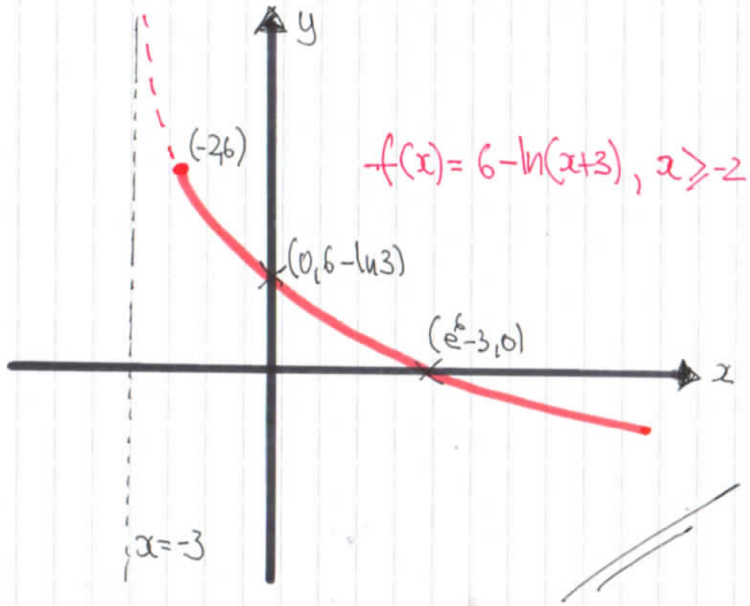
SKETCHING THE GRAPH OF $f: x \mapsto 6 - \ln(x+3)$ FOR ITS GIVEN DOMAIN

- $x = -2$ $y = 6 - \ln 1$ IE $(-2, 6)$
- $y = 0$ $0 = 6 - \ln(x+3)$

$\ln(x+3) = 6$

$x + 3 = e^6$

$x = e^6 - 3$ IE $(e^6 - 3, 0)$



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b) LET $y = 6 - \ln(x+3)$, FOR SIMPLICITY

$\Rightarrow y = 6 - \ln(x+3)$

$\Rightarrow \ln(x+3) = 6 - y$

$\Rightarrow x+3 = e^{6-y}$

$\Rightarrow x = e^{6-y} - 3$

$\therefore f^{-1}(x) = e^{6-x} - 3$

	f	f^{-1}
D	$x \geqslant -2$	$x \leqslant 6$
R	$f(x) \leqslant 6$	$f^{-1}(x) \geqslant -2$

$\therefore$ DOMAIN: $x \leqslant 6$

RANGE: $f^{-1}(x) \geqslant -2$

c) FINALLY THE COMPOSITION

$$f(g(x)) = f\left(e^{x^2} - 3\right)$$

$$= 6 - \ln\left[\left(e^{x^2} - 3\right) + 3\right]$$

$$= 6 - \ln\left(e^{x^2}\right)$$

$$= 6 - x^2$$

IYGB - SYNOPTIC PAPER G - QUESTION 19

b) LET $y = 6 - \ln(x+3)$, FOR SIMPLICITY

$$\Rightarrow y = 6 - \ln(x+3)$$
$$\Rightarrow \ln(x+3) = 6 - y$$
$$\Rightarrow x + 3 = e^{6-y}$$
$$\Rightarrow x = e^{6-y} - 3$$

$$\therefore f^{-1}(x) = e^{6-x} - 3$$

	f	f^{-1}
D	$x \geq -2$	$x \leq 6$
R	$f(x) \leq 6$	$f^{-1}(x) \geq -2$

$\therefore$ DOMAIN: $x \leq 6$

RANGE: $f^{-1}(x) \geq -2$

c) FINALLY THE COMPOSITION

$$f(g(x)) = f\left(e^{x^2} - 3\right)$$
$$= 6 - \ln\left[\left(e^{x^2} - 3\right) + 3\right]$$
$$= 6 - \ln\left(e^{x^2}\right)$$
$$= 6 - x^2$$

IYGB - SYNOPTIC PAPER G - QUESTION 20

a) OBTAIN THE GRADIENT FUNCTION

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-4}{2}$$

AT THE POINT A(2,4) THE VALUE OF $t=-1$ SINCE

$$2t+4=2$$
$$2t=-2$$
$$t=-1$$

GRADIENT AT A(2,4)

$$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{3(-1)^2-4}{2} = -\frac{1}{2}$$

EQUATION OF TANGENT IS GIVEN BY

$$y-y_0 = m(x-x_0)$$
$$y-4 = -\tfrac{1}{2}(x-2)$$
$$2y-8 = -x+2$$
$$x+2y=10$$

AS REQUIRED

b) SOLVING SIMULTANEOUSLY THE EQUATION OF THE TANGENT AND THE EQUATION OF THE CURVE IN PARAMETRIC

$$\Rightarrow x+2y=10$$
$$\Rightarrow (2t+4)+2(t^3-4t+1)=10$$
$$\Rightarrow 2t+4+2t^3-8t+2=10$$
$$\Rightarrow 2t^3-6t-4=0$$
$$\Rightarrow t^3-3t-2=0$$
$$\Rightarrow (t+1)^2(t-2)=0$$

POINT OF TANGENCY MUST BE A REPEATED SOLUTION

← QUICK CHECK $(t-2)(t^2+2t+1)$
$= t^3+2t^2+t-2t^2-4t-2$
$= t^3-3t-2$

$\therefore$ $t=2$ YIELDS B(8,1)

IYGB - SYNOPTIC PAPER G - QUESTION 21

STARTING WITH $\frac{dV}{dt} = 2t$, NOTING $8.1\ell = 8100\text{ cm}^3$ & 2 MINUTES = 120 s

$$\Rightarrow dV = 2t\,dt$$

$$\Rightarrow \int_{V=8100}^{V} 1\,dV = \int_{t=0}^{t=120} 2t\,dt$$

$$\Rightarrow \left[V\right]_{8100}^{V} = \left[t^2\right]_0^{120}$$

$$\Rightarrow V - 8100 = 120^2 - 0$$

$$\Rightarrow V = 14400 + 8100$$

$$\Rightarrow \underline{V = 22500}$$

NEXT WE HAVE

$$\Rightarrow \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{72h} \times 2t$$

$$\Rightarrow \boxed{\frac{dh}{dt} = \frac{t}{36h}}$$

$V = 36h^2$

$\frac{dV}{dh} = 72h$

$\frac{dh}{dV} = \frac{1}{72h}$

WHEN $V = 22500$, $V = 36h^2$

$22500 = 36h^2$

$h^2 = 625$

$\underline{h = 25}$

FINALLY WE HAVE

$$\left.\frac{dh}{dt}\right|_{\substack{t=120 \\ h=25}} = \frac{120}{36 \times 25} = \frac{2}{15} \simeq 0.133\text{ cm s}^{-1}$$