Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Integration

$\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\displaystyle\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ $\quad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A)+P(B)-P(A \cap B)$

$P(A \cap B) = P(A)P(B\mid A) = P(B)P(A\mid B)$ **or** $P(A\mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X=r) = {}^nC_r\,p^r q^{n-r}$ where $q = 1-p$

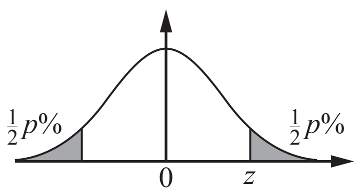
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 **(i)** Find the sum of all the even numbers from 2 to 1000. **[3]**

(ii) The sum of n consecutive even numbers, starting at 2, is less than 110.

(A) Show that $n^2 + n < 110$. **[1]**

(B) Find the set of possible values of n given that $n \neq 0$. **[3]**

2 **(i)** On the same axes, sketch the graphs of $y = x$ and $y = |2x - 1|$. **[2]**

(ii) **In this question you must show detailed reasoning.**

Solve the inequality $|2x - 1| > x$. **[4]**

3 Fig. 3 shows the curve with equation $y = \sqrt{1 + \ln x}$.

Fig. 3

Determine the exact x-coordinate of the point where the curve meets the x-axis. **[2]**

4 Fig. 4 shows rectangle ABCD. The point A lies on the y-axis and D is the point (2, 1). The equation of BC is $y = 3x + 5$.

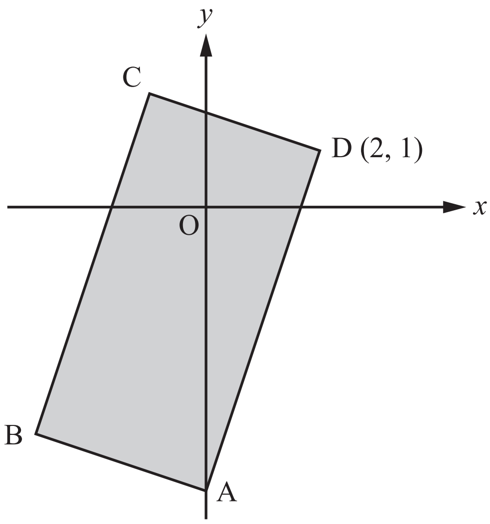


Fig. 4

(i) Determine the coordinates of A. **[3]**

(ii) Determine the area of ABCD. **[7]**

5 The equation of a curve is $y = \dfrac{a}{(x+b)^2}$. Fig. 5 shows the curve for particular values of the constants a and b.

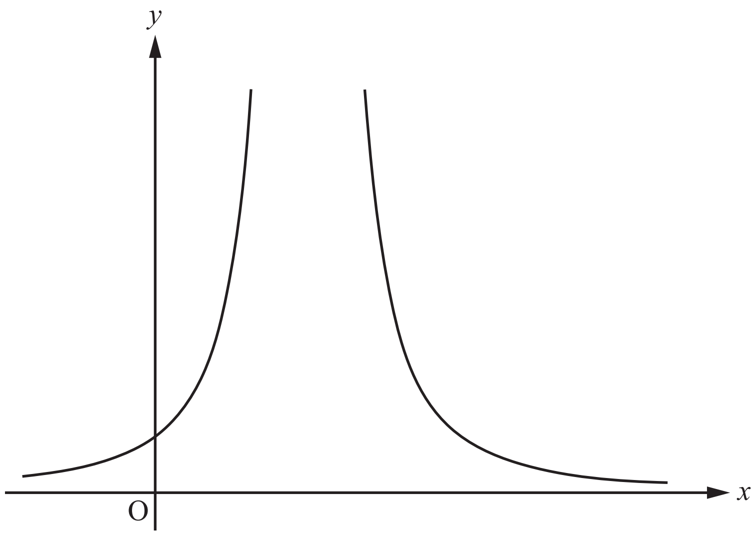


Fig. 5

(i) Write down the equations of the asymptotes, in terms of a and/or b where necessary. **[2]**

(ii) Joe says "For all values of a and b, the curve lies above the x-axis." Determine whether Joe's statement is true or false. **[2]**

(iii) The curve goes through the points (1, 3) and (3, 3). Find the values of a and b. **[4]**

6 **In this question you must show detailed reasoning.**

Fig. 6 shows the curves $y = \sin x$ and $y = \cos x$ for $-\pi \leqslant x \leqslant \pi$.

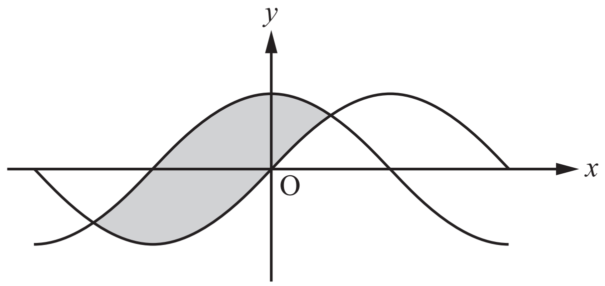


Fig. 6

(i) Show that the area of the region enclosed between the curves is given by $\int_a^b (\cos x - \sin x)\,dx$, where the values of a and b are to be determined. **[4]**

(ii) Find the exact value of the area of this region. **[3]**

7 Fig. 7 shows a circle with centre A and radius 6 cm, and a circle with centre B and radius 2 cm. The two circles touch. DCE is a common tangent to the circles. ABE is a straight line.

