OCR
Oxford Cambridge and RSA

Practice Paper – Set 4

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK   100

This document consists of 16 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a   Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g   Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i   If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question	Answer	Marks	AOs		Guidance
1	$\dfrac{A}{1-x}+\dfrac{B}{3+2x}$ seen	B1	1.1a		
	$14+6x=A(3+2x)+B(1-x)$	M1	1.1	allow 1 slip eg sign error	
	$A=4$ and $B=2$	A1	1.1		
		[3]			
2	Attempt to find Quotient	M1	1.1a	Allow **M1** for $x^2\pm 2x$ seen in quotient and $\pm 2x^2\pm 2x$ seen in long division.	
	Quotient is x^2-2x+2	A1	1.1		
	Remainder is -1	B1	1.1		
		[3]			
3	$a+4d=11$ and $a+10d=20$	B1	1.1		
	Equations solved simultaneously to obtain values for a and d.	M1	3.1a	FT their equations	
	$a=5$ and $d=1.5$	A1	1.1		
	$\dfrac{40}{2}\big(2\times 5+(40-1)\times 1.5\big)$	M1	1.1	FT their a and d	
	1370	A1	1.1	cao	
		[5]			

	Question		Answer	Marks	AOs		Guidance
4	(a)		$1+(-1)(3x)+(-1)(-2)\dfrac{(3x)^2}{2!}+(-1)(-2)(-3)\dfrac{(3x)^3}{3!}$	M1	1.1	allow sign errors	
				A1	1.1		
			$1-3x$	A1	1.1	allow recovery from omission of	
			$+9x^2-27x^3$	[3]		brackets	
4	(b)		$\left\vert x\right\vert <⅓$	B1	1.1	$-⅓<x<⅓$	
				[1]			
5	(a)		$\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\dfrac{0.2}{0.6}=\dfrac{1}{3}$	B1	1.1	Accept use of Venn diagram	
				[1]			
5	(b)		$\mathrm{P}(A\cup B)=0.9$	B1	2.1	Accept use of Venn diagram	
			$\mathrm{P}(A\cap B')=0.9-0.6=0.3$	B1	1.1		
			$\dfrac{0.3}{0.4}=0.75$	B1	1.1		
				[3]			
5	(c)		⅓ ≠ ¾ or ⅓ ≠ ½ or ¾ ≠ ½ so not independent	B1	2.2a		
				[1]			

Question			Answer	Marks	AOs		Guidance
6			**DR** $$\frac{dy}{dx} = \frac{\cos x \times \dfrac{1}{x+2} - \ln(x+2)(-\sin x)}{(\cos x)^2}$$ $x = 0$ $y = \ln 2$ $x = 0, \dfrac{dy}{dx} = \dfrac{1}{2}$ So equation of tangent is $y = \frac{1}{2}x + \ln 2$	**M1** **A1** **B1** **M1** **A1** **[5]**	**3.1a** **1.1** **1.1** **1.1** **1.1**	Use of quotient rule	
7	(a)		Opportunity sampling	**B1** **[1]**	**1.2**		
7	(b)		Each sample of this size does not have an equal probability of being selected since not all the customers will come at 11.00 am on a Monday	**E1** **[1]**	**2.4**	allow eg Some of Qasim's customers will be at work and never be able to visit café at 11am on Monday	
7	(c)		Positive skew	**B1** **[1]**	**1.2**		
7	(d)		Median is 10th value = 30 $43 - 22$ $= 21$	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	For their 15th value – their 5th value	NB other conventions for finding the quartiles are acceptable as long as the method is clear.

