IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper Q

Difficulty Rating: 4.10/0.7895

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 21 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Write each of the following expressions a single simplified surd.

a) $\sqrt{343}-\sqrt{28}$. **(2)**

b) $\sqrt{45}+\dfrac{20}{\sqrt{5}}$. **(2)**

Detailed workings must be shown in this question.

Question 2

A circle C has its centre at the point with coordinates $(5,4)$ and its radius is $3\sqrt{2}$.

a) Find an equation for C. **(2)**

The straight line L has equation

$$y=x+1.$$

b) Determine, as exact surds, the coordinates of the points of intersection between C and L. **(5)**

c) Show that the distance between these points of intersection is 8 units. **(3)**

Question 3

$$-53\;-\;44\;-\;35\;-\;26\;-\;\ldots\;+\;1000.$$

The above series has 118 terms.

Find the sum of the **last** 18 terms of the series. **(4)**

Question 4 (****)

Find the coefficient of x^5 in the binomial expansion of

$$(1-x)^5(1+x)^6.$$ **(6)**

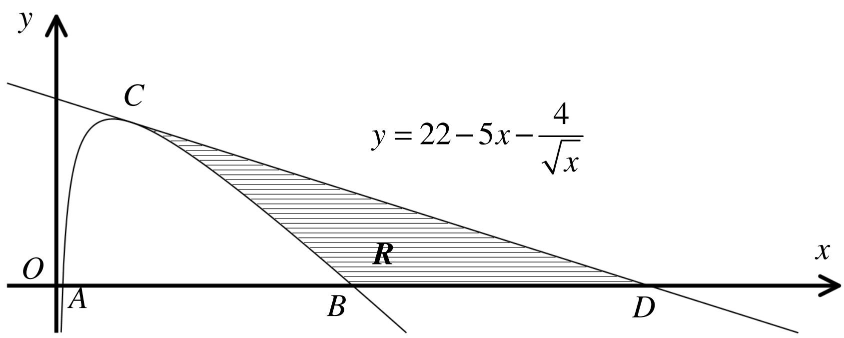
Question 5

Use small angle approximations to show that if x is measured in radians then

$$\frac{1+\cos x}{1+\sin\left(\frac{1}{2}x\right)} \approx A+Bx,$$

where A and B are constants to be found. **(4)**

Question 6



The figure above shows part of the curve with equation

$$y = 22 - 5x - \frac{4}{\sqrt{x}}, \quad x > 0$$

The curve meets the x axis at A and B. The point C lies on the curve where $x = 1$.

a) Verify that the coordinates of B are $(4,0)$. **(1)**

b) Find an equation of the tangent to the curve at C. **(5)**

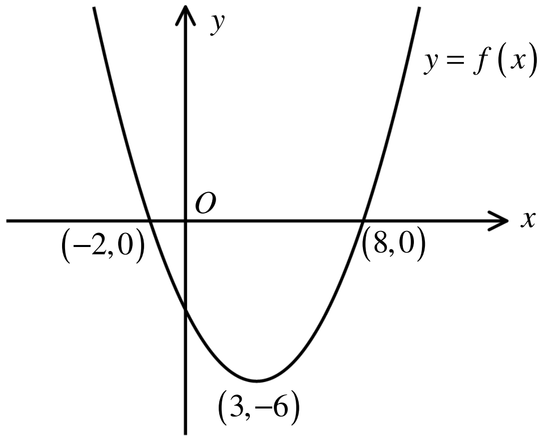
The tangent to the curve at C crosses the x axis at the point D.

The finite region R, shown shaded in the above figure, is bounded by the curve, the tangent to the curve at C and the x axis.

c) Determine the exact area of R. **(7)**

Question 7

A curve with equation $y = f(x)$ meets the x axis at the points with coordinates $(-2,0)$ and $(8,0)$, and has a stationary point at $(3,-6)$, as shown in the figure below.



a) If the graph of $y = 2f(x+\alpha)$ passes through the origin, determine the possible values of α. **(2)**

b) If the stationary point on the graph of $y = \beta f(x+2)$ has coordinates $(\gamma, 2)$, state the value of β and the value of γ. **(2)**

Question 8

A triangle has angles θ, φ and ψ, where ψ is an obtuse angle.

