OCR
Oxford Cambridge and RSA

Practice Paper – Set 3

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 16 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1	(*A*)		periodic since 1, 0, –1, 0, repeats	**B1** **[1]**	**1.2**		
	(*B*)		convergent, since difference between terms is decreasing	**B1** **[1]**	**2.4**		
	(*C*)		divergent, since magnitude of difference between terms increasing	**B1** **[1]**	**1.2**	allow "difference between terms increasing"	
2			$5\log_a x$ or $\log_a x^{-1}$ oe soi $6\log_a x$	**M1** **A1** **[2]**	**1.1** **1.1**		
3	(i)		$k(1\times2 + 2\times3 + 3\times4 + 4\times5 + 5\times6) = 1$ $k = \frac{1}{70}$	**M1** **A1** **[2]**	**1.1** **1.1**		
3	(ii)		$\frac{2+6+12}{70}$ $= \frac{2}{7}$	**M1** **A1** **[2]**	**1.1** **1.1**	FT their k	Allow $1-$ P($X \leq 3$) method
4			$\mathbf{b} - \mathbf{a} = 6\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{6^2 + (-6)^2 + (-3)^2}$ $\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$	**M1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	allow one slip FT their $\mathbf{b} - \mathbf{a}$ accept eg $\frac{6}{9}\mathbf{i} - \frac{6}{9}\mathbf{j} - \frac{3}{9}\mathbf{k}$	or in column vector form allow use of $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ oe or in column vector form

Question			Answer	Marks	AOs		Guidance
5	(i)		$1+(-2)\frac{x}{2}+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{2}\right)^3$	M1	1.1a	allow sign errors and one coefficient error	ignore extra terms
			$1-x+\frac{3}{4}x^2-\frac{1}{2}x^3$	A1 A1 [3]	1.1 1.1	three out of four terms correct all four terms correct	
	(ii)		$-2 < x < 2$ oe	B1 [1]	2.3		
6			$\cos\theta=\frac{9^2+5^2-12^2}{2\times5\times9}$	M1	3.1		
			$-\frac{38}{90}$ oe soi	A1	1.1	May be implied by 114.97…degrees or 2.0066917…radians	
			115°	A1 [3]	1.1		
7	(i)		Positive skew	B1 [1]	1.2		
7	(ii)		Layout correct	M1	1.1a		
			scale on axis and range of boxplot correct	A1	1.1		
			their IQR and median shown	M1	1.1		
			IQR is 19 to 40 and median is 28	A1 [4]	1.1	allow 20.5 to 41 but not 19 to 41 or 20.5 to 40	
8	(i)		Each possible group of [pupils] (of size n) which could be taken from the population has the same chance of being picked	E1 [1]	2.4	allow each pupil has equal chance of being sampled AND choosing one [pupil] to be sampled does not affect another being sampled	

Question			Answer	Marks	AOs		Guidance
8	(ii)		eg It is not possible to select the last 23 pupils on the list. eg The different starting points on the list do not all have the same probability of being selected eg It is not possible to select some samples eg the first 50 on the list	**E1** **E1** **[2]**	**2.4** **2.4**	accept any two valid reasons	
8	(iii)		eg Randomly select a starting point on the list between 1 and 31. Select every 12^{th} item on the list after that.	**E1** **[1]**	**2.4**	Accept any coherent statement which involves the selection of a random starting point, values being selected at regular intervals and results in all values being available for selection	
9	(i)		f(2) = 16 – 2 = 14. Since f(x) passes through (2, 14), $f^{-1}(x)$ must pass through (14, 2)	**E1** **[1]**	**2.4**		
9	(ii)		$f'(x) = 4x^3 - 1$ $f'(2) = 31$ $\frac{1}{31}$	**B1** **M1** **A1** **[3]**	**2.1** **1.1** **1.1**	 Accept BC	
10	(i)		the pattern of data points suggest a straight line oe the value of r indicates that the fit is close oe	**E1** **E1** **[2]**	**3.3** **2.4**		

