Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Practice Paper – Set 4

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $q = 1 - p$

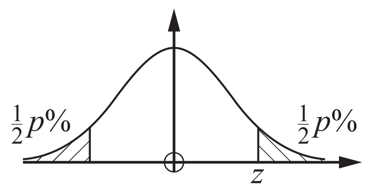
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (24 marks)

1 Find the set of values of k for which the equation $2x^2 + kx + 8 = 0$ has distinct real roots. **[3]**

2 A block of mass 5 kg is placed on a rough horizontal table. The coefficient of friction between the table and the block is 0.3. A horizontal force P N is applied to the block but the block does not move.

Find the greatest possible value of P. **[4]**

3 Fig. 3 shows a rod AB which is 0.9 m long and hangs vertically from a smooth hinge at A. The rod can rotate about A in a vertical plane. Forces of 100 N and 60 N act at right angles to AB in this plane. Their points of application are 0.3 m and 0.75 m respectively below A.

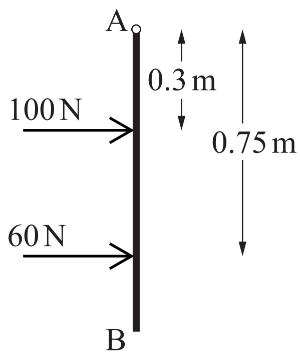


Fig. 3

(a) Find the combined moment of these forces about A. **[2]**

The rod is held in equilibrium by a force of F N which is also at right angles to the rod in the same vertical plane.

(b) Find the least possible value of F. **[2]**

(c) Explain how the rod can be in equilibrium when the resultant of these three forces is not zero. **[2]**

4 In this question the positive x and y directions are east and north respectively.

A model boat sails from the origin with initial velocity $3\,\text{m}\,\text{s}^{-1}$ due west and moves with acceleration $\begin{pmatrix}-0.1\\0.2\end{pmatrix}\text{m}\,\text{s}^{-2}$ for 25 s.

(a) Show that the velocity of the boat after 25 s is $\begin{pmatrix}-5.5\\5\end{pmatrix}\text{m}\,\text{s}^{-1}$. **[3]**

(b) Find the cartesian equation of the path of the boat. **[3]**

5 Fig. 5 shows triangle ABC where AB = 7 cm, BC = 8 cm and AC = 5 cm. The curve is an arc of a circle with centre C and radius r cm.

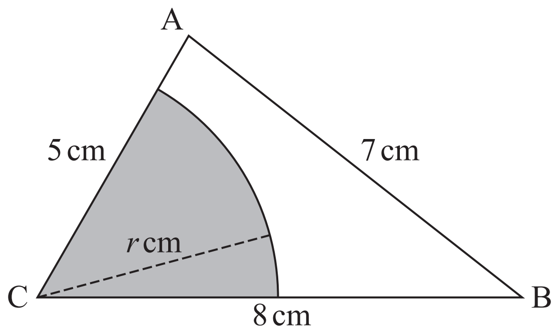


Fig. 5

Exactly half the area of the triangle is shaded. Find the value of r. **[5]**

Answer **all** the questions.

Section B (76 marks)

6 Bob wishes to find an estimate for $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$, where $\mathrm{f}(x) = \sqrt{x^{\frac{3}{2}} + 3}$, using the trapezium rule with 4 strips. Fig. 6 is a screenshot of a spreadsheet Bob created to help him. In rows 2 to 6, the values in columns B and C have been multiplied to give the value in column D. The value in D7 is the sum of the values from D2 to D6.

	A	B	C	D
1	x	f(x)	multiplier	multiple of f(x)
2	0	1.732051	1	1.7321
3	0.5	1.831271	2	3.6625
4	1	2	2	4
5	1.5	2.199345	2	4.3987
6	2	2.414214	1	2.4142
7				16.2075

Fig. 6

(a) Calculate the estimate for $\int_0^2 \sqrt{x^{\frac{3}{2}} + 3}\,\mathrm{d}x$ that Bob should obtain by using the trapezium rule with 4 strips. **[2]**

(b) You are given that the graph of $y = \mathrm{f}(x)$ is concave upwards for $0 \leqslant x \leqslant 2$. Explain what you can deduce about the estimate for the integral obtained in part **(a)**. **[1]**

7 A cyclist is travelling in a straight line. She has a velocity of $3\,\mathrm{m\,s^{-1}}$ when passing O. After 4 s she reaches A which is 24 m from O. After a further 6 s she reaches B which is 80 m beyond A.

