OCR

Oxford Cambridge and RSA

Practice Paper – Set 3

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 15 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

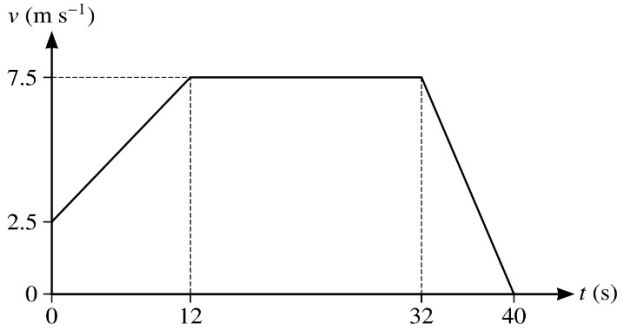
Question			Answer	Marks	AOs	Guidance	
1			Either $4x-5=3$ giving $x=2$ Or $4x-5=-3$ $4x=2$ giving $x=0.5$	**B1** **M1** **A1** **[3]**	**1.1** **1.1a** **1.1**	For answer $x = 2$ www May be implied	
2	(i)		The formula gives the gradient of the chord PQ	**B1** **[1]**	**2.4**		
2	(ii)		Column F suggests that the limit as $h \to 0$ is 28, and this is the gradient of (the tangent to) the curve at P	**B1** **[1]**	**2.4**	Comment must include reference to gradient and the idea of a limit	Not essential to mention that the limit is $f'(2)$
3			Vertical motion using $u=0$ Using $s=ut+\frac{1}{2}at^2$ with $s=-1.6$ to find t $-1.6=-4.9t^2 \Rightarrow t=\frac{4}{7}$ Horizontal distance is $21\times\frac{4}{7}=12$ m	**M1** **M1** **A1** **B1** **[4]**	**3.3** **3.1b** **1.1** **1.1**	May be implied For complete method to find t Allow for 0.571 or better **FT** their t; dependent on 2nd M mark	Allow $+s$ with $+a$

Question			Answer	Marks	AOs	Guidance	
4	(i)		$(\text{cosec}\,\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$	M1	2.1	Using $\text{cosec}\,\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$	
			$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$	M1	2.1	Using $\sin^2\theta = 1-\cos^2\theta$	
			$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$	A1 [3]	2.1	**AG** Factorising must be shown	
4	(ii)		$\frac{1-\cos\theta}{1+\cos\theta} = \frac{1}{3} \Rightarrow 3-3\cos\theta = 1+\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$	M1	1.1a	Attempt to rearrange and find $\cos\theta$	
			$\theta = 60°,\ 300°$	A1 A1 [3]	1.1 1.1	For one correct value for θ For second correct value; do not allow if additional values in range given, but ignore values outside range	

5			Equilibrium when P applied: $P - F = 0 \Rightarrow F = P$ Newton II when $3P$ applied: $3P - F = 5 \times 2$ So $2P = 10$, giving $P = 5$	B1 M1 A1 [3]	3.1b 3.1b 1.1	Friction force may appear as μR oe F in terms of P not needed here	

Question			Answer	Marks	AOs	Guidance	
6	(i)		$\text{ff}(x) = \text{f}\left(\frac{2x-3}{x-2}\right) = \frac{2\left(\frac{2x-3}{x-2}\right)-3}{\frac{2x-3}{x-2}-2}$	M1	1.2	Using $\frac{2x-3}{x-2}$ in the formula for f(x)	
			$= \frac{2(2x-3)-3(x-2)}{2x-3-2(x-2)}$	M1	1.1a	Attempt to simplify the fractions	
			$= \frac{x}{1} = x$	A1 [3]	2.1	**AG** Clear argument needed	
6	(ii)		$\text{f}^{-1}(x) = \text{f}(x) = \frac{2x-3}{x-2}$	B1 [1]	2.2a	Allow for statement that f is a self-inverse function	
6	(iii)		New equation is $y = \text{f}(x+1)-1$	M1	1.2	soi	
			$y = \frac{2(x+1)-3}{(x+1)-2} - 1 = \frac{2x-1-(x-1)}{x-1}$	M1	1.1a	Attempt to combine into a single fraction	
			$y = \frac{x}{x-1}$	A1 [3]	1.1		

