OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Practice Paper – Set 3

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B\,|\,A) = \mathrm{P}(B)\mathrm{P}(A\,|\,B)$ **or** $\mathrm{P}(A\,|\,B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n,p)$ then $\mathrm{P}(X=r) = {}^n\mathrm{C}_r\,p^r q^{n-r}$ where $q = 1-p$

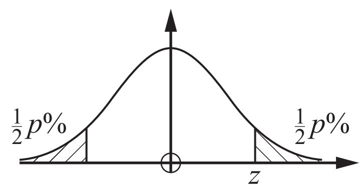
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu,\sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu,\dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (24 marks)

1 State, with a reason, whether each of the following sequences are convergent, divergent, periodic, or none of these.

(*A*) $u_n = \sin\left(\frac{n\pi}{2}\right)$, $n = 1, 2, 3, \ldots$ **[1]**

(*B*) 100, 150, 175, 187.5, 193.75, 196.875, … **[1]**

(*C*) $u_{k+1} = (-2)^k u_k$, $u_1 = 3$ **[1]**

2 Write $\log_a x^5 - \log_a\left(\frac{1}{x}\right)$ in the form $k\log_a x$, where k is a constant to be determined. **[2]**

3 The discrete random variable X takes the values $r = 0, 1, 2, 3, 4$ with probabilities

$$\mathrm{P}(X = r) = k(r+1)(r+2).$$

(i) Calculate the value of the constant k. **[2]**

(ii) Calculate $\mathrm{P}(X < 3)$, giving your answer as a fraction in its lowest terms. **[2]**

4 You are given that $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix}$.

Find a unit vector which is parallel to $\overrightarrow{\mathrm{AB}}$. **[3]**

5 **(i)** Find the first four terms in the expansion of $\left(1 + \frac{x}{2}\right)^{-2}$. **[3]**

(ii) State the range of values of x for which this expansion is valid. **[1]**

6 Fig. 6 shows a triangle with angle θ marked.

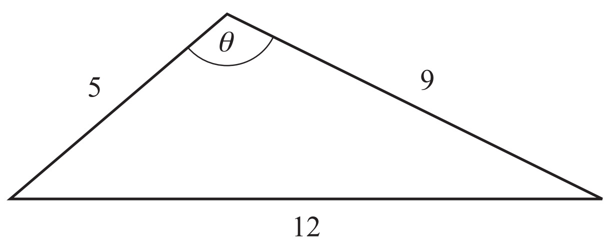


Fig. 6

Calculate the size of angle θ, giving your answer correct to the nearest degree. **[3]**

7 The stem-and-leaf diagram in Fig. 7 shows the numbers of customers at a village post office on the days it was open in March 2017.

0	9
1	1 3 5 7 8 9
2	2 4 5 6 6 7 8 9
3	1 3 4 5 8
4	0 2 5 9
5	1 3
6	
7	5

Key 4 | 0 represents 40 customers

Fig. 7

(i) Describe the shape of the distribution. **[1]**

(ii) Draw a box plot to represent the data. **[4]**

Section B (76 marks)

8 A class representative is investigating whether pupils at his school believe that school meals are satisfactory. The class representative has an alphabetical list of all 619 pupils in the school saved on a spreadsheet. He decides to select a sample of 50 pupils, and considers two different sampling procedures.

Procedure A.
Assign a distinct random number to each pupil. Select the 50 pupils with the smallest random numbers.

Procedure B.
Generate a 2-digit random number. Use this random number to select a starting point on the list according to the rule shown in Fig. 8. Select the pupil identified by this rule, and then select every 12th pupil on the list after this, stopping when a sample of 50 has been obtained.

Random number	00–10	11–20	21–30	31–40	41–50	51–70	71–80	81–99
Starting point	1	2	3	4	5	6	7	8

Fig. 8

(i) Explain why procedure **A** will generate a simple random sample. **[1]**

(ii) Identify **two** features of procedure **B** that prevent it from generating a simple random sample. **[2]**

(iii) Describe how you could generate a random sample of size 50 from the 619 pupils using systematic sampling. **[1]**

9 You are given that

$$f(x) = x^4 - x, \; x > 1.$$

The graphs of $y = f(x)$, $y = x$ and $y = f^{-1}(x)$ are shown in Fig. 9.