It is further given that $\sin\psi = 0.9703$ and $\tan(\theta - \varphi) = 0.2493$.

Calculate, in degrees, the value of each of the angles θ, φ and ψ. **(7)**

Question 9

$$y = \frac{3x^2 - 10x + 2}{(1-2x)(x-2)^2}, \quad x \in \mathbb{R}, \quad x > 2.$$

Find the exact value of $\frac{dy}{dx}$ at $x = 3$. **(8)**

Question 10

A curve is defined parametrically by the equations

$$x = 3\cos 2t, \quad y = 6\sin 2t, \quad 0 \le t < 2\pi.$$

Express $\frac{d^2y}{dx^2}$ in terms of y. **(6)**

Question 11

The function f is defined in a suitable domain of real numbers and satisfies

$$f(x) = \ln\left(\frac{\mathrm{e} - x}{\mathrm{e} + x}\right).$$

a) Show that f is odd. **(3)**

b) Determine the largest possible domain of f. **(4)**

c) Solve the equation

$$f(x) + f(x+1) = 0.$$ **(7)**

Question 12

It is known that the cubic equation

$$x^3 - 2x = 5, \quad x \in \mathbb{R},$$

has a single real solution α, which is close to 2.1.

Four iterative formulas based on rearrangements of this equation are being considered for their suitability to approximate the value of α to greater accuracy.

- $x_{n+1} = \frac{1}{2}\left(x_n^3 - 5\right)$
- $x_{n+1} = \sqrt[3]{2x_n + 5}$
- $x_{n+1} = \dfrac{5}{x_n^2 - 2}$
- $x_{n+1} = \sqrt{2 + \dfrac{5}{x_n}}$

a) Use a differentiation method, and **without** carrying any direct iterations, briefly describe the suitability of these four formulas.

In these descriptions you must make a reference to rates of convergence or divergence, and cobweb or staircase diagrams. **(12)**

b) Use one of these four formulas to approximate the value of α, correct to 6 decimal places **(3)**

Question 13

A runner took part in a 40 km walk.

He walked the first 16 km at an average speed x km h^{-1}.

He walked the rest of the race at an average speed of 2 km h^{-1} less than the average speed of his the first 16 km.

Given that the **total** time for the walk was 6 hours, determine the value of x. **(7)**

Question 14

The functions f and g are defined as

$$f(x) = |a - 2x| + a, \ x \in \mathbb{R}$$

$$g(x) = |3x + a|, \ x \in \mathbb{R},$$

where a is a positive constant.

a) Sketch in the same set of axes the graph of $f(x)$ and the graph of $g(x)$.

The sketch must include the coordinates of any points where the graphs meet the coordinate axes. **(4)**

b) Determine, in terms of a, the coordinates of any points of intersection between the two graphs. **(5)**

c) Find a simplified expression for $gf(x)$. **(3)**

d) Solve, in terms of a, the equation $gf(x) = 10a$. **(4)**

Question 15

Anton is planning to save for a house purchase deposit over a period of 5 years.

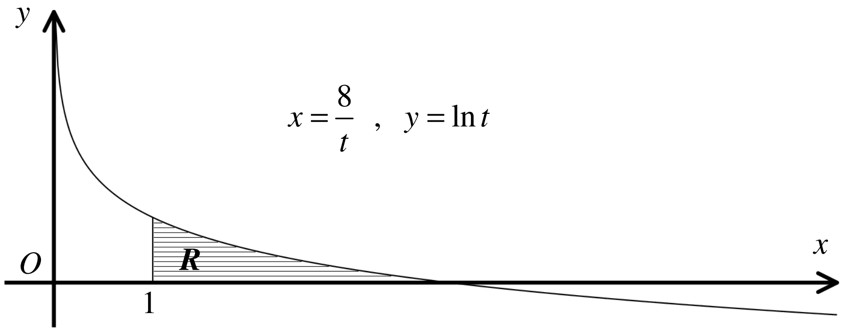
He opens an account known as a "Homesaver" and plans to pay into this account £200 at the start of every month, and continues to do so for 5 years.