7			Resolve $\rightarrow$: $X - 45 = 0 \Rightarrow X = 45$	B1	1.1a		
			Resolve $\uparrow$: $Y - 27 - 40 = 0 \Rightarrow Y = 67$	B1	1.1a		
			Moments about A: $Xd - 27 \times 60 = 0$	M1	3.4	oe; any moments equation must be correct, apart from sign errors	
			$d = \frac{27 \times 60}{45} = 36$	A1 [4]	1.1	**FT** their value for X	Answer for d must correspond to cm units; do not accept 0.36 or 0.36 m.

Question			Answer	Marks	AOs	Guidance	
8	(i)			B1 B1 B1 [3]	3.3 1.1 1.1	Graph with three straight lines with positive, zero and negative gradients Vertical axis labelled with 2.5 and 7.5 marked Horizontal axis labelled with 12, 32 and 40 marked (oe, e.g.separate time intervals indicated)	
8	(ii)		Total distance = area under graph $=\frac{1}{2}\times(2.5+7.5)\times12+(20\times7.5)+\frac{1}{2}\times8\times7.5$ = 240 m Average speed is $\frac{240}{40}$ = 6 m s^{-1}	M1 A1 M1 A1 [4]	3.1b 1.1 1.1a 1.1	Any complete method Dividing their distance by their total time **FT**	

Question			Answer	Marks	AOs	Guidance	
9	(i)		Circle is $(x-4)^2+(y+5)^2=25$ Centre is (4, –5) Radius is 5	**M1** **A1** **A1** **[3]**	**1.1a** **2.2a** **2.2a**	Attempting to complete the square for both x and y	
9	(ii)		The y-coordinate of the centre is –5 and the radius is 5 so the perpendicular distance of the centre from the x-axis is equal to the radius so the axis is a tangent	**B1**	**2.1**	For a complete argument with reference to both the radius and the value of the y-coordinate	A diagram without explanation is not sufficient for the mark
			Alternative solution $y = 0$ in equation gives $x^2-8x+16=0 \Rightarrow (x-4)^2=0$ so $x = 4$ is a repeated root, i.e. the x-axis is a tangent	 **B1**		 Clear argument based on the repeated root needed	
				[1]			
9	(iii)		Gradient of radius to P is $\frac{-1-(-5)}{7-4}=\frac{4}{3}$	**M1**	**3.1a**		
			Gradient of tangent is $-\frac{3}{4}$	**M1**	**3.1a**	For use of $m_1m_2=-1$	
			Equation of tangent is $y-(-1)=-\frac{3}{4}(x-7)$	**M1**	**1.1a**	Must use their tangent gradient	
			Rearranging: $3x + 4y = 17$	**A1** **[4]**	**1.1b**	cao and must be in this form	

Question			Answer	Marks	AOs	Guidance	
10	(i)		$3x^2 + 3y^2\frac{dy}{dx} = y + x\frac{dy}{dx}$	M1	1.1a	For attempted implicit differentiation	
				B1	1.1a	Correct use of product rule on RHS	
			$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$	M1	1.1a	Collecting terms and factorising	
			$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$	A1	1.1	Correct expression for $\frac{dy}{dx}$	
				[4]			
10	(ii)		$x = 0$ and $y = 0$ gives $\frac{0}{0}$ which cannot be evaluated	B1	2.4	Or for stating that the curve goes through the origin twice, with different gradients	
				[1]			
10	(iii)		Maximum occurs where $\frac{y - 3x^2}{3y^2 - x} = 0 \Rightarrow y - 3x^2 = 0$	M1	3.1a	For equating derivative to zero and attempting to rearrange	
			So P lies on $y = 3x^2$ giving $k = 3$	A1	2.2a	**FT** provided it is the equation of a suitable parabola	
				[2]			