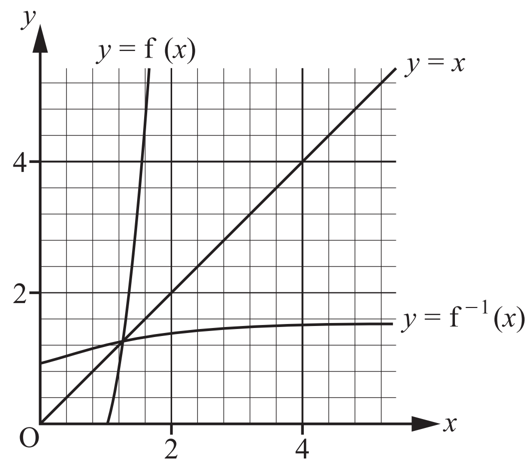


Fig. 9

(i) Show that the inverse function, $f^{-1}(x)$, passes through the point (14, 2). **[1]**

(ii) Find the gradient of $f^{-1}(x)$ at the point (14, 2). **[3]**

10 At the start of the January term year 11 students at Amplesides College sat mock examinations in GCSE mathematics papers 4 and 5. A teacher collected the results and presented them in a scatter diagram, which is shown in Fig. 10. A line of best fit has been added.

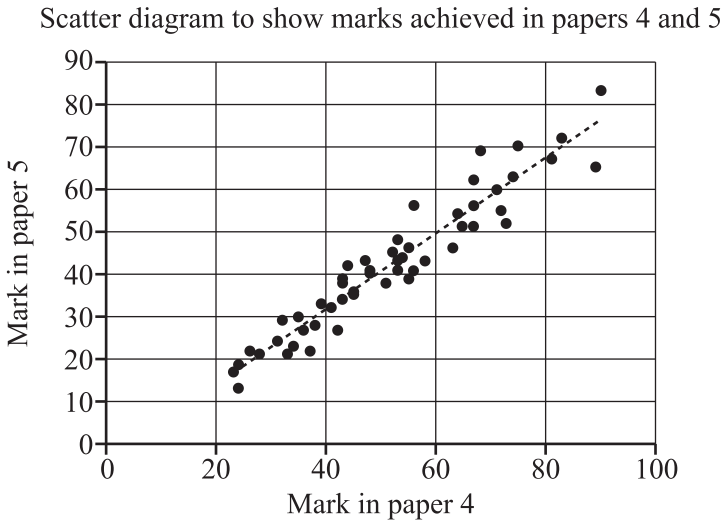


Fig. 10

The correlation coefficient for the data is 0.9566.

(i) Give **two** reasons why it is reasonable to model the relationship between the mark achieved in paper 4 and the mark achieved in paper 5 by a straight line. **[2]**

The equation of the line of best fit is $y = 0.89x - 3.76$, where y is the mark achieved in paper 5 and x is the mark achieved in paper 4.

(ii) Tina achieved a mark of 83 in paper 4, but was absent for paper 5. Calculate an estimate of the mark she would have achieved in paper 5. **[1]**

(iii) Dave was absent for paper 4. He achieved a mark of 8 in paper 5. Calculate an estimate of the mark Dave would have achieved in paper 4. **[2]**

(iv) Explain why the estimate of Tina's mark for paper 5 is more reliable than the estimate of Dave's mark for paper 4. **[1]**

11 **(i)** Express $\sqrt{2}\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **[3]**

(ii) You are given that

$$f(x) = \frac{5}{2 + \sqrt{2}\cos x - \sin x} \text{ for } 0 \leqslant x \leqslant 2\pi.$$

Find the minimum value of f(x), giving your answer in the form $a + b\sqrt{c}$ where a, b and c are integers to be determined. **[3]**

12 The spreadsheet output in Fig. 12.1 gives some information about all the countries that won more than 10 gold medals in the London 2012 Olympic Games.

	A	B	C	D	E	F	G
1	**Country**	**population**	**GDP per capita (US$)**	**Total GDP (US$)**	**Gold medals**	**Silver medals**	**Bronze medals**
2	United States	318 892 103	52 800	1.68×10^{13}	46	29	29
3	China	1 355 692 576	9800	1.33×10^{13}	38	27	22
4	United Kingdom	63 742 977	37 300	2.38×10^{12}	29	17	19
5	Russia	142 470 272	18 100	2.58×10^{12}	24	25	33
6	Korea, South	49 039 986	33 200	1.63×10^{12}	13	8	7
7	France	66 259 012	35 700	2.37×10^{12}	11	11	12
8	Germany	80 996 685	39 500	3.20×10^{12}	11	19	14

Fig. 12.1

(i) Give a spreadsheet formula for calculating the value in cell D2 using other cell values in Fig. 12.1. **[1]**

The statistics in Fig. 12.2 are for all the countries in the pre-release data.

	Population	**GDP per capita (US$)**	**Total GDP (US$)**
Lower Quartile	4.587×10^5	4525	2.09×10^9
Median	5.623×10^6	13 750	7.73×10^{10}
Upper Quartile	2.116×10^7	31 750	6.72×10^{10}

Fig. 12.2

(ii) Explain whether or not the statements below are consistent with the information given in Fig. 12.1 and Fig 12.2.