Determine whether modelling the motion as having constant acceleration is consistent with these values. **[5]**

8 Fig. 8 shows the velocity-time graph of a car that is travelling in a straight line as it manoeuvres then drives away. Its velocity $v\,\mathrm{m\,s^{-1}}$ at time t s is given by $v = 0.1t^3 + 0.9t^2 - 1$.

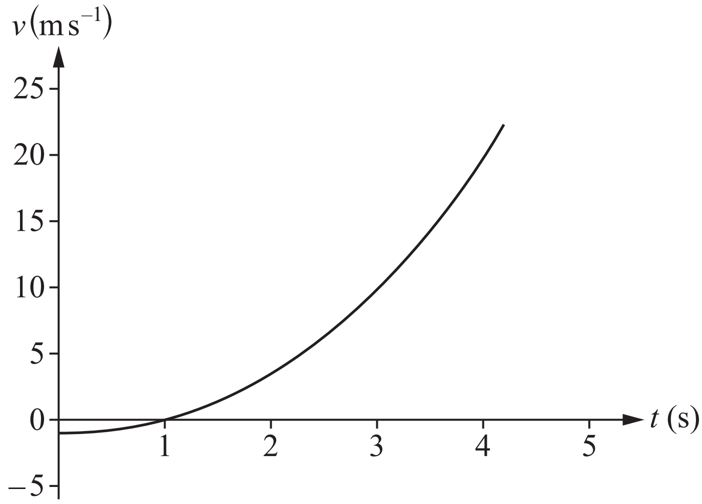


Fig. 8

(a) Describe two features of the motion of the car in the first 4 seconds. **[2]**

(b) **In this question you must show detailed reasoning.**

Calculate the total distance travelled in the first 4 seconds. **[5]**

(c) Find an expression for the acceleration of the car in terms of t. **[2]**

9 A model train consists of an engine of mass 0.3 kg and a truck of mass 0.2 kg. The train is pulled up a smooth plane inclined at 20° to the horizontal by a force of 2.5 N. This force is applied to the engine and acts parallel to a line of greatest slope of the plane.

(a) Show that the acceleration of the train is $1.65\,\mathrm{m\,s^{-2}}$ correct to 3 significant figures. **[3]**

(b) Find the tension in the coupling between the engine and the truck. **[2]**

(c) Find the distance travelled by the train as it accelerates from $1\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$. **[2]**

10 C is a circle with equation $x^2 + y^2 - 20x + 12y + 100 = 0$.

(a) Show that C touches the x-axis. **[3]**

The point A has coordinates (–1, –7) and B has coordinates (11, 5).

(b) Show that the equation of the perpendicular bisector of AB is $y = 4 - x$. **[3]**

(c) A circle with the same centre as C passes through A. Deduce from part **(b)**, or show otherwise, that this circle also passes through B. **[2]**

11 Solve the differential equation $5x\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - y - 6$ given that $y = 8$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. **[10]**

12 A goalkeeper kicks a football from ground level on a level playing field. The ball is in the air for 3.5 s.

(a) State a modelling assumption in the standard projectile model. **[1]**

(b) Calculate the vertical component of the initial velocity of the ball. **[2]**

(c) Calculate the maximum height of the ball. **[2]**

(d) The ball lands 42 m from its original position. Calculate

(i) the initial speed of the ball, **[3]**

(ii) the angle that the initial velocity makes with the ground. **[2]**

13 The function $\mathrm{f}(x)$ for the domain $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$ is defined by $\mathrm{f}(x) = \mathrm{e}^{kx}\sin 2x$, where k is a non-zero constant. Fig. 13 shows the graph of $y = \mathrm{f}(x)$.

