OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Integration

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ $\quad\left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X=r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1-p$

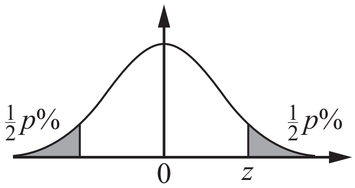
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (22 marks)

1 Find $\dfrac{dy}{dx}$ given that $y = 3x^2 \sin 2x$. **[3]**

2 A non-uniform rod 0.8 m long rests horizontally on smooth pegs A and B at each end of the rod. The contact forces at A and B are 10 N and 15 N respectively, as shown in Fig. 2.

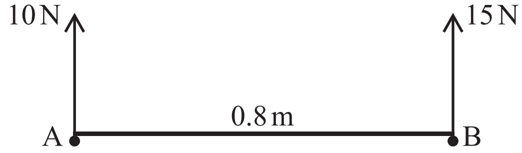


Fig. 2

Calculate the distance of the centre of mass of the rod from A. **[3]**

3 ABCD is a parallelogram. The points A, B and C have position vectors

$$\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$$

respectively.

(i) Find the position vector of the point D. **[3]**

(ii) Find the exact distance AC. **[3]**

4 A block slides from rest down a line of greatest slope of a smooth plane inclined at 20° to the horizontal. Calculate the speed of the block when it has travelled 2 m. **[4]**

5 Using the substitution $x = e^u$, find $\int (\ln x)^2 \, dx$. **[6]**

Answer **all** the questions

Section B (78 marks)

6 **In this question you must show detailed reasoning.**

Fig. 6 shows the velocity-time graph for a car as it travels along a straight road. The car sets off from some traffic lights and stops momentarily at a road junction. The velocity $v\,\mathrm{m\,s^{-1}}$ of the car at time t s after leaving the traffic lights is modelled by

$$v = 0.025t^3 - 0.8t^2 + 6.4t \text{ for } 0 \leqslant t \leqslant 20.$$

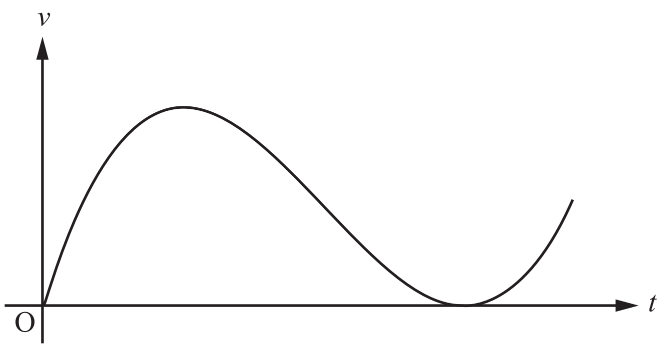


Fig. 6

Calculate the distance from the traffic lights to the road junction. **[6]**

7 Fig. 7 shows an isosceles triangle ABC, in which AC = BC = a and angle ACB = θ.

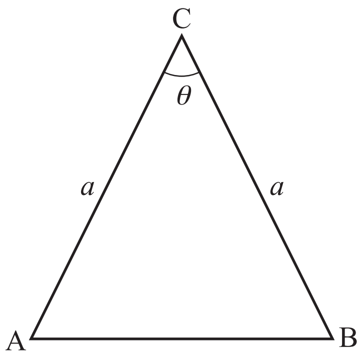


Fig. 7

(i) Use the cosine rule to write down an expression for AB^2 in terms of a and θ. **[1]**

(ii) D is the midpoint of AB. Use an expression for AD in terms of a and θ to show that

$$\cos\theta \equiv 1 - 2\sin^2(\tfrac{1}{2}\theta).$$ **[3]**

The identity in part **(ii)** is valid for all values of θ. However, the argument in parts **(i)** and **(ii)** does not prove the identity for all values of θ.

(iii) State the values of θ for which the argument is valid. **[1]**

8 The function f(x) is defined by $f(x) = x^4 - 3x^3 + 3x^2$ for $x \in \mathbb{R}$.

(i) Show that the only stationary point on the curve $y = f(x)$ is a minimum point at the origin. **[7]**

(ii) Explain why f(x) does not have an inverse function. **[1]**

The function g(x) is defined by $g(x) = x^4 - 3x^3 + 3x^2$ for $0 \leqslant x \leqslant 2$.

(iii) State the domain and range of the inverse function $g^{-1}(x)$. **[2]**

9 Fig. 9 shows a block of mass 2 kg resting on a rough horizontal table. It is attached to a ball of mass m kg by a light inextensible string that passes over a smooth pulley at the edge of the table. The ball hangs vertically below the pulley. The coefficient of friction between the block and the table is 0.6.

The system is held in equilibrium by a force of $5\sqrt{2}$ N acting on the block at 45° above the horizontal. The block is on the point of sliding towards the pulley.

