Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 14 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

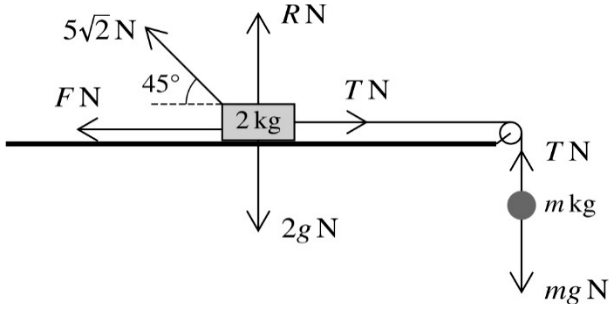
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			Product rule with $u = 3x^2$ and $v = \sin 2x$	M1	1.1a	Need not be written explicitly	
			$\frac{dy}{dx} = 6x\sin 2x + 6x^2 \cos 2x$	A1	1.1b	For either of the two terms correct	
				A1 [3]	1.1b	For completely correct answer	
2			Vertical equilibrium: weight = 10 + 15 = 25 N Moments about A: 25x = 15 × 0.8 x = 0.48 m	B1 M1 A1 [3]	1.1a 3.1b 1.1b	soi or take moments about other points	
3	(i)		$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$	M1	3.1a		Or $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$
			$\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$	M1	3.1a	Use of $\|\|^{gm}$ and vector subtraction soi	$\overrightarrow{CD} = \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$
			$\overrightarrow{OD} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix}$	A1 [3]	1.1b		
3	(ii)		$\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}$	B1	1.1a		Or $\overrightarrow{CA} = \begin{pmatrix} 3 \\ -1 \\ -7 \end{pmatrix}$
			$AC = \sqrt{9+1+49}$	M1	1.1a		
			$= \sqrt{59}$	A1 [3]	1.1b		
4			Resolve down the slope, using Newtons' second law	M1	3.3		
			$mg\sin 20° = ma$	A1	1.1b		
			$v^2 = 0^2 + 2(g\sin 20°)s$	M1	1.1a	Use of $v^2 = u^2 + 2as$	
			$v = \sqrt{13.407...} = 3.66 \text{ m s}^{-1}$	A1 [4]	1.1b		

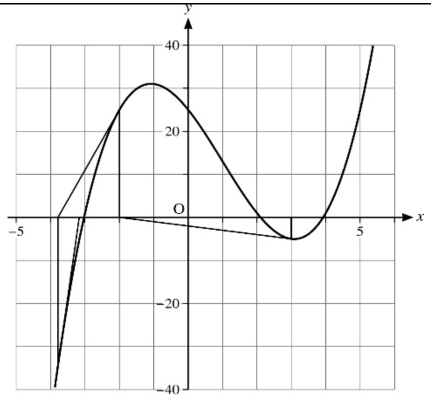
Question			Answer	Marks	AOs	Guidance	
5			Use $x = e^u$ and $\frac{dx}{du} = e^u$	M1	1.1b	Using substitution including dx	
			$\int (\ln x)^2 \, dx = \int (\ln e^u)^2 e^u \, du = \int u^2 e^u \, du$	A1	1.1b	Simplifying correctly	
			Use integration by parts	M1	3.1a	First use of integration by parts	
			$= u^2 e^u - \int 2ue^u \, du$	A1	1.1b	First stage all correct	
			$= u^2 e^u - 2\left(ue^u - \int e^u \, du\right)$	M1	1.1a	Second use of integration by parts	
			$= u^2 e^u - 2ue^u + 2e^u + c$				
			$= x(\ln x)^2 - 2x \ln x + 2x + c$	A1	1.1b	Must be in terms of x for final mark	
				[6]			

Question			Answer	Marks	AOs	Guidance	
6			**DR**				
			$0.025t^3 - 0.8t^2 + 6.4t = 0$	M1	3.4	Equating v to 0 for time at junction	
			$0.025t(t^2 - 32t + 256) = 0 \Rightarrow 0.025t(t-16)^2 = 0$				
			t = 0 or 16	A1	2.1	Factorising seen; method must be clear	
			Distance is $\int_0^{16} (0.025t^3 - 0.8t^2 + 6.4t) \, dt$	M1	3.4	Limits not required for this mark	
			$= \left[0.025\frac{t^4}{4} - 0.8\frac{t^3}{3} + 6.4\frac{t^2}{2} \right]_0^{16}$	A1	1.1b	Correct integration, and limits soi	
			$= \left(0.025 \times \frac{16^4}{4} - 0.8 \times \frac{16^3}{3} + 6.4 \times \frac{16^2}{2}\right) - (0)$	M1	2.1	Use of limits seen; substitution of limits into integral must be seen	$\frac{2048}{5} - \frac{16\,384}{15} + \frac{4096}{5}$
			Distance = 137 m (3 sf)	B1	1.1b	Allow for any method www	$\frac{2048}{15} = 136.53\ldots$
				[6]			