Statement A
Countries with larger populations are more likely to win Olympic gold medals.

Statement B
Countries with larger total GDP are more likely to win Olympic gold medals. **[3]**

There were approximately 10 500 Olympic competitors in 2012. The population of the world was approximately 7 000 000 000 in 2012.

A geography student assumed that Olympic competitors are randomly and uniformly scattered across the population of the world. The student calculated that the population of a country with two Olympic competitors would be approximately 1 300 000.

(iii) Use your knowledge of the large data set to explain whether the geography student's assumption is realistic. **[1]**

13 **(i)** Find $\int\left(\frac{x}{1+\sqrt{x}}\right)\mathrm{d}x$. You may use the substitution $u = 1+\sqrt{x}$. **[7]**

(ii) Hence show that $\int_0^1\left(\frac{x}{1+\sqrt{x}}\right)\mathrm{d}x = A - \ln B$, where A and B are constants to be determined. **[2]**

14 **In this question you must show detailed reasoning.**

Each evening Statto goes for a walk on the same circular route. Over a long period of time Statto noticed that the mean time taken to complete the walk was 56 minutes and the standard deviation was 4 minutes.

A few months ago Statto was ill and was unable to complete the walk for a month. Since then Statto's partner thinks that the mean time Statto takes to complete the evening walk has increased. Over a period of weeks Statto's partner collects a random sample of the times taken by Statto to complete the evening walk.

Statto's partner uses software to generate the summary statistics in Fig. 14.

Statistics ▼	
n	19
Mean	57.4737
σ	3.8848
s	3.9912
Σx	1092
Σx²	63048
Min	52
Q1	54
Median	57
Q3	60
Max	65

Fig. 14

Use information from Fig. 14 to conduct a hypothesis test to determine whether there is any evidence at the 5% level to suggest that the mean time Statto takes to complete the evening walk has increased. **[7]**

15 **In this question you must show detailed reasoning.**

A bag contains blue discs and red discs. There are 15 blue discs and an unknown number of red discs. There are more red discs than there are blue discs. A disc is taken at random from the bag and not replaced. A second disc is then taken at random from the bag.

You are given that the probability that two discs of the same colour are taken is the same as the probability that two discs of different colours are taken.

Calculate the probability that 2 blue discs are taken, given that two discs of the same colour are taken. **[8]**

16 Rose packs eggs in boxes of 6, which she then sells at her farm. During the process some eggs are cracked, and Rose randomly selects a sample of boxes and records the number of cracked eggs in each box. The results are summarised in Fig. 16.

Number of cracked eggs	0	1	2	3	4	5	6
Number of boxes	163	103	28	5	0	0	1

Fig. 16

(i) Calculate the mean number of cracked eggs per box. **[1]**

Rose believes that the number of cracked eggs per box may be modelled by a binomial distribution.

(ii) State a modelling assumption that is necessary for a binomial distribution to be used to model the number of cracked eggs per box. **[1]**

Rose defines p as the probability that a particular egg is cracked.

(iii) Use your answer to part **(i)** to find the value of p. **[2]**

(iv) Calculate the expected frequencies of cracked eggs per box according to Rose's model, giving your answers correct to 1 decimal place. **[3]**

(v) Comment on whether Rose's model is a good fit for the data. **[1]**

Instead of selling eggs at the farm, Rose decides to sell them to a wholesaler. The eggs are now selected randomly and packed in open trays of 24. Rose believes that this will result in a change in the probability of an egg being cracked. She selects a tray at random and finds that 5 eggs are cracked.

(vi) **In this question you must show detailed reasoning.**

Conduct a hypothesis test to determine whether there is any evidence at the 5% level to suggest that the proportion of cracked eggs has changed. **[7]**

17 When a container is filled with water to a depth of y cm, the volume of water, V cm^3, in the container is modelled by the formula

$$V = 20y + 3y^2 - 0.1y^3.$$

When the container is filled to its maximum possible depth of 10 cm, the volume of water in the container is 400 cm^3.

Water is poured into the container after t seconds at a rate which is modelled by the differential equation

$$\frac{dV}{dt} = \frac{k}{\sqrt{t+1}},$$

where k is a positive constant. Initially the container is empty and after 3 seconds it is full.

(i) Express V in terms of t. **[5]**

(ii) Calculate the time T taken, according to the model, until the container is filled to a depth of 5 cm. **[2]**

(iii) Calculate the rate at which the depth of water in the container is increasing at this time T. **[3]**

A student states that, according to the model, the water will overflow.

(iv) Determine whether the student's statement is correct. **[2]**

END OF QUESTION PAPER