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Examiners' report

MATHEMATICS B (MEI)

H640

For first teaching in 2017

H640/01 Summer 2022 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper 1 series overview

Overall, the paper produced a very good spread of marks and most candidates seemed to finish the paper in the time allowed. There was no real evidence that two years of lockdown had any lasting effect on their learning, although some weak algebraic manipulation was seen in several places, and many candidates had not learned the cosine rule and how to manipulate the formula.

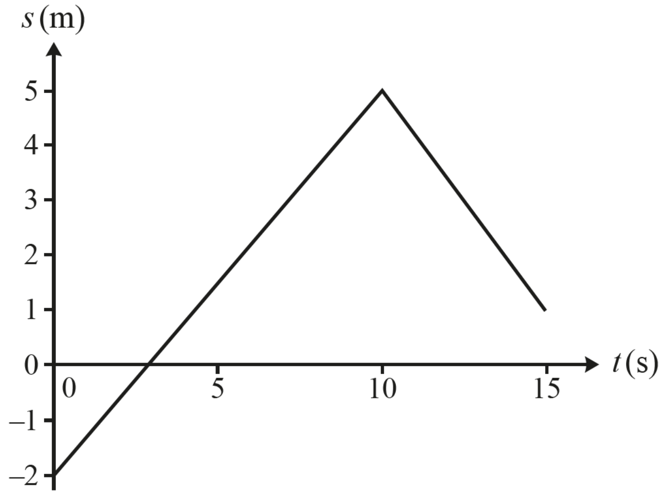
Candidates who did well on this paper generally did the following:	**Candidates who did less well on this paper generally did the following:**
• set out their working logically • worked accurately and concisely • gave sufficient explanation of their thinking • used their calculator sensibly.	• worked in fragments that were hard to follow • made arithmetic errors and mistakes in algebraic manipulation • relied too little or too much on their calculator.

Section A overview

Section A is designed to minimise the amount of reading necessary to answer the questions and many questions were quite accessible to most candidates.

Question 1 (a)

1 A particle moves along a straight line. The displacement s m at time t s is shown in the displacement-time graph below. The graph consists of straight line segments joining the points $(0, -2)$, $(10, 5)$ and $(15, 1)$.



(a) Find the distance travelled by the particle in the first 15 s. **[2]**

This was quite a simple opening question which most candidates were able to answer although a few took the graph to be a velocity-time graph and attempted to calculate areas to answer the question.

Question 1 (b)

(b) Calculate the velocity of the particle between $t = 10$ and $t = 15$. **[2]**

Most candidates correctly attempted to find the gradient of the graph. The most common error was to ignore the negative sign, despite seeing the negative gradient of that part of the graph. For some this was an arithmetic error and for some it was a confusion between velocity and speed.

Misconception

Velocity is a vector quantity which for motion in a line may be a negative number. Speed is the magnitude of the velocity and is always a positive number.