Question			Answer	Marks	AOs	Guidance	
7	(i)		$AB^2 = 2a^2 - 2a^2\cos\theta$	B1 [1]	1.1b		
7	(ii)		$AD = a\sin\left(\frac{1}{2}\theta\right)$ $AB = 2AD \Rightarrow AB^2 = 4AD^2$, so $2a^2 - 2a^2\cos\theta = 4\left(a\sin\left(\frac{1}{2}\theta\right)\right)^2$ $1-\cos\theta = 2\sin^2\left(\frac{1}{2}\theta\right) \Rightarrow \cos\theta = 1-2\sin^2\left(\frac{1}{2}\theta\right)$	B1 M1 E1 [3]	3.1a 2.1 2.1	Must be in terms of a and θ Must handle squared term correctly **AG** Result must be clearly shown	
7	(iii)		Proved for $0 < \theta < 180°$ (as θ is angle in a triangle)	E1 [1]	2.3	Allow 'between' in words	Or $0 < \theta < \pi$
8	(i)		At stationary point: $4x^3 - 9x^2 + 6x = 0$ $x(4x^2 - 9x + 6) = 0 \Rightarrow x = 0$ or $4x^2 - 9x + 6 = 0$ Discriminant of quadratic is $(-9)^2 - 4\times 24 = -15 < 0$ So $x = 0$ gives the only stationary point $f(0) = 0^4 - 3\times 0^3 + 3\times 0^2 = 0$, so the point is the origin $f''(x) = 12x^2 - 18x + 6 \Rightarrow f''(0) = 6 > 0$ So the origin is a minimum point	M1 M1 M1 A1 B1 M1 A1 [7]	1.1a 1.1a 1.1a 2.2a 2.1 1.1a 2.2a	Attempt to differentiate & equate to 0 Factorising the cubic oe (quadratic formula, completing the square, complex roots from calculator) $f(0)$ or y must be shown to be zero Or other valid method, e.g. sign of $f'(x)$ for $x < 0$ and $x > 0$ **AG** Must establish minimum clearly	
8	(ii)		f is not a one-to-one function	B1 [1]	2.4		
8	(iii)		$g(0) = 0$ and $g(2) = 4$ Domain of $g^{-1}(x)$ is range of $g(x)$, so $0 \le x \le 4$ Range of $g^{-1}(x)$ is domain of $g(x)$, so $0 \le g^{-1}(x) \le 2$	 B1 B1 [2]	 2.2a 1.2	 Must not be in terms of y Must not be an interval for x	

Question			Answer	Marks	AOs	Guidance	
9	**(i)**			**B1** **B1** **[2]**	**1.1a** **3.3**	Weights and normal reaction labelled Tension correct in both parts of the string and friction in the correct direction	Condone absence of units N in diagram Friction force could be shown as μR or $0.6R$ and tension could be shown as mg
9	**(ii)**		$R + 5\sqrt{2}\sin 45° = 2g$ $R = 2g - 5 \quad (=14.6)$ $F = 0.6(2g - 5)$ $F = 8.76$ N	**M1*** **A1** **M1 dep*** **A1** **[4]**	**3.4** **1.1b** **1.1a** **1.1b**	Resolve vertically for the block Numerical evaluation not needed here Use of $F = \mu R$, but do not allow if $R = 2g$	Allow sin or cos here
9	**(iii)**		$T = F + 5\sqrt{2}\cos 45° \quad (=13.76)$ $T = mg$ $m = \frac{13.76}{g} = 1.40$	**M1** **A1** **M1** **A1** **[4]**	**3.4** **1.1b** **1.1a** **1.1b**	Resolve horizontally for the block Correct equation; evaluation not required for this mark Resolve vertically for the ball	Allow sin or cos here soi