Question			Answer	Marks	AOs		Guidance
10	(ii)		70	B1 [1]	3.4	from 0.89×83 – 3.76 (= 70.11)	
10	(iii)		(10 + 3.76) ÷0.89 13	M1 A1 [2]	3.4 1.1	(= 13.21…)	
10	(iv)		The approximation for Tina's mark is obtained by interpolation, whereas the approximation for Dave's mark is from extrapolation	E1 [1]	3.5b	allow eg should use the equation for x on y to estimate Dave's mark	
11	(i)		$R = \sqrt{3}$ $\tan^{-1}(\frac{1}{\sqrt{2}})$ $\alpha = 0.615$	B1 M1 A1 [3]	1.1 1.1 1.1	0.61547970…rounded to 2 or more significant figures	
11	(ii)		$f(x) = \frac{5}{2+\sqrt{3}\cos(x+0.62)}$ At min value, $\cos(x + 0.62) = 1$ soi $10 – 5\sqrt{3}$	M1 M1 A1 [3]	3.1a 2.1 1.1	FT their R BC rationalising	
12	(i)		=B2*C2	E1 [1]	2.4	must have "="	

Question			Answer	Marks	AOs		Guidance
12	(ii)		**A** All populations are in the upper quartile **B** All total GDPs are in the upper quartile Both statements are consistent with the data.	**E1** **E1** **B1** **[3]**	**2.4** **2.4** **2.2a**		
12	(iii)		No because some countries do not take part in the Olympics OR having more Olympic athletes in a country may encourage others	**E1** **[1]**	**2.2a**		

Question			Answer	Marks	AOs		Guidance
13	(i)		$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	B1	1.1		
			$x = (u-1)^2$	M1	2.1	allow sign error	
			$\int \frac{(u-1)^2}{u} \times 2(u-1)\,du$	M1	3.1a	FT their x and their derivative	
			$2\int\left(u^2 - 3u + 3 - \frac{1}{u}\right)du$ oe	A1	1.1	must be in a form ready to integrate	
			$2\left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right]$	A1 A1	1.1 1.1	three terms correct all four terms correct	
			$\frac{2(1+\sqrt{x})^3}{3} - 3(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 2\ln(1+\sqrt{x})$ $[+c]$	A1 [7]	3.2a	allow full marks if + c omitted	
13	(ii)		Evaluation of F[1] – F[0]	M1	2.1		
			$\frac{5}{3} - \ln 4$	A1 [2]	1.1		

Question			Answer	Marks	AOs		Guidance
14			H_0: $\mu = 56$				
			H_1:$\mu > 56$	**B1**	**1.1**	Both hypotheses	
			μ is the population mean time taken by Statto to complete his walk	**B1**	**2.5**		
			Use of N(56, $\frac{4^2}{19}$)	**M1**	**3.3**		
			0.9459 BC	**A1**	**1.1**		
			1 – 0.9459 > 0.05	**M1**	**3.4**	FT their probability	
			No evidence to reject H_0 at 5% level oe	**A1**	**1.1**		
			There is no evidence to suggest at the 5% level that the mean time taken by Statto to complete his walk has increased	**E1**	**2.2b**		
				[7]			

Question	Answer	Marks	AOs		Guidance
15	$\frac{15}{n}\times\frac{14}{n-1}+\frac{n-15}{n}\times\frac{n-1-15}{n-1}=2\times\frac{15}{n}\times\frac{n-15}{n-1}$	**M1** **M1** **A1**	**3.1b** **2.1** **1.1**	P(BB) + P(RR) = P(one of each) Two of three terms correct All correct	*n* is the total number of discs
	Multiply through to obtain quadratic in *n*	**M1**	**2.1**		
	$n^2 - 61n + 900 = 0$	**A1**	**1.1**		
	n = 36 (not 25) since more red discs	**B1**	**3.2b**	so there are 21 red discs	
	$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$	**M1**	**2.1**	FT their 44	
	$\frac{1}{3}$	**A1**	**1.1**	Accept decimal equivalent to 2 or more dp	
	OR	**[8]**			
	$\frac{15}{15+r}\times\frac{14}{14+r}+\frac{r}{15+r}\times\frac{r-1}{14+r}=2\times\frac{15}{15+r}\times\frac{r}{14+r}$	**M1** **M1** **A1**		P(BB) + P(RR) = P(one of each) Two of three terms correct All correct	*r* is the number of red discs
	Multiply through to obtain quadratic in *r*	**M1**			
	$r^2 - 31r + 210 = 0$	**A1**			
	r = 21 (not 10) since *r* > 15	**B1**			
	$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$	**M1**		FT their 21 (> 15)	
	$\frac{1}{3}$	**A1** **[8]**		Accept decimal equivalent to 2 or more dp	
		[8]			