Question 2

2 Express $\dfrac{13-x}{(x-3)(x+2)}$ in partial fractions. **[3]**

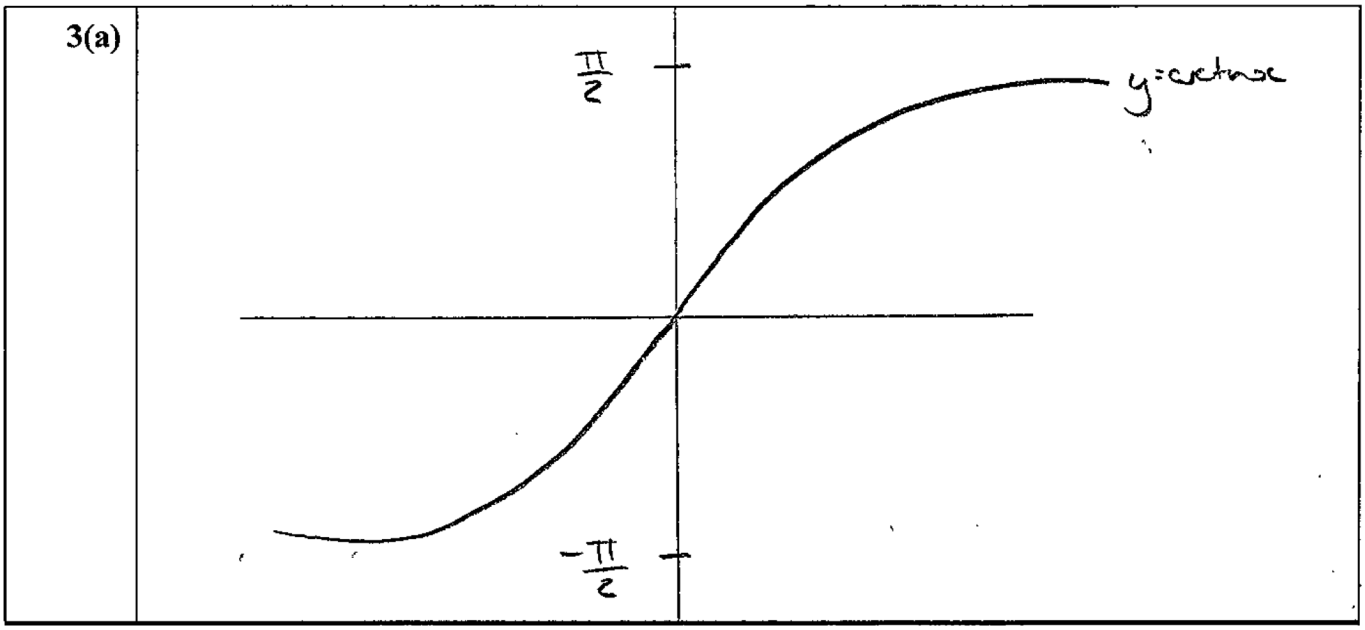
Almost all candidates could do this correctly with a few candidates making mistakes with signs.

Question 3 (a)

3 **(a)** Sketch the graph of $y = \arctan x$ where x is in radians. **[2]**

This was often not well answered as some candidates drew $y = -\tan x$, $y = \cot x$ or $x = \tan y$ (with several branches of the graph). For those who knew what to draw, the mark for the shape of the graph was much easier where candidates had drawn the horizontal asymptotes. Some candidates seemed to use their graphical calculator and although they drew the correct shape, often did not realise the exact values needed for the labels of the asymptotes.

Exemplar 1



In this response, it was not clear whether the graph was part of a graph with a minimum and a maximum point, or whether the candidate had intended to show asymptotes. Had dotted lines been seen, the examiner would have given full marks recognising the intention to draw asymptotes.

Question 3 (b)

(b) In this question you must show detailed reasoning.

Find all points of intersection of the curves $y = 3\sin x\cos x$ and $y = \cos^2 x$ for $-\pi \leqslant x \leqslant \pi$. **[6]**

Many candidates made a good attempt at a full solution of this question, but many divided by $\cos x$ and so lost two roots. Many also did not remember to find the *y*-coordinates of the points of intersection.

Assessment for learning

This question was labelled as requiring detailed reasoning. This does not mean non-calculator and some candidates used their calculator to find all the roots. However, marks were not awarded for roots stated without supporting evidence, so $\cos x = 0$ had to be seen to award the marks for $\pm\frac{\pi}{2}$ and $\tan x = \frac{1}{3}$ or equivalent for the other roots.

Question 4

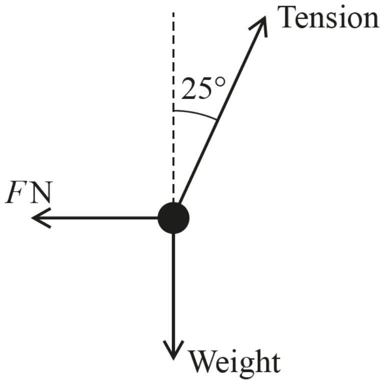
4 Using an appropriate expansion show that, for sufficiently small values of x,

$$\frac{1-x}{(2+x)^2} \approx \frac{1}{4} - \frac{1}{2}x + \frac{7}{16}x^2.$$ **[4]**

Most candidates realised this question required the use of a binomial expansion. Some candidates were unsure if taking out a factor of 2 from the bracket became ¼, ½, 2 or 4, but many candidates managed the expansion and multiplication successfully. Candidates were not penalised for not mentioning what happened to the term in x^3 obtained in the expansion but it was pleasing where it was seen.