The account pays 0.5% compound interest **per month**, with the interest credited to the account at the end of every month.

a) Show clearly that at the **end** of the third month the balance of the account will be £606.02. **(2)**

b) Calculate the total amount in Anton's "Homesaver" account after 5 years. **(6)**

Question 16



The figure above shows part of the curve with parametric equations

$$x = \frac{8}{t}, \; y = \ln t, \; t > 0$$

The finite region R is bounded by the curve, the x axis and the straight line with equation $x = 1$.

a) Show that the area of R is given by

$$\int_{t_1}^{t_2} \frac{8 \ln t}{t^2} \, dt$$

where the t_1 and t_2 are constants to be found. **(5)**

b) Evaluate the above parametric integral to determine, in exact simplified form, the area of R. **(6)**

c) Find a Cartesian equation of the curve and hence verify the answer of part **(b)**. **(8)**

Question 17 **(****+)**

Find, in exact form where appropriate, the solutions of the following equation

$$6e^{3x} + 1 = 7e^{2x}.$$ **(8)**

Question 18

Relative to a fixed origin O, the points A and B have position vectors $3\mathbf{i}-9\mathbf{j}$ and $2\mathbf{i}+10\mathbf{j}$, respectively.

The point M is the midpoint of OB and the point N lies on OA so that $\overrightarrow{OA} = 3\overrightarrow{ON}$.

The point P is the point of intersection of AM and BN.

Determine the ratio $\overrightarrow{NP} : \overrightarrow{PB}$. **(9)**

Question 19

A curve has equation

$$y = 2x^3 + \frac{k}{x} - 19, \quad x > 0,$$

where k is a positive constant.

If the y coordinate of the stationary point of this curve is 45, find the value of k. **(9)**

Question 20

$$f(x) = \frac{\sin 3x}{\cos x} + \frac{\cos 3x}{\sin x}, \quad x \in \mathbb{R}, \quad x \neq \frac{n\pi}{2}, \; n \in \mathbb{Z}.$$

a) Show clearly that

$$f(x) \equiv 2\cot 2x.$$ **(3)**

b) Solve the trigonometric equation

$$\frac{1}{4}f(x) + 1 = \tan x, \quad 0 \le x < 2\pi.$$ **(7)**

Question 21

Water is pouring into a long vertical cylinder at a constant rate of $2400 \text{ cm}^3\text{s}^{-1}$ and leaking out of a hole at the base of the cylinder at a rate proportional to the square root of the height of the water already in the cylinder.

The cylinder has constant cross sectional area of 4800 cm^2.

a) Show that, if H is the height of the water in the cylinder, in cm, at time t seconds, then

$$\frac{dH}{dt} = \frac{1}{2} - B\sqrt{H},$$

where B is positive constant. **(5)**

The cylinder was initially empty and when the height of the water in the cylinder reached 16 cm water was **leaking out of the hole**, at the rate of $120 \text{ cm}^3\text{s}^{-1}$.

b) Show clearly that

$$\frac{dH}{dt} = \frac{80-\sqrt{H}}{160}.$$ **(3)**

c) Use the substitution $u = 80 - \sqrt{H}$, to find

$$\int \frac{1}{80-\sqrt{H}}\, dH.$$ **(6)**

d) Solve the differential equation in part **(b)** to find, to the nearest minute, the time it takes to fill the cylinder from empty to a height of 4 **metres.** **(5)**

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