	Question		Answer	Marks	AOs		Guidance
7	(e)		Any two reasonable statements eg				
			Medians similar, so age of typical customer found to be the same	B1	2.4		
			IQR much smaller in larger sample, suggesting less variability	B1	2.4		
			range is larger, (but these values are both outliers and could be atypical)	[2]			
8	(a)		eg median $\approx$ mean eg extreme values and/or quartiles symmetrical about mean oe	B1 [1]	2.4		
8	(b)		$P(X < 46.4) = 0.24956..\approx 0.25$ $P(X < 66.1) = 0.75262…\approx 0.75$	M1	3.3	BC	or invNorm(0.25, 56.2,14.5) = 46.4199…$\approx$ 46.4
			So both values are consistent	A1 [2]	2.4	Must follow from correct calc. values	invNorm(0.75, 56.2, 14.5) = 65.98..$\approx$ 66.1
8	(c)		0.00987574855…	B1 [1]	1.1	BC to 3 or more sf	
8	(d)		Use of N(112.4, 29^2) soi	M1	3.1a	BC	
			0.780065376569…	A1 [2]	1.1	to 3 or more sf	

Question			Answer	Marks	AOs	Guidance
8	**(e)**		their $\dfrac{0.78006..}{0.00987...+0.78006...}$	**M1**	**2.1**	their $\dfrac{0.00987..}{0.00987...+0.78006...}$
			0.98750…so approximately a 99% chance the phone is Wilson's so their father's statement is justifiable	**A1** **[2]**	**2.2b**	0.0125…so approximately a 1% chance the phone is Chidera's so their father's statement is justifiable
9	**(a)**		$k(25 + 9 + 1 + 1 + 9 + 25) = 1$	**M1**	**3.3**	
			$k = \dfrac{1}{70}$	**A1** **[2]**	**1.1**	
9	**(b)**		Roxanne's model			

Roxanne's model

days	0	1	2	3	4	5
f_e	7	7	7	7	7	7

Correct for 1 and 4 days, but otherwise a poor fit

Alex's model

days	0	1	2	3	4	5
f_e	15	5.4	0.6	0.6	5.4	15

The expected frequencies are all within 2 of the observed frequencies, so Alex's model appears to be a good fit.

Marks / AOs for 9(b): **B1** 3.4, **B1** 3.5a, **M1** 3.4, **A1** 1.1, **B1** 2.4, **B1** 3.5a, **[6]**

Guidance for 9(b): **A0** if expected frequencies are rounded to nearest integer

Question			Answer	Marks	AOs		Guidance
10			$\bar{X} = 130$ seen	**B1**	**1.1**	BC	
			$H_0 : \mu = 142$ $H_1 : \mu < 142$	**B1**	**1.1**		
			μ is the population mean length of time of meetings	**B1**	**2.5**		
			use of $N\left(142, \dfrac{17.4^2}{9}\right)$	**M1**	**3.3**	soi	
			$P(\bar{X} < 130) = 0.01927...$	**A1**	**1.1**	BC	
			$0.01927... < 0.05$	**M1**	**3.4**	**FT** comparison of their probability	
			So [reject H_0], there is evidence at the 5% level to **suggest** that the mean length of meeting time has decreased.	**A1**	**2.2b**		
				[7]			
11	**(a)**		$[\cos^2 x =]$ ½(1 + cos2x)	**B1**	**1.1a**		
				[1]			

Question			Answer	Marks	AOs		Guidance
11	(b)		$R = 10$	B1	1.1		
			$\theta = \arctan(0.75)$ isw or 0.643501… to 3 or more sf	B1	1.1		
				[2]			
11	(c)		DR substitution of results from parts (a) and (b) in the equation	M1	2.1		
			$6\sin2x + 8\cos2x = 5$	A1	1.1		
			$\arccos(\frac{5}{R})$ found FT their R	M1	3.1a		
			0.845, 3.99,	A1	1.1	if **A0A0** allow **A1** for all four values correct to a different precision	
			2.94, 6.08 cao	A1	1.1		
				[5]			
12	(a)		$a = \frac{1}{2}(17.03 - 7.47) = 4.78$	B1	3.1b	$\sin\theta = 1$ for max $\sin\theta = -1$ for min B1	BC Sufficient reasoning needed to justify given answers
			$c + 4.78 = 17.03$ so $c = 12.25$	B1	3.3	$17.03 = a + c, \; 7.47 = -a + c$ B1	
				[2]			
	(b)		$\frac{2\pi}{365} \times 172 + b = \frac{\pi}{2}$ or $\frac{2\pi}{365} \times 355 + b = \frac{3\pi}{2}$	M1	3.3		
			$b = -1.39$	A1	1.1		
				[2]			