Question			Answer	Marks	AOs	Guidance	
11	(i)		$v = 0.1$ when $t = 1$ gives $a + b = 0.1$	M1	3.3	Using given information to find an equation linking a and b	
			$\frac{dv}{dt} = 4at^3 + 3bt^2$	M1	3.1b		
			Maximum v when $t = 1$ gives $4a + 3b = 0$	M1	3.3	Equating the derivative to zero to find an equation linking a and b	
			Solving simultaneous equation for a and b	M1	1.1a	Method may be implied, e.g. if **BC**	
			$a = -0.3$ and $b = 0.4$	A1 [5]	1.1	cao	
11	(ii)		Changes direction when $v = 0$, so $-0.3t^4 + 0.4t^3 = 0 \Rightarrow 0.3t^4 = 0.4t^3 \Rightarrow t = \frac{4}{3}$ (as $t > 0$)	M1	3.4	For equating v to 0 and solving for t (may be **BC**); ignore any inclusion of $t = 0$ at this point	
			Particle changes direction when $t = \frac{4}{3}$	A1 [2]	1.1	cao	
11	(iii)		Model is not suitable for large values of t as the object's velocity would increase without limit	B1 [1]	3.5b	oe, e.g. 'velocity gets very large'	

Question			Answer	Marks	AOs	Guidance	
12	(i)		Using $\sin\theta \approx \theta$ with $\theta = 2x^2$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with $\theta = 5x$ gives $2x^2 - \left(1 - \frac{1}{2}(5x)^2\right) = 0$	M1	2.1	Allow slip in $(5x)^2$	
			$\frac{29}{2}x^2 = 1 \Rightarrow x = \sqrt{\frac{2}{29}} = 0.2626\text{K}$	M1	2.1	Attempt to solve for x	
			So estimate is 0.26 to 2 decimal places	A1	2.1	**AG** Must be rounded to 2 dp following either exact answer or answer to more than 2 dp seen	
				[3]			
12	(ii)		Denoting $\sin 2x^2 - \cos 5x$ by f(x): f(0.255) = –0.1618… and f(0.265) = –0.1033…	M1	2.1	Search for sign change using their 2dp value ± 0.005	oe, e.g. comparing values of $\sin 2x^2$ and $\cos 5x$ at end-points
			These are both negative so the root does not lie between 0.255 and 0.265, so the estimate is not correct to 2 decimal places	A1	2.2a	Complete argument from correct figs	
				[2]			
12	(iii)		$f'(x) = 4x\cos 2x^2 + 5\sin 5x$	M1	1.1a	Attempt to find $f'(x)$	
				M1	1.1a	Use of chain rule for $\sin 2x^2$	
				A1	1.1	Derivative fully correct	
			$x_1 = 0.2 - \dfrac{\sin(2\times 0.2^2) - \cos(5\times 0.2)}{4\times 0.2\cos(2\times 0.2^2) + 5\sin(5\times 0.2)}$ $= 0.2 - \dfrac{-0.460\,387\,611\,9}{5.004\,796\,289} = 0.291\,989\,28\text{K}$	M1	2.1	Use of iterative formula to find x_1	
			so $x_1 = 0.291\,989\,3$ correct to 7 dp	A1	2.1	**AG** Answer must follow from clear working	
				[5]			
12	(iv)		$x_2 = 0.282\,338\,3$	M1	1.1a	Finds at least one more iteration	
			$x_3 = 0.282\,285\,3$	A1	1.1	For correct x_3	
			Root is 0.282 correct to 3sf	B1	2.2b		
				[3]			