Question			Answer	Marks	AOs	Guidance	
10	(i)		**DR** $8\cos x + 5\sin x = R(\cos x\cos\alpha + \sin x\sin\alpha)$, so $8 = R\cos\alpha$ and $5 = R\sin\alpha$ $R = \sqrt{8^2+5^2} = \sqrt{89}$ $\alpha = \arctan\left(\frac{5}{8}\right)$ $8\cos x + 5\sin x = \sqrt{89}\cos\left(x - \arctan\left(\frac{5}{8}\right)\right)$	 **M1** **B1** **A1** **[3]**	 **1.1a** **1.1b** **1.1b**	 Equating coefficients Accept 9.43 or better Accept 0.559 or better (No penalty for omission of this step)	
10	(ii)		**DR** $\cos\left(x - \arctan\left(\frac{5}{8}\right)\right) = \frac{6}{\sqrt{89}}$, so $x - \arctan\left(\frac{5}{8}\right) = 0.88149\ldots$ or $2\pi - 0.88149\ldots$ $x = 1.4401$ $x = 5.9603$	 **M1** **A1** **A1** **[3]**	 **1.1a** **1.1a** **1.1a**	 Method leading to at least one solution If a rounded value from (i) used max. A1 only	

Question			Answer	Marks	AOs	Guidance	
11	(i)		$f(x)=x^3-3x^2-10x+25 \Rightarrow f'(x)=3x^2-6x-10$, so the N-R formula gives $x_{n+1}=x_n-\frac{x_n^3-3x_n^2-10x_n+25}{3x_n^2-6x_n-10}$	E1 [1]	2.1	**AG** Must be clear that denominator is derivative of numerator	
11	(ii)	(*A*)	Not valid: the sequence may decrease further, far enough to change the first 3 figures	B1 [1]	2.3	Reason for 'not valid' needed	
11	(ii)	(*B*)	$f(3.915)=-0.1255...$ and $f(3.925)=2.03\ldots\times10^{-4}$ Change of sign shows that there is a root in the interval (3.915, 3.925) so the root is 3.92 to 2dp	M1 A1 [2]	2.1 2.2a	Both calculations Complete argument needed	Allow use of any two values closer to 3.92 that give sign change
11	(iii)	(*A*)	$x_0=3 \Rightarrow x_1=-2$ $x_2=-3.785\,71\ldots$ and $x_3=-3.168\,34\ldots$	B1 B1 [2]	1.1b 1.1b	Correct first iteration x_2 and x_3 correct to at least 3dp	
11	(iii)	(*B*)	 The initial value is close to a stationary point, so the tangent meets the *x*-axis far from the required root, and the sequence converges to the wrong root	B1 B1 [2]	2.3 2.4	Explanations do not need to include a sketch; if a sketch is included, ignore any inaccuracies if correct explanation is given; sketch with no explanation scores 0 'close to stationary point' oe seen 'converges to wrong root' oe seen	
11	(iv)		Choose starting value near the root and not near a stationary point, eg take $x_0=2$	E1 [1]	2.1		

Question			Answer	Marks	AOs	Guidance	
12	(i)		For AB: use of $s = ut + \frac{1}{2}at^2$ with s = 64, t = 4 gives	M1	3.3		Allow use of a different sign convention (eg negative a) provided it is used consistently throughout and is explained
			$64 = 4u + \frac{1}{2}a \times 4^2$	A1	1.1b	Correct (unsimplified) equation	
			For AC: s = 96 and t = 8	B1	3.1b	For both 96 and 8 seen	
			$96 = 8u + \frac{1}{2}a \times 8^2$	M1	3.4	Forming second equation in u and a	
			Solve simultaneously ($16 = u + 2a$, $12 = u + 4a$)	M1	1.1a	May be implied if calculator used	
			u = 20 and a = –2	A1	1.1b	Both correct; allow deceleration = 2	
			Alternative solution				
			For AB: use of $s = ut + \frac{1}{2}at^2$ with s = 64, t = 4 gives	M1			Allow use of a different sign convention (eg negative a) provided it is used consistently throughout and is explained
			$64 = 4u + \frac{1}{2}a \times 4^2$	A1		Correct (unsimplified) equation	
			For BC: speed at B is $u + 4a$	B1		Use of $v = u + at$ for AB	
			$32 = 4(u + 4a) + \frac{1}{2}a \times 4^2$	M1		Forming second equation in u and a	
			Solve simultaneously ($16 = u + 2a$, $8 = u + 6a$)	M1		oe BC	
			u = 20 and a = –2	A1		Both correct; allow deceleration = 2	
				[6]			
12	(ii)		u = 20, v = 0, a = –2 gives $0 = 20^2 - 2 \times 2 \times s$	M1	3.4	Allow for any *suvat* equation(s) leading to a value for s	
			s = 100 so truck comes to rest 4 m beyond C	A1	1.1b		
				[2]			