Question			Answer	Marks	AOs		Guidance
12	(c)		$t = 244$ used in their formula	M1	3.4	BC	
			$Y = 13.81$ which is fairly close to 13.75 (out by 3.6 minutes)	A1 [1]	3.5a		
12	(d)		$a = 8.51$ and $c = 12.63$	B1 [1]	3.5b		
12	(e)		New model gives 15.40 hrs, which is not a good fit	B1 [1]	3.5a	NB 15.39828…	
13	(a)		If a particular data value is not available, the code #N/A has been included in the Large Data Set – this is to prevent some software reading a blank as a zero.	E1 [1]	2.4		
13	(b)		The populations of African countries vary considerably. The "mean of means" does not take weighting into account.	E1 [1]	2.4		
13	(c)		The heights of the bars are proportional to the frequencies in the frequency diagram, the areas of the bars are proportional to frequency in a histogram (or the heights of the bars are proportional to frequency density)	E1 [1]	2.4		

Question			Answer	Marks	AOs		Guidance
13	(d)		Frequency densities of 1.3, 2.4, 3.8 and 0.733	B1	1.1		
			Horiziontal and vertical scales correct and correctly labelled	B1	1.1		
			Correct diagram with no gaps between bars	B1 [3]	1.1		
13	(e)		eg there may some association between the variables, but it is not clear what sort,	B1	2.4	or eg in the subsection of scatter where median life expectancy is 17, there appears to be no correlation	
			so using a value of one variable to predict a value of the other variable is unlikely to be reliable	B1 [2]	2.2b		
14	(a)		$\frac{1}{24} + \frac{23}{24} \times \frac{1}{23} + \frac{23}{24} \times \frac{22}{23} \times \frac{1}{22}$	M1	2.1	$\frac{1}{24}$ seen	
				M1	2.4	addition of three terms	
			$= \frac{1}{8}$	A1 [3]	1.1		
14	(b)		Use of B$(24, \frac{1}{8})$	M1	3.3		
			0.139(009338…) to 3, 4 or 5 sf	A1 [2]	1.1		

Question			Answer	Marks	AOs		Guidance
14	(c)		$\left(\dfrac{1}{8}\right)\left(\dfrac{7}{8}\right)^{23}$	M1	3.1a	soi NB 0.0057955….	
			$(0.0057955….)^2 = 0.000033588489…$ to 3, 4 or 5 sf	A1 **[2]**	1.1		
14	(d)		$[6000 \div 120] = 50$	B1	3.1b		
			$[1 - 0.13900..]^{49} \times 0.0057955...$	M1	3.1a	FT their part **(b)**	
			$0.00000376653…$	A1	1.1		
			so Mark is wrong – it's more like 4 in a million	A1 **[4]**	3.2a		

Question			Answer	Marks	AOs		Guidance
15	**(a)**		$\ln a = b$	**B1**	**1.1**		
			Area B: $\displaystyle\int_0^b \mathrm{e}^y \mathrm{d}y = \mathrm{e}^b - \mathrm{e}^0$	**M1**	**3.1a**	or $a \times b - $ their $a\ln a - a + 1$	
			$= a - 1$	**A1**	**1.1**		
			Area A: $\displaystyle\int_1^a \ln x \; \mathrm{d}x = \big[x\ln x - x\big]_1^a$	**M1**	**2.1**		
			$= a\ln a - a + 1$	**A1**	**1.1**		
			Their $a - 1 = $ their $a\ln a - a + 1$	**M1**	**3.1a**		
			$a\ln a - 2a + 2 = 0$	**A1**	**1.1**		
				[7]			
15	**(b)**		Evaluation of their f(4.921554 − δ) and their f(4.921554 + δ)	**M1**	**2.1**	$\delta \leq 0.000\,000\,5$	
			eg − 0.000000079882 and 0.000000513742 seen to 2 or more sf plus correct conclusion: sign change, so Heidi **is** correct	**A1** **[2]**	**2.2a**		

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