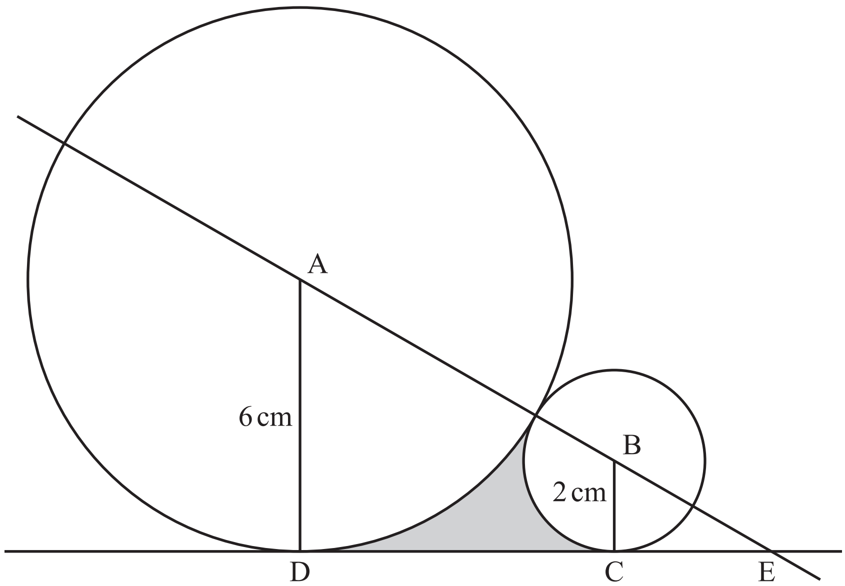


Fig. 7

(i) Give a reason why angles ADE and BCE are right angles. **[1]**

(ii) Show that angle DAB is $\frac{1}{3}\pi$ radians. **[2]**

(iii) Show that trapezium ABCD has area $16\sqrt{3}\ \text{cm}^2$. **[4]**

(iv) Calculate the area of the shaded region enclosed by the two circles and the common tangent DCE. Give your answer correct to 3 decimal places. **[3]**

8 In this question you must show detailed reasoning.

Fig. 8 shows the curve with parametric equations

$$x = 7\cos\theta + 2\cos 2\theta, \quad y = 2 + \sin\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

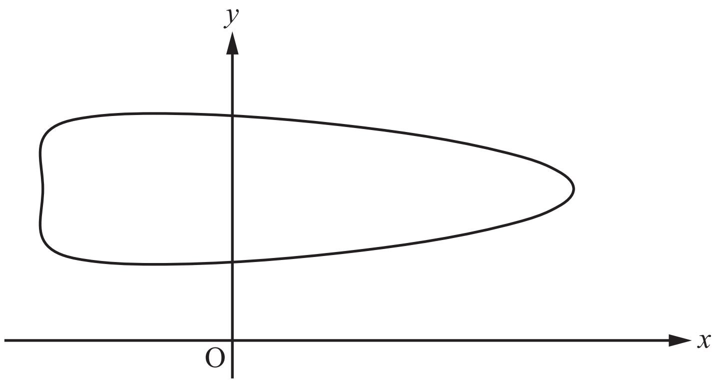


Fig. 8

(i) Find the coordinates of the point on the curve with the greatest y-coordinate. **[4]**

(ii) Determine the exact y-coordinates of the points where the curve crosses the y-axis. **[6]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

9 **(i)** Describe the transformation that maps the curve $y = \mathrm{f}(x)$ onto the curve $y = \mathrm{f}\left(x - \frac{b}{3a}\right)$. **[2]**

(ii) Given that α is a root of $\mathrm{f}\left(x - \frac{b}{3a}\right) = 0$, write down a root of $\mathrm{f}(x) = 0$ in terms of α, a and b. **[1]**

10 **(i)** Show how the substitution $x = y - \frac{m}{3y}$ can be used to transform $x^3 + mx = n$ into a quadratic equation in y^3. **[3]**

(ii) Show that, when $m > 0$, the resulting quadratic equation in y^3 has distinct real roots. **[2]**

11 Let a_1 and a_2 be the two values of y referred to in line 38 with $a_1^3 = \frac{5+\sqrt{29}}{2}$ and $a_2^3 = \frac{5-\sqrt{29}}{2}$.

(i) Show that $a_2^3 = -\frac{1}{a_1^3}$. **[1]**

(ii) Deduce that $a_1 - \frac{1}{a_1} = a_2 - \frac{1}{a_2}$ as stated in line 38. **[2]**

12 When solving the equation $x^3 + bx^2 + cx + d = 0$ by depressing the cubic and using Vieta's substitution, it can be shown that the resulting quadratic in y^3 must have real roots if $3c > b^2$.

(i) Show that $y = x^3 + bx^2 + cx + d$ has no stationary points if $3c > b^2$. **[3]**

(ii) Deduce that $x^3 + bx^2 + cx + d = 0$ has exactly one real root if $3c > b^2$. **[1]**

END OF QUESTION PAPER