OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Practice Paper – Set 1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Turn over

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $q = 1-p$

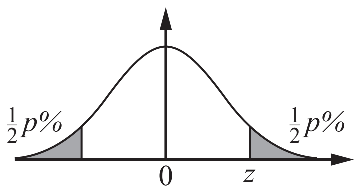
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 Differentiate $(3x^2+5)^4$. **[3]**

2 **In this question you must show detailed reasoning.**

The shaded region in Fig. 2 is bounded by the curves $y = \sin 3x$ and $y = 3\sin 3x$.

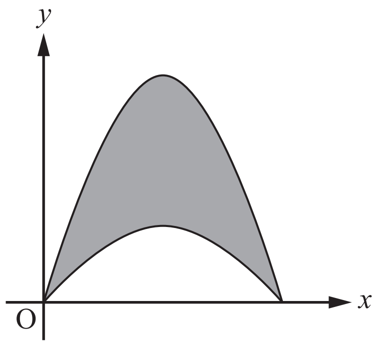


Fig. 2

Show that the area of this region is $\frac{4}{3}$. **[4]**

3 The sequence of positive numbers $a_1, a_2, a_3, \ldots$ is a geometric sequence. Prove that the sequence $\ln a_1, \ln a_2, \ln a_3, \ldots$ is an arithmetic sequence. **[3]**

4 A car travels along a straight track for 5 seconds. Its displacement s metres after t seconds is given by

$$s = 3t + 0.1t^3.$$

Show that the car does not have constant acceleration. **[3]**

5 Two chess pieces are placed on a uniform straight ruler. The ruler balances horizontally on a pivot.

- The ruler AB is of length 30 cm.
- The pivot P is at the centre of the ruler.
- The first chess piece, of mass 20 grams, is at A.
- The second chess piece, of mass 50 grams, is x cm from B.

This is shown in Fig. 5.

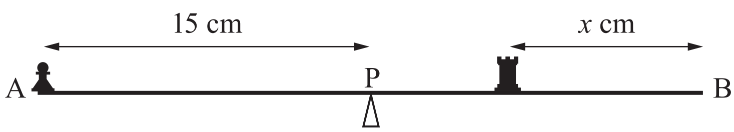


Fig. 5

Calculate the value of x. **[4]**

6 Fig. 6.1 shows a shape consisting of an isosceles triangle ABC and a semicircle with AC as diameter. Angle ABC $= \frac{1}{6}\pi$ radians and AB = BC = 10 cm.

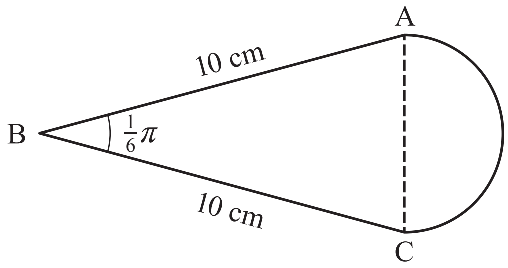


Fig. 6.1

(i) Calculate the total area of the shape. **[4]**

Fig. 6.2 shows a sector DEF of a circle of radius 10 cm. Angle DEF $= \theta$ radians.

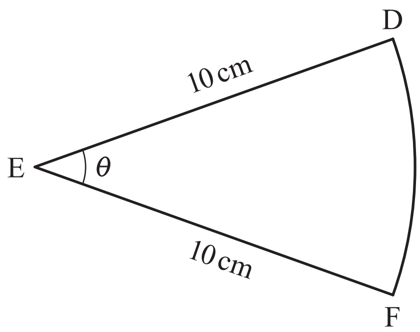


Fig. 6.2

(ii) The shapes in Fig. 6.1 and Fig. 6.2 have the same area. Find θ, giving your answer correct to 3 significant figures. **[2]**

Answer **all** the questions

Section B (77 marks)

7 A globe hangs on the end of a light wire that makes an angle of θ with the vertical. It is held in place by a light horizontal string, as shown in Fig. 7.

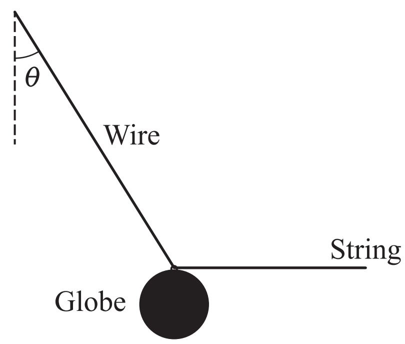


Fig. 7

Find the set of values of θ for which the tension in the string is less than the weight of the globe. **[4]**

8 **(i)** Use the binomial expansion to show that $(1-2x)^{-\frac{1}{2}} \approx 1+x+\frac{3}{2}x^2$ for sufficiently small values of x. **[2]**

(ii) For what values of x is the expansion valid? **[1]**

(iii) Find the expansion of $\sqrt{\dfrac{1+2x}{1-2x}}$ in ascending powers of x as far as the term in x^2. **[3]**

(iv) Use $x=\frac{1}{20}$ in your answer to part **(iii)** to find an approximate value for $\sqrt{11}$. **[2]**

9 Arjun is trying to hit a can with a stone. The can is standing on a narrow wall 4 m away from him. The can is 10 cm tall and its base is 1.9 m above the ground, which is level. Arjun throws the stone at the can with a speed of $8\,\mathrm{m\,s^{-1}}$ at an angle of 35° above the horizontal. The point of projection is 1 m above the ground.