Question			Answer	Marks	AOs		Guidance
16	(i)		0.6	B1 [1]	1.1	BC	
16	(ii)		each egg has the same (constant) probability of being cracked	E1 [1]	2.4	OR the probability of any particular egg being cracked is independent of any other egg being cracked oe	
16	(iii)		np = their 0.6 $p = 0.1$	M1 A1 [2]	3.1b 1.1		
16	(iv)		159.4, 106.3, 29.5, 4.4, 0.4, 0.0, 0.0	M1 M1 A1 [3]	3.4 1.1 1.1	Use of B(6, 0.1) soi Multiplication by 300 All correct	If unsupported, allow **B3** for all frequencies correct, **B2** if one error, **B1** if two errors
16	(v)		The theoretical frequencies are close to the observed values, so Rose's model is a good fit.	B1 [1]	3.2b		

Question			Answer	Marks	AOs		Guidance
16	(vi)		H_0: $p = 0.1$ H_1: $p \neq 0.1$	**B1**	**1.1**	Both hypotheses	
			p is the probability that an egg selected at random is cracked	**B1**	**2.5**		
			use of B(24, 0.1)	**M1**	**3.3**		
			$P(X \leq 5) = 0.9723…$BC	**A1**	**1.1**		
			$1 – 0.9723 > 0.025$	**M1**	**3.4**	Comparison of their $1 – P(X \leq 5)$ with 0.025	
			not significant oe	**A1**	**1.1**		
			There is no evidence at the 5% level to suggest that the probability that an egg selected at random is cracked is not 0.1	**E1** **[7]**	**2.2b**		
17	(i)		$\int dV = \int \left(\frac{k}{\sqrt{t+1}} \right) dt$	**M1**	**3.1a**	Separation of variables. Allow omission of integral signs and or dt or dV	
			$V = 2k(t+1)^{\frac{1}{2}} + c$	**M1**	**1.1**	Allow one error eg omission of + c	
			Substitution of both conditions	**M1**	**2.1**	$0 = 2k + c$ $400 = 4k + c$	
			$k = 200$ or $c = – 400$	**A1**	**3.3**	For either of these	
			$V = 400\sqrt{t+1} – 400$ oe	**A1** **[5]**	**1.1**	All correct	

Question			Answer	Marks	AOs		Guidance
17	(ii)		$y = 5$, $V = 162.5$ T = 0.9775(390625…) BC	**B1** **B1** **[2]**	**3.4** **1.1**	 Rounded to 2, 3 or 4 sf, awrt 0.9775	
17	(iii)		$\frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$ used $\frac{200}{\sqrt{1+0.9775...}} = (20 + 6 \times 5 - 0.3 \times 5^2)\frac{dy}{dt}$ 3.346(40522876…)	**M1** **M1** **A1** **[3]**	**3.1a** **2.1** **1.1**	 Allow one slip in derivative Rounded to 2, 3 or 4 sf, awrt 3.35	
17	(iv)		*Either* at $t = 3$, $\frac{dV}{dt} = 100\text{cm}^3\text{s}^{-1}$ *or* $\frac{dy}{dt} = 2\text{cms}^{-1}$ Therefore volune/height still increasing so the student is correct	**B1** **E1** **[2]**	**3.4** **3.5a**		

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