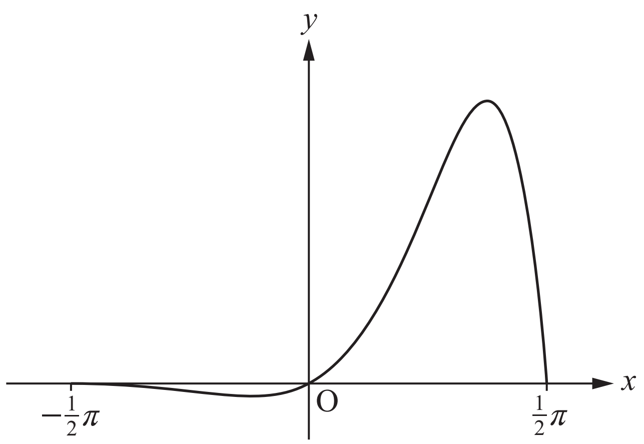


Fig. 13

Given that the maximum value of $\mathrm{f}(x)$ occurs when $x = \frac{3}{8}\pi$, find the exact minimum value of $\mathrm{f}(x)$. **[8]**

14 Chione uses the equation $V = 22000 - a\sqrt{t}$ to model the value of her caravan, where V is the value in pounds t years after purchase.

(a) The model must give the value of the caravan after 4 years as £12 000. Find the value of a. **[1]**

(b) Find the rate at which the value is changing when the caravan is 4 years old. **[3]**

(c) Explain the limitations of this model for large values of t. **[1]**

Chione creates a second model $V = be^{-0.15t} + c$ in which the initial value is £22 000 and the value after a long time tends to £1000.

(d) Find the values of the constants b and c. **[2]**

Chione wishes to find the times other than $t = 0$ at which the two models give the same value.

(e) Show that these times satisfy the equation $t = -\frac{20}{3}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$. **[3]**

Chione uses fixed point iteration with this equation and $t_0 = 5$. The table shows some of her values.

t_0	5
t_1	
t_2	
t_3	5.157 893 470
t_4	5.187 562 897
t_5	5.210 144 461

(f) Find the missing values t_1 and t_2. **[2]**

Fig. 14 shows the graphs of $y = t$ and $y = -\frac{20}{3}\ln\left(1 - \frac{5}{21}\sqrt{t}\right)$.

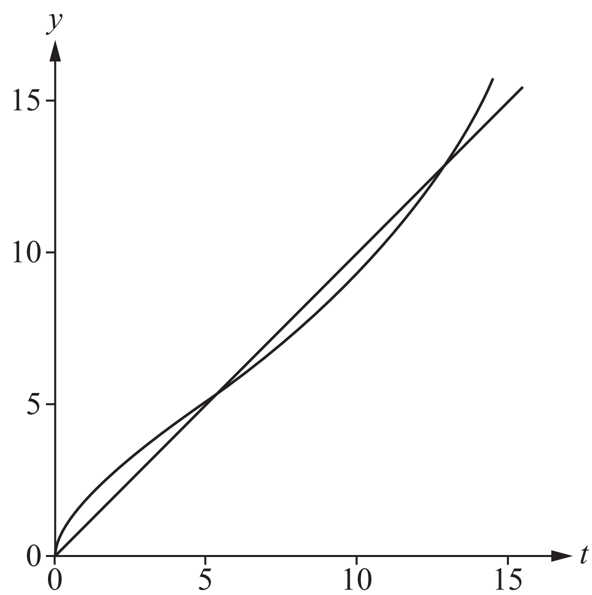


Fig. 14

(g) Sketch the staircase diagram for Chione's iteration on the diagram in the Printed Answer Booklet. **[1]**

(h) Chione notices that t_4 and t_5 both round to 5.2 so argues that the root of the equation is 5.2 correct to 1 decimal place. Comment on the validity of her argument. **[1]**

(i) Determine whether this fixed point iteration can be used to find the root near to 12. **[2]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE