OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Practice Paper – Set 4

Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1-p$

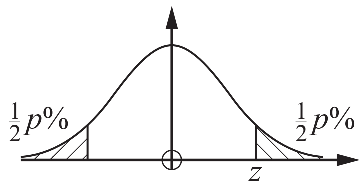
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 Write down the roots of the equation $(x+2)(x^2-25)=0$. **[2]**

2 **In this question you must show detailed reasoning.**

Find the set of values of x for which the line $y=5x-6$ lies below the curve $y=x^2$. **[4]**

3 The population of a small country is modelled using the formula $P=5\times1.02^n$ where P is the population in millions and n is the number of years after the start of the year 2000.

(a) According to the model, what is the population of the country at the start of the year 2000? **[1]**

(b) Explain fully what the model implies about how the population changes over time. **[2]**

(c) **In this question you must show detailed reasoning.**

According to the model, in what year will the population reach 10 million? **[3]**

(d) Show that, according to the model, the graph of $\log_{10}P$ against n will be a straight line. **[2]**

4 The function $\mathrm{f}(x)=\frac{1}{x}$ is defined for the domain $\{x : x\in\mathbb{R}, x\neq 0\}$. The function $\mathrm{g}(x)$ is defined for all real x. The composite function $\mathrm{fg}(x)$ is defined by $\mathrm{fg}(x)=\frac{1}{3x-6}$.

(a) Write down an expression for $\mathrm{g}(x)$. **[1]**

(b) What is the domain of $\mathrm{fg}(x)$? **[1]**

(c) Describe a sequence of two transformations that would map the graph of $y=\mathrm{f}(x)$ onto the graph of $y=\mathrm{fg}(x)$. **[4]**

5 Using a suitable substitution, or otherwise, show that $\int_3^8 \frac{2x}{2x-1}\,\mathrm{d}x = 5+\frac{1}{2}\ln 3$. **[6]**

6 **In this question you must show detailed reasoning.**

(a) Find the exact coordinates of the stationary point of the curve $y=x\ln x$. **[5]**

(b) Show that the stationary point is a minimum turning point. **[2]**

7 A curve has parametric equations

$$x = 2\cot\theta, \quad y = 2\sin^2\theta,$$

for values of θ for which both x and y are defined.

(a) For what values of θ, in the interval $0 \leqslant \theta \leqslant 2\pi$, is x undefined? **[2]**

(b) Show that the cartesian equation of the curve is $y = \dfrac{A}{x^2 + B}$ where A and B are positive integers to be determined. **[3]**

(c) Ali says that the curve lies completely above the x-axis. Determine whether Ali is correct. **[2]**

8 The function $\mathrm{h}(x) = \sin\left(\dfrac{1}{x}\right)$ is defined for the domain $x > \dfrac{2}{\pi}$.

(a) Differentiate $\mathrm{h}(x)$ with respect to x. **[2]**

(b) Find the values between which $\dfrac{1}{x}$ lies for $x > \dfrac{2}{\pi}$. **[2]**

(c) Show that $\mathrm{h}(x)$ is a decreasing function. **[3]**

9 **In this question you must show detailed reasoning.**

Fig. 9 shows the line $y = x$ and the curve $y = \frac{4}{3}x^3 + a$ where a is a constant. The line is a tangent to the curve, touching the curve in the first quadrant.

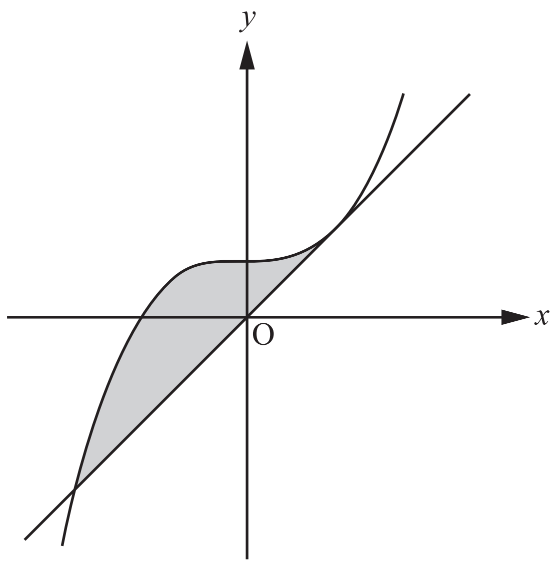


Fig. 9

(a) **(i)** Show that, at the point of contact between the line and the curve, $4x^2 = 1$. **[2]**

(ii) Hence find the value of a. **[4]**

(b) Find the area of the shaded region between the curve and the line. **[7]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

10 Assume that the length of each side of the equilateral triangle shown in Fig. C1.1 is one unit.

(a) Find the perimeter of the second iteration, shown in Fig. C1.3. **[1]**

(b) Find an expression for the perimeter of the *n*th iteration. **[3]**

(c) Show that, as stated in line 11, the perimeter of the Koch snowflake is not finite. **[1]**

11 Assume now that the area of the equilateral triangle shown in Fig. C1.1 is one unit of area.

(a) Find the area of the first iteration, shown in Fig. C1.2. **[2]**

(b) Find the increase in area between the iterations shown in Figs C1.2 and C1.3. **[1]**

(c) Find the area of the Koch snowflake. **[4]**

12 Show that the fractal dimension of the dragon curve is 2, as stated in line 58. **[3]**

END OF QUESTION PAPER

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