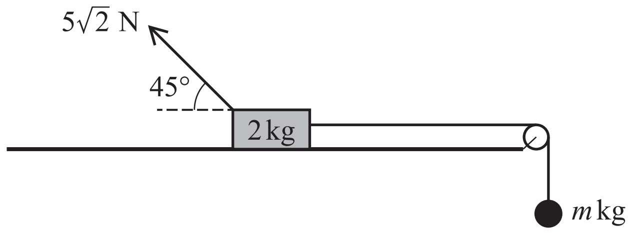


Fig. 9

(i) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the block and the ball. **[2]**

(ii) Calculate the frictional force acting on the block. **[4]**

(iii) Calculate the value of m. **[4]**

10 **In this question you must show detailed reasoning.**

(i) Express $8\cos x + 5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) Hence solve the equation $8\cos x + 5\sin x = 6$ for $0 \leqslant x < 2\pi$, giving your answers correct to 4 decimal places. **[3]**

11 Joe uses the Newton-Raphson method to try to solve the equation $x^3 - 3x^2 - 10x + 25 = 0$.

(i) Show that the formula Joe should use is $x_{n+1} = x_n - \dfrac{x_n^3 - 3x_n^2 - 10x_n + 25}{3x_n^2 - 6x_n - 10}$. **[1]**

(ii) Joe uses $x_0 = 4$ in this formula to find a root and obtains the following values:

$$x_1 = 3.928\,571\,4,$$
$$x_2 = 3.924\,992\,8.$$

Joe states that the root must be 3.92 to 2 decimal places and argues that this is because both x_1 and x_2 begin with 3.92.

(*A*) Comment on the validity of Joe's argument. **[1]**

(*B*) Use a sign change argument to show that Joe's statement is correct. **[2]**

The graph of $y = x^3 - 3x^2 - 10x + 25$ in Fig. 11 shows that there is a root between 2 and 3.

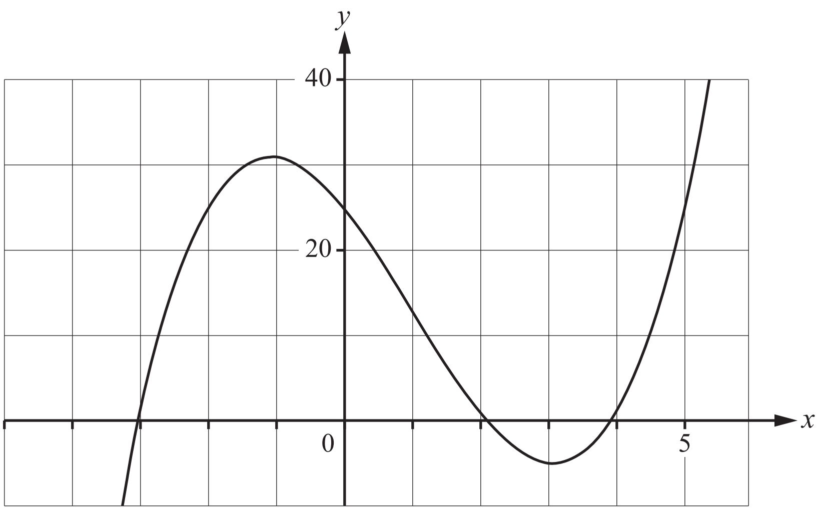


Fig. 11

(iii) Joe uses $x_0 = 3$ to attempt to find this root.

(*A*) Find x_1, x_2 and x_3. **[2]**

(*B*) Explain why the Newton-Raphson method fails to give the required root in this case. **[2]**

(iv) Explain what Joe should do to find the root of the equation between 2 and 3. **[1]**

12 A truck is travelling along a straight road ABC, and is slowing down at a constant rate. The truck takes 4 s to travel the 64 m from A to B and it takes another 4 s to travel the 32 m from B to C.

(i) Find

- the speed of the truck at A,
- the acceleration of the truck. **[6]**

(ii) Find how far beyond C the truck travels before coming to rest. **[2]**

13 In a chemical reaction, compound B is formed from compound A and other compounds. The mass of B at time t minutes is x kg. The total mass of A and B is always 1 kg. Sadiq formulates a simple model for the reaction in which the rate at which the mass of B increases is proportional to the product of the masses of A and B.

(i) Show that the model can be written as $\frac{dx}{dt} = kx(1-x)$, where k is a constant. **[1]**

Initially, the mass of B is 0.2 kg.

(ii) Solve the differential equation, expressing x in terms of k and t. **[7]**

After 15 minutes, the mass of B is measured to be 0.9 kg.

(iii) Find the value of k, correct to 3 significant figures. **[2]**

(iv) Find the mass of B after 30 minutes. **[1]**

(v) Explain what the model predicts for the mass of A remaining for large values of t. **[1]**

14 In this question, **i** is a horizontal unit vector and **j** is a unit vector directed vertically upwards.

A particle is projected from the origin with an initial velocity of $(u_1\mathbf{i} + u_2\mathbf{j})\,\text{m s}^{-1}$, and moves freely under gravity. Its position vector **r** m at time t s is given by

$$\mathbf{r} = (u_1\mathbf{i} + u_2\mathbf{j})t - 5t^2\mathbf{j}.$$

(i) Write down the value of g used in this model. **[1]**

(ii) Explain what is meant by the statement that g is not a universal constant. **[1]**

The position vector of the particle when it reaches its maximum height is (14**i** + 20**j**) m.

(iii) Determine the initial velocity of the particle, giving your answer as a vector. **[7]**

(iv) The particle hits a building which is 21 m away from the origin in the **i** direction. Calculate the height above the level of the origin at which the particle hits the building. **[3]**

END OF QUESTION PAPER