Question 5 (a)

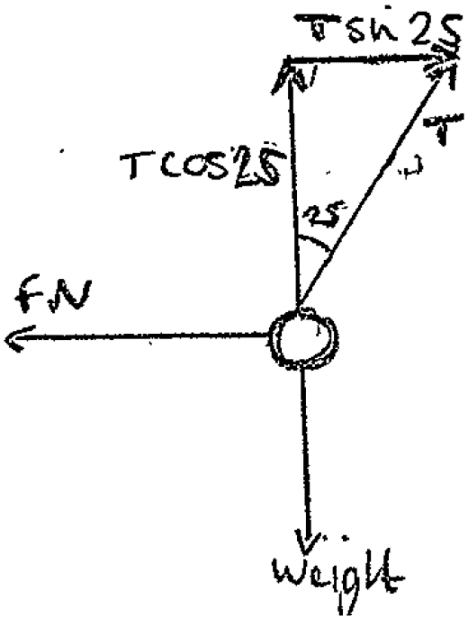
5 A sphere of mass 3 kg hangs on a string. A horizontal force of magnitude F N acts on the sphere so that it hangs in equilibrium with the string making an angle of 25° to the vertical. The force diagram for the sphere is shown below.



(a) Sketch the triangle of forces for these forces. **[2]**

This three force problem lends itself very well to using a triangle of forces to find T and F, and is a specification point in its own right. There were clearly a substantial number of candidates who did not know how to draw a closed loop of vectors to represent forces in equilibrium. A few candidates who drew a triangle of forces did not include an angle to specify the direction of T so lost the second mark.

Exemplar 2



This graph shows the components of the force T added to the given diagram. Although there is a triangle on the page, this is not what is meant by the term triangle of forces in this context.

Generally, where candidates add components of a force to an existing force diagram, they should use dotted arrows to distinguish the components from the actual forces.

Question 5 (b)

(b) Hence or otherwise determine each of the following:

- the tension in the string
- the value of F. **[3]**

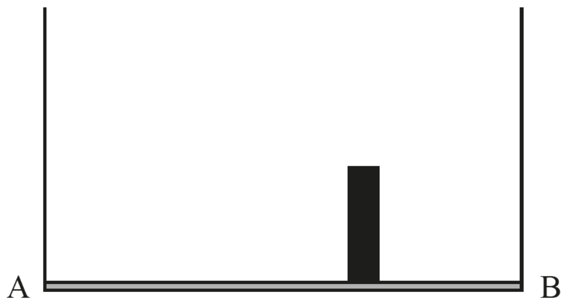
This was very well answered. Those who had had a good triangle of forces used it successfully to answer this. Most other candidates resolved horizontally and vertically to get the correct values for full credit here.

Section B overview

Section B consists of longer questions with more reading and interpretation needed. Some of the questions here were quite accessible while others had a more challenging modelling or problem solving component.

Question 6 (a)

6 A shelf consists of a horizontal uniform plank AB of length 0.8 m and mass 5 kg with light inextensible vertical strings attached at each end. A stack of bricks each of mass 2.3 kg is placed on the plank as shown in the diagram.



(a) Explain the meaning of each of the following modelling assumptions.

- The stack of bricks is modelled as a particle.
- The plank is modelled as uniform. **[2]**

Most understood what was required but felt that combining the bricks into a single object answered the first part – many would include irrelevant statements such neglecting air resistance. The second part was usually better with many stating that the centre of mass would be in the middle. Some said that the plank didn't bend, which is correct but that it is its rigidity and not its uniformity used. Others talked about the weight acting at a single point in both cases or that the forces would be in equilibrium. Statements involving the phrase "mass acts…" were not given a mark as mass is a passive quantity.

Assessment for learning

Several modelling assumptions are often listed together. Candidates need to be able to think about each separately and describe the effect of one particular assumption.

Question 6 (b)

Either of the strings will break if the tension exceeds 75 N.

(b) Find the greatest number of bricks that can be placed at the centre of the plank without breaking the strings. **[2]**

Candidates who balanced forces often did better here than those who took moments. The most common errors were to miss out the weight of the plank itself or to forget that there were two strings supporting the plank.

Question 6 (c)

(c) Find an expression for the moment about A of the weight of a stack of n bricks when the stack is at a distance of x m from A. State the units for your answer. **[2]**

Many candidates did not realise that the question required just a single moment, but the mark scheme allowed the mark to be given for the correct term seen in an equation whether the rest of the equation was correct or not. Many candidates did not give the correct units.

Question 6 (d)

(d) Calculate the greatest distance from A that the largest stack of bricks can be placed without a string breaking. **[3]**

The previous part question was intended to hint to candidates that a moments equation was needed here, and most realised this. The main error here came where candidates equated the moments of the weights to the tension in the string at B.

Assessment for learning

Candidates must form equations that are dimensionally correct. Here this means that every term in their equation must be a moment. An equation that equates a moment to a force can never be correct.

Question 7 (a)

7 In this question the x- and y-directions are horizontal and vertically upwards respectively and the origin is on horizontal ground.

A ball is thrown from a point 5 m above the origin with an initial velocity $\begin{pmatrix} 14 \\ 7 \end{pmatrix}$ m s^{-1}.

(a) Find the position vector of the ball at time t s after it is thrown. **[3]**

Writing this projectile question with vector notation made this question much more difficult for candidates. A good number thought they had to integrate the velocity giving them a model with no gravitational term. Others used $s = ut + \frac{1}{2}at^2$ with *u* as *a* vector and a as a scalar. This meant that all marks were lost.

Assessment for learning

An equation that adds a scalar term to a vector can never be correct.

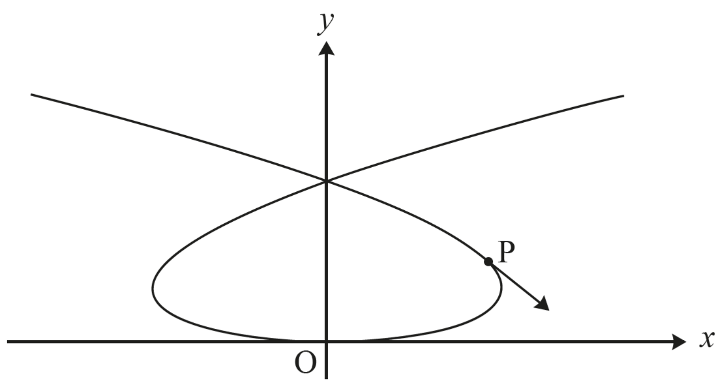
Question 7 (b)

(b) Find the distance between the origin and the point at which the ball lands on the ground. **[3]**

Some candidates succeeded here by restarting the question, ignoring their work with vectors and working with the two components separately. A common mistake was to assume that the time taken to reach the top would be half the time to reach the ground, but this is not the case where the ball does not start at ground level. A few candidates who had found the correct horizontal distance then went on to find the distance from the origin to the landing point, but his was ignored by examiners and the mark given for the correct horizontal distance seen earlier in their solution.

Question 8 (a)

8 A particle moves in the x-y plane so that its position at time t s is given by $x = t^3 - 8t,\ y = t^2$ for $-3.5 < t < 3.5$. The units of distance are metres. The graph shows the path of the particle and the direction of travel at the point P (8, 4).



(a) Find $\frac{dy}{dx}$ in terms of t. **[3]**

This question was very well answered. A few candidates tried to find and work with the cartesian equation of the curve but none were successful at finding the derivative in terms of *t*.

Question 8 (b)

(b) Hence show that the value of $\frac{dy}{dx}$ at P is -1. **[2]**

Most candidates used the y-coordinate to discover that the value of the parameter at P is ± 2. Some substituted $t = 2$ and found they did not have the correct value, so used $t = -2$ instead. The final mark was not given unless it was clear that the negative value of *t* gives the coordinates of P. Some candidates worked the other way round, solving $\frac{dy}{dx} = -1$ but often did not then establish that $t = -2$ gives both the coordinates of P.

Question 8 (c)

(c) Find the time at which the particle is travelling in the direction opposite to that at P. **[2]**

Most candidates misunderstood that travelling in the opposite direction happened at another point with gradient -1. Most solved the equation $\frac{dy}{dx} = 1$ which was awarded M0A0.

Question 8 (d)

(d) Find the cartesian equation of the path, giving x^2 as a function of y. **[3]**

This question was answered best by candidates who squared the expression for x first and substituted for t^2 in their equation. Many candidates used $y = \sqrt{t}$ but this only then gives the part of the graph where x is positive. Only where $y = \pm\sqrt{t}$ was used, did the response get full marks. It was also common to see errors in squaring the expression for x, as many candidates gave the sum of the squares of the two separate terms.