Question			Answer	Marks	AOs	Guidance	
13	(i)		$\frac{dx}{dt}$ is the rate at which x is increasing Mass of B is x, so mass of A is $(1 - x)$ $\frac{dx}{dt} \propto x(1-x)$, so $\frac{dx}{dt} = kx(1-x)$	**B1** **[1]**	**2.1**	Must indicate where terms come from **AG**	
13	(ii)		$\int \frac{1}{x(1-x)}\,dx = \int k\,dt$	**M1**	**3.1a**	Separation of variables	
			$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$	**M1**	**3.1a**	Find partial fractions (may be implied)	
			$1 \equiv A(1-x) + Bx \Rightarrow A = 1,\ B = 1$	**A1**	**2.2a**		
			$\int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx = \int k\,dt \Rightarrow \ln x - \ln(1-x) = kt + c$	**M1***	**1.1a**	Condone sign error, but must have two ln terms and $+c$	
			$t = 0,\ x = 0.2 \Rightarrow c = \ln\frac{1}{4}$ (oe)	**M1 dep***	**3.1a**	Use of initial conditions; may be done after equation is rearranged	
			$\frac{4x}{1-x} = e^{kt}$ (oe)	**M1 dep***	**1.1b**	Rearrange equation to remove logs; may be done before finding c	
			$x = \frac{e^{kt}}{4 + e^{kt}}$	**A1** **[7]**	**2.5**	oe, but must be of the form $x = f(t)$	
13	(iii)		$t = 15,\ x = 0.9 \Rightarrow 36 = e^{15k}$ (oe) $k = 0.239$ to 3sf	**M1** **A1** **[2]**	**3.3** **1.1b**	Substitute values in their solution	
13	(iv)		$t = 30 \Rightarrow$ mass of B is $\frac{e^{0.239\times30}}{4 + e^{0.239\times30}} = 0.997$ kg	**B1** **[1]**	**3.4**		
13	(v)		As $t \to \infty,\ x \to 1$ and so $1 - x \to 0$, so the model predicts there is a very small amount of A remaining when t is large	**B1** **[1]**	**3.5a**	May evaluate x for large t (eg t = 100)	

Question			Answer	Marks	AOs	Guidance	
14	(i)		10 (m s^{-2})	B1 [1]	3.4		
14	(ii)		g varies according to location	B1 [1]	1.2		
14	(iii)		$\mathbf{v}=\left(u_1\mathbf{i}+u_2\mathbf{j}\right)-10t\mathbf{j}$	M1	3.1b	Differentiation of **r** to find **v**	
			Maximum height when $u_2-10t=0$	M1	3.1b	Equating their **j** component to zero	
			$t=\frac{u_2}{10}$	A1	1.1b		
			$(u_1\mathbf{i}+u_2\mathbf{j})\frac{u_2}{10}-5\mathbf{j}\left(\frac{u_2}{10}\right)^2=14\mathbf{i}+20\mathbf{j}$	M1	1.1a	Equate **r** with their t to given vector	
			$\frac{u_2^2}{10}-\frac{5u_2^2}{100}=20\Rightarrow u_2=20$	A1	1.1b	cao, from equating **j** components	
			$u_1\times\frac{20}{10}=14\Rightarrow u_1=7$	M1	3.1b	Equating **i** components to find u_1	
			Initial velocity is (7**i** + 20**j**) m s^{-1}	A1	2.5	Must be in vector form	Accept $\begin{pmatrix}7\\20\end{pmatrix}$
			Alternative solution				
			Vertical motion has $u=u_2, v=0, a=-10, s=20$	M1			
			$0=u_2^2+2\times(-10)\times 20$	M1		Use of *suvat* equation(s) leading to u_2	
			$u_2=20$	A1		cao	
			$0=20-10t$	M1		Use of *suvat* equation(s) leading to t	
			t = 2	A1			
			Horizontal motion: $14=u_1\times 2\Rightarrow u_1=7$	M1		Use of their t in constant speed eqn	
			Initial velocity is (7**i** + 20**j**) m s^{-1}	A1		Must be in vector form	Accept $\begin{pmatrix}7\\20\end{pmatrix}$
				[7]			
14	(iv)		$21=7t\Rightarrow t=3$	M1	3.1b	Finding t from horizontal motion	
			Height at $t=3$ is $20\times3-5\times3^2$	M1	1.1a	Use of $s=ut+\frac{1}{2}at^2$ with their t	
			= 15 m	A1 [3]	3.2a		

BLANK PAGE