OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Practice Paper – Set 1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1-p$

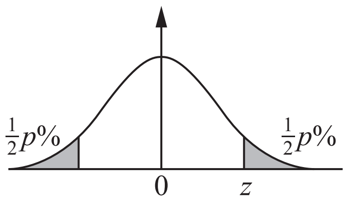
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 You are given that $\text{gf}(x) = |3x-1|$, for $x \in \mathbb{R}$.

(i) Given that $\text{f}(x) = 3x-1$, express $\text{g}(x)$ in terms of x. **[1]**

(ii) State the range of $\text{gf}(x)$. **[1]**

(iii) Solve the inequality $|3x-1| > 1$. **[4]**

2 **In this question you must show detailed reasoning.**

Fig. 2 shows the line $2x+y=6$. The line crosses the x-axis at A and the y-axis at B.

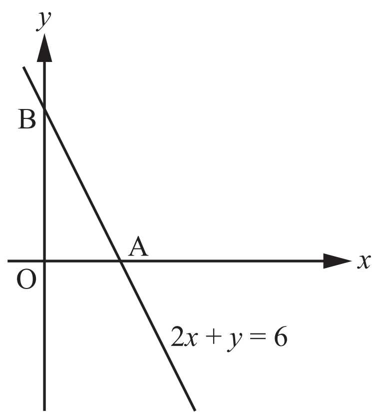


Fig. 2

Find the equation of the quadratic curve which touches the x-axis at A and passes through B. **[5]**

3 Point P lies on the positive x-axis and point Q lies on the positive y-axis. Triangle OPQ is isosceles and its area is 18 square units. Fig. 3 shows the circle which passes through points O, P and Q.

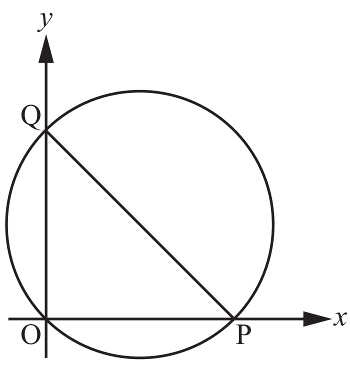


Fig. 3

Find the equation of the circle. **[5]**

4 Prove from first principles that the derivative of x^3 is $3x^2$. **[5]**

5 **(i)** **In this question you must show detailed reasoning.**

Determine the exact values of k for which the curves $y = x^2 - kx$ and $y = 3(k+1) + kx - x^2$ touch. **[6]**

(ii) Determine whether or not there is a value of k for which the curves cross on the y-axis. **[4]**

6 Points A, B and C have position vectors $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $4\mathbf{i} - \mathbf{j}$ and $5\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ respectively.

Find the position vector of the point D such that ABCD is a parallelogram. **[4]**

7 **In this question you must show detailed reasoning.**

(i) For the curve $y = x^4 - 3x^2 + 2$, find the equation of the normal at the point (1, 0). **[4]**

(ii) For the curve $y = x^4 - 3x^2 + c$, L is the normal at the point where $x = 1$. Determine the value of c for which L passes through the origin. **[3]**

8 **In this question you must show detailed reasoning**.

A geometric series has first term $(b^2 - 13)$, common ratio $\frac{1}{b}$ and sum to infinity -6.

Find the possible values of the common ratio. **[9]**

9 **(i)** Express $2\cos\theta + 3\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is a positive constant given in exact form. **[4]**

(ii) Determine the set of values of k for which the curve $y = k + 2\cos x + 3\sin x$ lies completely above the x-axis. **[4]**

(iii) Explain why the curve $y = \dfrac{1}{k + 2\cos x + 3\sin x}$ lies completely above the x-axis for the set of values of k found in part **(ii)**. **[1]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

10 Use integration by parts to show that $\int_1^8 \ln x \, dx = 8\ln 8 - 7$, as given in line 13. **[4]**

11 Starting from Fig. C1.3, show that $\ln 8! > 8\ln 8 - 7$, as given in line 19. **[3]**

12 **(i)** Prove the following.

(A) The function $\ln x$ is increasing throughout its domain. **[2]**

(B) The graph of $y = \ln x$ is concave downwards. **[2]**

(ii) Explain why the approximation to $\int_1^n \ln x \, dx$ using the trapezium rule must be an underestimate of the exact value of the integral. **[1]**

13 ${}^{2n}C_n = \frac{(2n)!}{n!n!}$ is sometimes called the nth central binomial coefficient. Using Stirling's formula as given in line 37, show that ${}^{2n}C_n \approx \frac{4^n}{\sqrt{\pi n}}$. **[3]**

END OF QUESTION PAPER

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