Question 9 (a)

9 In this question, the vectors **i** and **j** are directed east and north respectively.

The velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ of a particle at time t s is given by $\mathbf{v} = kt^2\mathbf{i} + 6t\mathbf{j}$, where k is a positive constant. The magnitude of the acceleration when $t = 2$ is $10\,\mathrm{m\,s^{-2}}$.

(a) Calculate the value of k. **[4]**

Most candidates were able to differentiate the vector to give an acceleration vector, although a few lost the **i** and **j** from the start. Many candidates wrote expressions in which vectors and scalars were mixed, such as $\begin{pmatrix} 4k \\ 6 \end{pmatrix} = 10$, but some candidates did recover from this and gained full marks. Missing brackets when squaring meant that some candidates did not square the 4.

Question 9 (b)

The particle is at the origin when $t = 0$.

(b) Determine an expression for the position vector of the particle at time t. **[2]**

Most candidates knew that integration was required here and did so correctly.

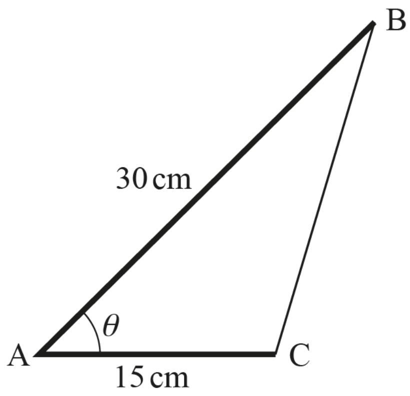
Question 9 (c)

(c) Determine the time when the particle is directly north-east of the origin. **[2]**

Most candidates understood that they needed to equate the components of their position vector, although a few used the velocity vector.

Question 10

10 A triangle ABC is made from two thin rods hinged together at A and a piece of elastic which joins B and C. AB is a 30 cm rod and AC is a 15 cm rod. The angle BAC is θ radians as shown in the diagram.



The angle θ increases at a rate of 0.1 radians per second.

Determine the rate of change of the length BC when $\theta = \frac{1}{3}\pi$. **[8]**

In this question, the angle theta is a variable, so the triangle is not generally right angled. When $\theta = \frac{\pi}{3}$ the triangle is momentarily right angled and the correct final answer can be obtained from invalid work, but this was not given the marks without correct working. Many candidates were not able to use the cosine rule correctly, but those who did often went on to score well, using the chain rule for differentiation with respect to theta. Quite a few candidates also successfully used implicit differentiation here. Most candidates understood the link between the derivatives involved and were given method marks for multiplying their derivatives and substituting $\theta = \frac{\pi}{3}$. A few good candidates lost the final A1 for not stating the units with their answer where this was felt to be necessary as one could work in metres throughout or centimetres.

Question 11

11 Given that k is a positive constant, show that $\int_k^{2k} \frac{2}{(2x+k)^2}\,dx$ is inversely proportional to k. **[6]**

There were some good solutions from candidates who integrated by inspection and also from those using the substitution $u = 2x + k$. There were a lot of candidates who found problems dealing with fractions with a factor of k in the denominator and some lost the final mark as they did not point out that their result showed inverse proportionality.

Question 12

12 Prove by contradiction that 3 is the only prime number which is 1 less than a square number. **[4]**

Candidates who did not know the premise of proof by contradiction could still gain SCB1 by showing the case for $2^2 - 1 = 3$ even if only highlighted in a list of different squares and primes they tried. Most knew how to start the proof gaining M1 generally by saying "assume that 3 is **not** the only prime that is one less than a square". Quite a few went on to write $p = n^2 - 1$ and factorise the difference of two squares. Only the best students then went on to earn both the 2 E marks for considering the general case for two factors and the specific case that one of those factors could be 1.

Some candidates tried methods they had encountered in other proofs such as separating the even and odd cases, or considering $\sqrt{\frac{m}{n}}$ without success. Some only made lists of square and/or prime numbers and could only get the SC mark..

Question 13 (a)

13 A toy train consists of an engine of mass 0.5 kg pulling a coach of mass 0.4 kg. The coupling between the engine and the coach is light and inextensible. The train is pulled along with a string attached to the front of the engine.

At first, the train is pulled from rest along a horizontal carpet where there is a resistance to motion of 0.8 N on each part of the train. The string is horizontal, and the tension in the string is 5 N.