Determine whether the stone hits the can. **[7]**

10 **(i)** Prove the identity $\sec\theta-\cos\theta \equiv \tan\theta\sin\theta$. **[4]**

(ii) Hence or otherwise solve the equation $\sec\theta-\cos\theta=\frac{1}{2}\tan\theta$ for $0 \leqslant \theta < 2\pi$. **[4]**

11 Austin pushes a curling stone of mass 18 kg on ice. He releases the stone and it slides 21 m in a straight line until coming to rest after 14 s. He models the motion by assuming that the frictional force is constant.

(i) Show that the speed of the stone at the moment that Austin releases it is $3\,\mathrm{m\,s^{-1}}$. **[2]**

(ii) Find the magnitude of the frictional force acting on the stone. **[3]**

(iii) Calculate the coefficient of friction between the stone and the ice, giving your answer correct to 2 significant figures. **[3]**

12 A toy car moves on a horizontal surface. Its velocity in $\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 1.5\mathbf{i} + 0.5t\mathbf{j}$$

where **i** and **j** are unit vectors east (x-direction) and north (y-direction) respectively and t is the time in seconds.

Initially the car is at the point 2 m north of the origin.

(i) Calculate the speed of the car after 3 seconds. **[2]**

(ii) Find the position vector of the car after t seconds. **[3]**

(iii) Show that the cartesian equation of the path of the car is $y = \frac{x^2}{9} + 2$. **[3]**

13 Gerry has been collecting data about the ladybird population in his locality, where they appear mainly during the spring, summer and autumn. He has carried out surveys on many days over a three-month period in the summer, and has recorded the values of N, the number of ladybirds seen, and t, the time in days after 1 January. Using graph-drawing software, he has found that the part of the quadratic curve shown in Fig. 13.1 is a good fit for his data.

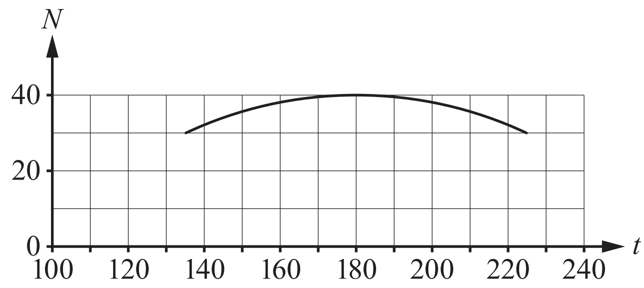


Fig. 13.1

This curve has a maximum point at (180, 40) and the point (135, 30) also lies on the curve. Gerry uses the equation of the curve in the form

$$N = a - b(t - c)^2$$

as a model for the ladybird population.

(i) Find the values of a and c and show that $b = \frac{2}{405}$. **[3]**

(ii) (A) Find the number of ladybirds predicted by the model when $t = 250$. **[1]**

(B) Explain why this predicted value may be unreliable. **[1]**

(iii) (A) Explain why the model is not valid for $t < 90$. **[1]**

(B) Give another range of values of t for which the model is not valid. **[1]**

With the software set to degrees, Gerry also produces the graph in Fig. 13.2 to model his data for suitable values of t. This graph is a transformation of $N = \cos t$; it passes through (90, 0) and has a maximum point at (180, 40).

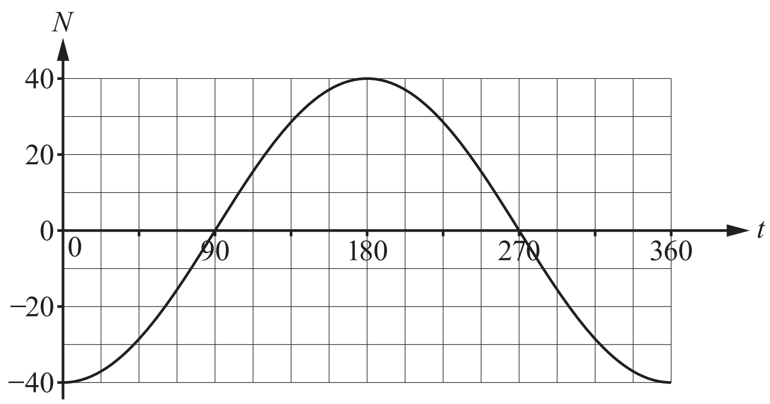


Fig. 13.2

(iv) Describe a sequence of transformations that maps the curve $N = \cos t$ to the curve in Fig. 13.2. **[2]**

(v) Write down the equation for N in terms of t for this model. **[1]**

(vi) (A) Explain how the model can be refined to have a period of 365 days. **[1]**

(B) What effect, if any, does this refinement have on the maximum point of the graph? **[1]**

14 A car pulls a caravan along a straight road uphill on a slope that makes an angle of 8° with the horizontal. The mass of the car is 1700 kg and the mass of the caravan is 1300 kg. The driving force is 9000 N and the car and caravan experience resistances of 600 N and 1200 N respectively. The coupling between the car and the caravan is light and parallel to the road.

(i) Draw a diagram showing all the forces acting on the car and all the forces acting on the caravan. **[3]**

(ii) Find the acceleration of the car and caravan as they move up the hill. **[4]**

(iii) Find the tension in the coupling between the car and the caravan. **[3]**

15 **(i)** Solve the differential equation

$$\frac{dy}{dx} = y(1+y)(1-x),$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$, where f is a function to be determined. **[9]**

(ii) By considering the sign of $\frac{dy}{dx}$ near (1, 1), or otherwise, show that this point is a maximum point on the curve $y = f(x)$. **[3]**

END OF QUESTION PAPER

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