Question			Answer	Marks	AOs	Guidance	
13	(i)		Resolve perpendicular to plane: $R = 2.7g\cos 25°$	B1	3.3	Allow sin/cos interchange if consistent error elsewhere	
			so $F = \mu R$ gives $F = 0.4 \times 2.7g\cos 25°$ (= 9.59…)	M1	3.4	For 0.4 × their R, but not if $R = 2.7g$	
				A1	1.1	For correct expression for F; evaluation not needed here	
			Newton II ‖ to plane: $24 - F - 2.7g\sin 25° = 2.7a$	M1	3.3	All forces needed and the 'weight' term must be a component; allow sign errors	
			a = 1.1945… so acceleration is 1.19 m s^{-2} (3sf)	A1	1.1	awrt 1.19	
				[5]			
13	(ii)		After 5 s, $v = 0 + 1.1945 \times 5$	M1	3.1b	Use of *suvat* leading to a value for v	
			$v = 5.9725$	A1	1.1		
			Newton II ‖ to plane: $-F - 2.7g\sin 25° = 2.7a$	M1	3.3	Both forces, with weight resolved	
			$a = -7.6943$	A1	1.1		
			$v = 0$ at distance s where $0 = 5.9725^2 - 2 \times 7.6943 \times s$	M1	1.1a	Use of *suvat* leading to a value for s	
			s = 2.3179… so distance travelled is 2.32 m (3sf)	A1	1.1	cao	
				[6]			

Question			Answer	Marks	AOs	Guidance	
14	(i)		$4\mathbf{i} + 6\mathbf{j} = -2\mathbf{j} + 8\mathbf{a}$	M1	1.1a	For use of vector equation $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, or alternatively for finding both acceleration components separately	
			$\mathbf{a} = 0.5\mathbf{i} + \mathbf{j}$ m s^{-2}	A1 [2]	2.5	Must be vector form; do not allow final answer as separate components	Do not allow for the magnitude of the acceleration unless final answer is labelled as such
14	(ii)		$\mathbf{v} = -2\mathbf{j} + (0.5\mathbf{i} + \mathbf{j})t$ is directed east when $-2 + t = 0$	M1	3.1b	For equating **j** component of velocity to zero, giving equation for t	
			$t = 2$, so boat sails east at time 2 s	A1 [2]	3.2a	cao	
14	(iii)		Displacement in 4 s is $(-2\mathbf{j}) \times 4 + \frac{1}{2}(0.5\mathbf{i} + \mathbf{j}) \times 4^2$	M1	3.1b	For any use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $t = 4$ and their **a** even if no clarity about displacement/position	Equation may include initial position term $\mathbf{r}_A$
			$= 4\mathbf{i}\ (+ 0\mathbf{j})$	A1	1.1		
			Position at time $t = 4$ gives $\mathbf{r}_A + 4\mathbf{i} = 5\mathbf{i} - 2\mathbf{j}$	M1	3.1b	For correct use of position vectors	May be earned earlier, if original equation is $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
			$\mathbf{r}_A = \mathbf{i} - 2\mathbf{j}$	A1 [4]	1.1	Must be in vector form	

Question			Answer	Marks	AOs	Guidance	
15			$\frac{dy}{dx} = 5x(-2e^{-2x}) + 5e^{-2x}$	M1	3.1a	For use of product rule	
				A1	1.1	For correct (unsimplified) derivative	
			$e^{-2x}(-10x + 5) = 0$	M1	1.1a	For equating derivative to zero and attempting to solve for x	
			$x = \frac{1}{2}$	A1	1.1	oe	
			Area $= \int_0^{\frac{1}{2}} 5x e^{-2x}\, dx$	M1	3.1a	Limits 0 and (their) $\frac{1}{2}$ must be seen at some stage for this mark to be awarded	
			$= \left[5x\left(-\frac{1}{2}e^{-2x}\right)\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}}\left(-\frac{5}{2}e^{-2x}\right) dx$	M1	1.1a	For integration by parts with $u = 5x$ and $v' = e^{-2x}$	
				A1	1.1	First stage all correct; ignore limits for this mark	
			$= -\frac{5}{4}e^{-1} + \left[-\frac{5}{4}e^{-2x}\right]_0^{\frac{1}{2}}$	A1	1.1	Integration completed correctly	
			$= -\frac{5}{2}e^{-1} + \frac{5}{4}$	A1	1.1	oe, but must be exact simplified form	Not e.g. 0.3303…
				[9]			