(a) Determine the velocity of the train after 1.5 s. **[4]**

Finding the acceleration caused a lot of difficulty, with incorrect resistances and/or masses. However, almost all candidates knew how to use the constant acceleration equations to find the final velocity. Follow-through was allowed for any calculated acceleration.

Question 13 (b)

The train is then pulled up a track inclined at 20° to the horizontal. The string is parallel to the track and the tension in the string is P N. The resistance on each part of the train along the track is R N.

(b) Draw a diagram showing all the forces acting on the train modelled as two connected particles. **[3]**

Most diagrams were partially correct. For the first mark, candidates needed to have correct weights and distinct normal reactions arrows indicating the tension in the coupling were often missing or the tension incorrectly labelled with P. Many candidates only earned the mark earned for the resistances. No marks were given to candidates who drew a diagram showing the whole train as a single object.

Misconception

Two objects resting on the same surface will not necessarily have the same normal reaction.

Do not mark the different normal reactions with the same label.

Question 13 (c)

(c) Find the equation of motion for the train modelled as a single particle. **[2]**

This question was designed to test understanding of the term "equation of motion" and it was clear that many candidates did not know what was required. Some gave the resultant force but did not equate this to mass multiplied by acceleration. Some used weight multiplied by acceleration and were awarded M0A0.

Misconception

Some candidates wrote down $F = ma$ and then felt that there needed to be a force called F in their equation; this is a misconception that is seen repeatedly in mechanics papers. Teachers might be better to talk in terms of resultant force $= ma$ instead.

Question 13 (d)

(d) The acceleration of the train when $P = 5.5$ is double the acceleration when $P = 5$.

Calculate the value of R. **[3]**

Most candidates made a good effort at this, with most of those who wrote down their two equations getting the correct relationship between the accelerations. Some did not use their equation of motion from part c and started again. Many candidates with correct equations did not solve the simultaneous equations correctly and might have been better to rearrange the equations and use their calculator for that.

Question 14 (a)

14 Alex places a hot object into iced water and records the temperature θ °C of the object every minute. The temperature of an object t minutes after being placed in iced water is modelled by $\theta = \theta_0 e^{-kt}$ where θ_0 and k are constants whose values depend on the characteristics of the object.

The temperature of Alex's object is 82 °C when it is placed into the water. After 5 minutes the temperature is 27 °C.

(a) Find the values of θ_0 and k that best model the data. **[3]**

This was generally well answered.

Question 14 (b)

(b) Explain why the model may **not** be suitable in the long term if Alex does not top up the ice in the water. **[1]**

The key point here is to point out the mis-match between the model prediction for the long term and the real-life situation. The model predicts that the object will cool to zero degrees, and this will not be the case if the surroundings warm up.

Exemplar 3

14(b)	the temperature of the water will get hotter because the ice will melt so the speed of cooling would decrease

This candidate makes a sensible comment explaining why not topping up the ice will change the rate of cooling in the real world. The key point is missing however, so this was awarded 0/1.

Question 14 (c)

(c) Show that the model with the values found in part **(a)** can be written as $\ln\theta = a - bt$ where a and b are constants to be determined. **[2]**

This was an easy 2 marks for some candidates as they could see the similarity to part (a) especially when there was equation underneath for Ben's model. A common error was to multiply the 2 log terms instead of adding or omitting the log in front of 82. A few candidates started with this new form of the model and the given data, and although this gives the correct values for the constants, it did not show the equivalence between the models so was not given the marks.

Question 14 (d)

Ben places a different object into iced water at the same time as Alex. The model for Ben's object is $\ln\theta = 3.4 - 0.08t$.

(d) Determine each of the following:

- the initial temperature of Ben's object
- the rate at which Ben's object is cooling initially. **[4]**

Almost all candidates managed to find the correct initial temperature but many did not realise that they needed the value of $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ when $t = 0$. Those who did often made errors in the differentiation of the exponential model. There were some very good solutions from candidates who used implicit differentiation of the equation for $\ln\theta$. Some candidates found the derivative in terms of *t*, but did not substitute $t = 0$.

Question 14 (e)

(e) According to the models, there is a time at which both objects have the same temperature.

Find this time and the corresponding temperature. **[3]**

Many candidates realised they had to solve the two exponential equations or the two log equations simultaneously and were given the method mark once they had attempted to solve their equations. There were a lot or algebraic or arithmetic slips which denied candidates the final 2 accuracy marks.

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