Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Practice Paper – Set 3

Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ $\quad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i-\bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X=r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1-p$

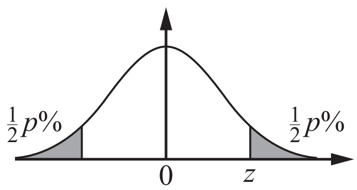
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu,\sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 Find the term in x^3 in the binomial expansion of $(2+x)^5$. **[2]**

2 An arithmetic sequence has third term 6 and ninth term 30. Find the sum of the first 100 terms. **[4]**

3 Prove that $x^2+x+2>1$ for all real values of x. **[3]**

4 **In this question you must show detailed reasoning.**

(i) Show that $x-3$ is a factor of $4x^3-12x^2-x+3$. **[1]**

(ii) Fig. 4 shows the curve $y=4x^3-12x^2-x+3$. Find the coordinates of the points where it crosses the x-axis. **[4]**

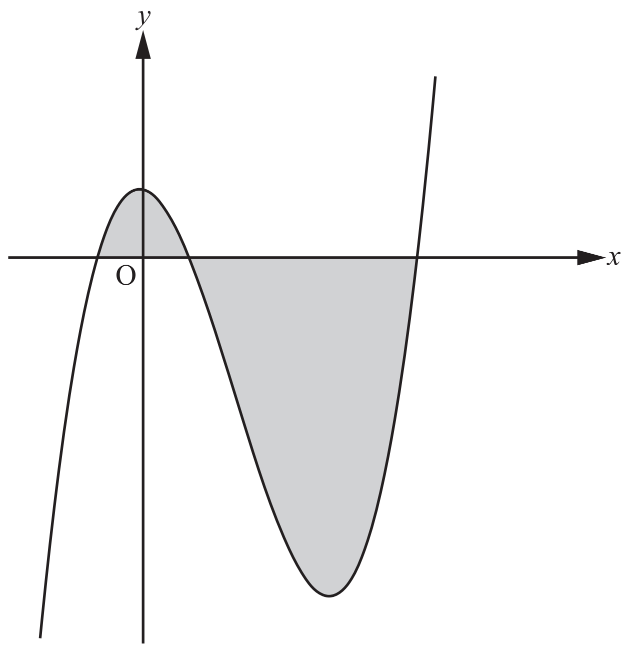


Fig. 4

(iii) The two regions bounded by the curve $y=4x^3-12x^2-x+3$ and the x-axis are shaded in Fig. 4. Determine the total area of the shaded regions. **[5]**

5 Fig. 5.1 shows the curve $y = \mathrm{e}^{1-x^2}$. Fig. 5.2 shows a spreadsheet used to calculate an estimate of $\int_0^2 \mathrm{e}^{1-x^2}\,\mathrm{d}x$ using the trapezium rule with four strips.

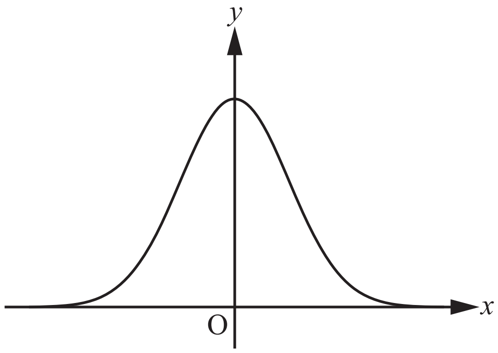


Fig. 5.1

	A	B	C
1	*x*		*y*
2	0	1	2.718282
3	0.5	0.75	2.117
4	1	0	1
5	1.5	-1.25	0.286505
6	2	-3	0.049787
7			

Fig. 5.2

(i) Show how the value in cell B3 is calculated. **[1]**

(ii) Complete the calculation to estimate $\int_0^2 \mathrm{e}^{1-x^2}\,\mathrm{d}x$, giving the answer correct to 3 significant figures. **[2]**

(iii) Show that the only stationary point on the curve is at (0, e). **[2]**

6 Fig. 6 shows a circle with centre at the origin passing through the point A with coordinates (2, 0). The point B in the first quadrant lies on this circle.

The area of sector AOB is $\frac{2}{3}\pi$.

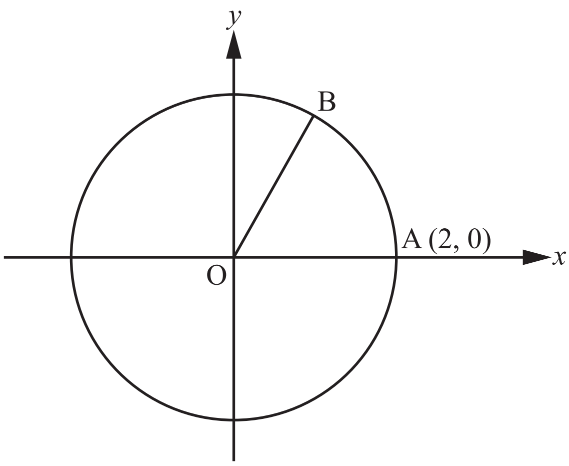


Fig. 6

(i) Find the exact coordinates of B. **[4]**

(ii) The circle in Fig. 6 is reflected in line AB. Find the equation of the image circle. **[3]**

7 Fig. 7 shows the relationship between the average weekly sales of a newspaper (y) and the time in years after 2010 (x). Each value of y is the average across the whole year. The graph of $\log_{10}y$ plotted against x is approximately a straight line.

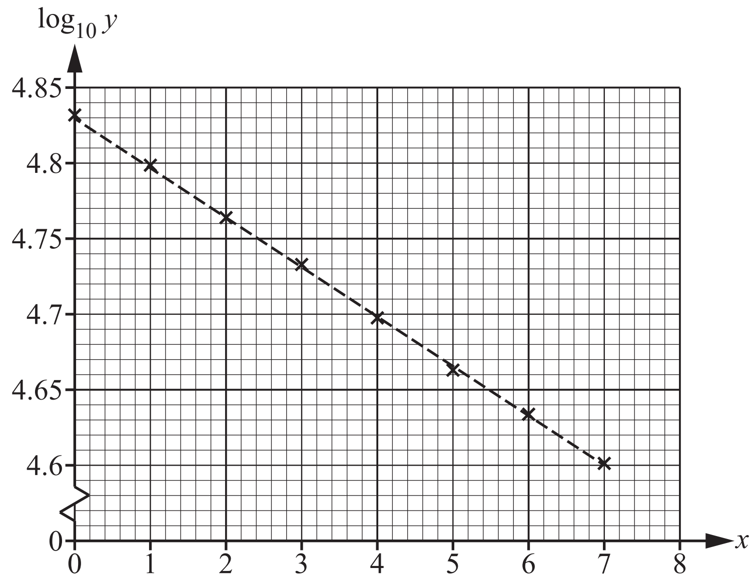


Fig. 7

(i) Show that the straight line is consistent with a model of the form $y = A \times 10^{kx}$, where A and k are constants. **[2]**

(ii) Use the straight line to estimate the values of A and k. Giving the answers correct to 3 significant figures. **[4]**

(iii) Predict the year in which average weekly sales will fall below 10 000. **[3]**

(iv) How reliable do you expect the prediction in part **(iii)** to be? Justify your answer. **[1]**

8 Use the substitution $u = x + 1$ to find $\int (5x+2)\sqrt{x+1}\,\mathrm{d}x$. Give your answer in the form $kx(x+1)^{p} + c$ where k, p and c are constants. **[7]**

9 Fig. 9 shows the curve with parametric equations

$$x = \tan\theta, \quad y = 1 + \cos 2\theta, \quad \text{for } 0 \leqslant \theta \leqslant \pi \text{ with } \theta \neq \tfrac{1}{2}\pi.$$

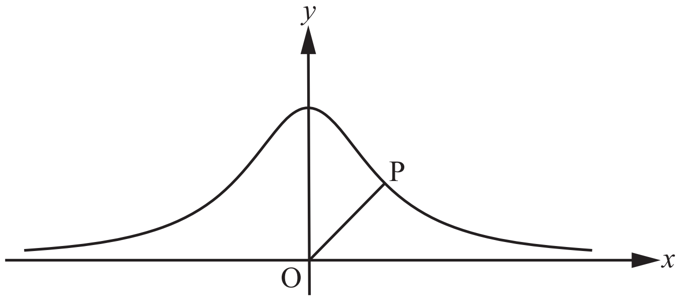


Fig. 9

(i) Explain why $\theta \neq \frac{1}{2}\pi$. **[1]**

(ii) Show that the maximum value of y for points on the curve is 2. **[1]**

(iii) Show that the cartesian equation of the curve is $y = \dfrac{2}{1+x^2}$. **[3]**

(iv) **In this question you must show detailed reasoning.**

The point P in the first quadrant lies on the curve. Find the coordinates of P given that OP is the normal to the curve at P. **[7]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

10 Starting from the formula Price elasticity of demand $= \dfrac{\text{\% increase in quantity demanded}}{\text{\% increase in price}}$, as given on line 19, show that, at point A in Fig. C1 the price elasticity of demand is $\dfrac{P}{mQ}$, where m is the gradient of the straight line. **[3]**

11 This question is about a straight line demand curve with equation $P = mQ + c$, where $m < 0$ and $c > 0$.

(i) Find an expression for the revenue in terms of Q, m and c. **[1]**

(ii) Hence show that, in this case, the maximum revenue occurs where the PED is -1, as stated on line 35. **[5]**

12 **(i)** The differential equation $\dfrac{dP}{dQ} = \dfrac{1}{k}\dfrac{P}{Q}$ is given on line 42. Find the general solution, giving Q as a function of P. **[3]**

(ii) Hence show that, when the PED is constant, a 5% increase in price results in the demand changing by a percentage which is independent of the original price, as stated in lines 43–44. **[3]**

END OF QUESTION PAPER